Adaptive Beamforming with Augmentable Arrays in Non-Stationary Environments

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***Abstract*—This paper introduces a near-field reduced rank adaptive beamforming method for nulling more interferers than sensors using fully augmentable arