Robust CFAR Detection in Clutter with Unknown Covariance Matrix

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Abstract — We conduct a simulation analysis for assessing the constant false alarm rate (CFAR) behavior of three coherent radar detectors in the presence of correlated Gaussian clutter with unknown covariance. We establish the conditions a detector must fulfill in order to ensure the CFAR property. We discuss several detectors with quasi CFAR property. The analysis of CFAR property and the performance analysis, which has been carried out in the presence of correlated Gaussian clutter with unknown covariance, shows that the proposed detectors exhibit a quite acceptable loss with respect to optimum Neyman-Pearson detector.

1. INTRODUCTION

A very challenging problem arising in radar signal processing is to achieve a reliable target detection in the presence of severe clutter backgrounds. When the clutter statistics are unknown or highly variable the false alarm rate of the known nonadaptive radar receivers cannot be controlled and target decisions become unreliable. This is due to the lack of robustness of the these receivers with respect to possible mismatched between the design and operating conditions as well as variations in the clutter statistics.

There are four (nonadaptive) matched subspace detectors (MSD) that form the basis for the subspace detector with CFAR property in the presence of correlated Gaussian clutter with unknown covariance. They arise from two types of generalizations of the matched filter detector. First, the inner product of the matched filter may be generalized to a projection of the measurement onto a higher dimensional signal subspace, thus producing a subspace detector [1,2,4]. Second, the detector may be normalized by an estimate of the noise power to make it have a CFAR with respect to the noise power. The four detectors are thus

1) the coherent MSD (i.e., matched filter), which is a normally distributed statistic that detects coherent signals by resolving the inner product of the measurement and signal;
2) the MSD, which is a $\chi^2$ statistic that detects subspace signals (including noncoherent signals) by computing the energy of the measurement in the signal subspace;
3) the coherent CFAR MSD, which is a $t$ statistic that detects coherent signals in noise of unknown variance by resolving the cosine of the angle the measurement makes with the signal;
4) the CFAR MSD, which is an $F$ statistic that detects subspace signals (including noncoherent signals) in noise of unknown variance by measuring the fraction of energy the measurement has in the signal subspace.

Each of the resulting four detectors is a generalized likelihood ratio tests (GLRT) for a concrete problem, and each is UMP-invariant, uniformly most powerful over the entire class of detectors invariant to an appropriate transformation group.

All of these detectors are compelling. They have clearly stated optimalities and invariances, and they have evocative geometrical interpretations. The MSDs use extra knowledge of the noise variance for some performance gain against the CFAR MSDs, which do not assume this knowledge. On the other hand, the CFAR MSDs compensate for this lack of knowledge by providing an extra invariance to data scaling, a property that the MSDs do not have.

The MSDs and CFAR MSDs all assume prior knowledge of noise covariance matrices. However, this information is often not known, meaning that, in practice, it must be estimated and then used correctly in an adaptive detector. These detectors, however, still need a sufficient amount of data to “learn” the clutter environment web enough for reliable detection of the signal. In a severely no stationary and/or no homogeneous environment such as that surrounding an airborne surveillance radar system, the detection performance of an adaptive system can fall below what can be expected in a
stationary and homogeneous environment, simply due to the lack a sufficient amount of data. In this case no-adaptive radar detection has been utilized. As shown in [5], under this conditions it is possible to obtain the detector (nonadaptive) with a constant probability of a false alarm rate (CFAR). In [5] there were obtained conditions at which the detector guarantees constant probability of a false alarm rate under unknown interference covariance matrix with Toeplitz’s or Hermitian structure.

Our aim in this paper is to adapt these detectors of [1,2,3,4] to unknown clutter covariance in order to produce a robust detector with CFAR property that may be applied to signal detection for radar, sonar, and data communication. For CFAR analysis and assessment of the performance of the detectors we shall use the method of Monte-Carlo simulation

2. DETECTION STRATEGIES

Let us consider the problem of detection of a pulse train composed of $N$ coherent pulses in additive clutter. It can be formulated in terms of the following binary hypotheses test:

$$
H_0: \, r = \sigma n, \\
H_1: \, r = u + \sigma n,
$$

(1)

where $r$, $u$, and $n$ denote the $N$-dimensional complex vectors of the samples from the baseband equivalents of the received signal, the wanted target echo, and the clutter with unknown covariance matrix. The useful signal $u$ is modeled as a coherent pulse train, namely $u = \alpha p$ where $p$ is the transmitted waveform and $\alpha$ an unknown complex parameter, accounting for both the channel propagation effects and the target radar cross section. The clutter vector $n$ is scaled by the level constant $\sigma$.

In the sequel we present several solutions, proposed during the last fifty years, for solving the problem (1).

1) Coherent MSD (matched filter). Assuming that $n$ is zero-mean, complex, Gaussian vector with known covariance matrix $M$, in [1] has been derived the following one-sided threshold test for testing $H_0$ versus $H_1$:

$$
\frac{p^H p r}{\sigma (p^H p)^{1/2}} > m_0 \\
\frac{m_1}{m_0}
$$

(2)

where $P_p$ is the projection matrix onto vector $p$; $m_0$ is the detection threshold; $(.)^H$ denotes conjugate transpose.

2) MSD [1], which is a $\chi^2$ statistic that detects subspace signals $\langle H \rangle$. In this case, the signal $p$ is known to lie in the linear subspace $\langle H \rangle$ spanned by the columns of matrix $H$. The signal is a linear combination of modes. It may be represented as $p = H\theta$ (3). Here $H$ is a known $N \times L$ matrix with columns $h_n$ and $\theta$ is a $L \times 1$ vector with elements $\theta_n$. MSD test may be represent as

$$
\chi^2 = r^H P_H r
$$

(4)

3) The coherent CFAR MSD [1], which is a $t$ statistic that detects coherent signals $p$ in noise of unknown variance $\sigma^2$. This test may be represent as

$$
t = \frac{p^H p r}{\sigma^2 p^H p} \frac{r^H (I - P_p) r}{(N-1)\sigma^2}
$$

(5)

Following the test (5) vector $r$ is projected onto orthogonal signal and clutter subspaces. In the signal subspace, the projection is correlated with $p^H(p^H p)^{1/2}$ to produce a matched filter output. In the clutter subspace the projection $(I-P_p)r$ is squared, divided by $(N-1)$, and rooted to produce an estimate of $\sigma$.

4) The CFAR MSD, which is an $F$ statistic that detects subspace signals (including noncoherent signals) in clutter of unknown variance is follows:

$$
F = \frac{r^H P_H r / \sigma^2 L}{r^H (I - P_H) r / \sigma^2 (N - L)},
$$

(6)

where $P_H$ is matrix projector onto subspace $\langle H \rangle$, $I$ is the unit matrix. This statistic is ratio of quadratic forms in projection matrices. It measures the ratio of the energy of $r$, per dimension, that lies in the subspace $\langle H \rangle$ to the energy of $r$, per dimension, that lies in the orthogonal subspace $I - P_H$.

We want to adapt these detectors of (2,4,5,6) to unknown clutter covariance in order to produce a robust approximately CFAR detectors. We will obtain a conditions for matrix projections $P_p$ and orthogonal matrix projection $P_{\perp}$ of CFAR test for unknown Hermitian covariance matrix of clutter $M$ and $\sigma$. We use the obtained in [5] the conditions which provide these properties:

$$
\text{tr}(P_p M) / \text{tr}(P_{\perp} M) = \text{const },
$$

(7)

here $\text{tr}$ designates a track of a matrix. According to this equation the sum of all elements in diagonal or in everyone sub-diagonal $k$ of projection matrix are identical to all projection matrices $P_p$ of subspace:
\[ \sum_{i=k}^{N-1} P_{h}^{k+i} \neq f(h) \]  

(8)

The example of such projection matrix for wanted signal \( p \) is resulted in [5]:

\[ K^{(a)} = a_{i}(a_{i}^{T}a_{i})^{-1}a_{i}^{T} \]  

(9)

here elements of vector \( a \) are:

\[ a_{i}(\tau) = \sum_{t=0}^{N-1} \exp[j(t \oplus i)/T_{p}] \exp[-j(t \oplus \tau)/T_{p}] \]  

(10)

the operation \( t \oplus i \) is called generalized (logical) shift in an argument \( t \) on a value \( i \). In this case the projection matrix for clutter subspace \( \langle H \rangle \) we choose as:

\[ K^{H} = \sum_{i=1}^{L} K^{(b_{i})} \]  

(11)

here

\[ b_{i}(\tau) = \sum_{t=0}^{N-1} \exp[j(t \oplus i)/T_{p}] \exp[-j(t \oplus \tau)/T_{p}] \]  

(12)

\( T_{p} \neq T_{H} \) and \( K^{(a)} \perp K^{H} \). For the coherent CFAR MSD case modified test may be represent as

\[ a_{i}^{H}K^{(a)}r / \sqrt{\sigma_{a_{i}}^{2}a_{i}^{H}a_{i}} > H_{0}^{1} \]

\[ \sqrt{r^{H}K^{(H)}r}/(N-1)\sigma_{r}^{2} < m_{i} \]  

(13)

\[ p^{H}(p^{H}p)^{1/2} \]

\[ r \]

\[ K^{(a)} \]

\[ X \]

\[ \div \]

\[ \sum \]

\[ K^{(H)} \]

\[ (\gamma)^{2}/(N-1) \]

The detector (13) is illustrated in Fig.1.

Further we shall analyze on a private examples the sensitivity of actual false alarm \( P_{f} \) for coherent MSD, coherent CFAR MSD and modified CFAR MSD (CFAR MMSD) to mismatch between design and actual value of one-lag clutter correlation coefficient \( \rho \). As to the clutter correlation properties, we assume that the returns from a given range-cell are exponentially correlated:

\[ M = ||m_{i}|| = 2\sigma^{2} || \exp[i-j] \]  

(14)

For a system with a single channel antenna, the Toeplitz covariance assumption implies the use of the uniformly spaced pulse train. For the angle domain processing with a multichannel antenna system, this assumption corresponds to the use of a linear array of uniformly spaced identical antennas. We chose as transmitted signal a rectangular (coherent) pulse train with zero Doppler-shift. Then the signal vector is given by:

\[ p = [1 \exp{i2\pi f_{s}} \ldots \exp{i(M-1)2\pi f_{s}}]^{T} \]  

(15)

where \( |f_{s}| < 0.5 \) is known constant. Since closed-form expressions of the probability density function of the test statistics of the above tests are not available (clutter covariance matrix is unknown), we extensively resort to computer simulation. Precisely, \( P_{f} \) and \( P_{d} \) are estimated through Monte Carlo counting techniques, based on 100/P_{f} and 100/P_{d} independent trials. To define the projection matrix \( K^{(a)} \) for signal subspace (N=16), we have chosen \( T_{p} = 1 \). In this case the projection matrix has rank-1. As to define the projection matrix \( K^{(H)} \) for clutter subspace, we have chosen \( T_{H1} = 1.143; T_{H2} = 2; T_{H3} = 1.33 \). In this case \( K^{(a)} \perp K^{(H)} \).

Fig.1 Block diagram of CFAR MMSD filter for detecting known form signal in clutter of unknown level and covariance matrix

Fig.2 FFT of subspaces axes \( a \) for \( T_{p} = 1, T_{H1} = 1.143, T_{H2} = 2, T_{H3} = 1.33 \). All axes of subspaces have the identical module of a FFT spectrum and they differ among themselves a phase FFT.
spectrum. On a fig.2 modules of FFT spectra are shown, these modules for different subspaces are not crossed, hence the chosen subspaces are orthogonal.

TABLE
Sensitivity of actual PF for coherent MSD, coherent CFAR MSD, coherent CFAR MMSD (proposed detector) to mismatch between design and actual value of one-lag clutter correlation coefficient \( \rho \)

<table>
<thead>
<tr>
<th>Actual Value ( \rho )</th>
<th>Coherent MSD</th>
<th>Coherent CFAR MSD</th>
<th>Coherent CFAR MMSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>21.7</td>
<td>5.0</td>
<td>0.5</td>
</tr>
<tr>
<td>0.4</td>
<td>16.7</td>
<td>3.4</td>
<td>0.55</td>
</tr>
<tr>
<td>0.6</td>
<td>10.9</td>
<td>2.3</td>
<td>0.6</td>
</tr>
<tr>
<td>0.8</td>
<td>4.2</td>
<td>1.4</td>
<td>0.75</td>
</tr>
<tr>
<td>0.9</td>
<td>1.1</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>0.95</td>
<td>0.08</td>
<td>0.51</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Note: \( N=16 \), design value of \( \rho = 0.9 \).

As to the sensitivity of the tests to the actual clutter correlation coefficient \( \rho \), we refer to Table, wherein the threshold is set with reference to \( \rho = 0.9 \).

Fig.3 ROC for coherent MSD, coherent CFAR MSD and coherent CFAR MMSD (proposed detector).

The receiver operating characteristics (ROC’s) for specified above detectors are given in Fig.3. In Fig.3, the probability of false alarm is fixed at \( P_F = 10^{-2} \), and for coherent CFAR MMSD (proposed detector) the dimension of the clutter subspace \( \langle H_n \rangle \) is varied: curve 1 corresponds to one-rank clutter subspace, curve 2 for 2-

rank clutter subspace, curve 3 for 3-rank clutter subspace. The curve 4 corresponds to the coherent CFAR MSD and the curve 5 to the coherent MSD.

4. CONCLUSION
In the proposed detector two orthogonal subspaces with properties CFAR have been used. We have conducted a simulation analysis in order to study the CFAR behavior of proposed detector. The simulation data in the table demonstrate, that the proposed detector has the lowest sensitivity of actual PF to the mismatch between design and actual value of one-lag clutter correlation coefficient. Obviously, for a prior unknown clutter covariance matrix it is impossible to create the optimum nonadaptive detector, but as is shown in Fig.3, in the proposed detector the value of the detection loss does not exceed 3 dB. Note if to increase a rank of a clutter projection matrix the detection losses will be on the decrease.

REFERENCES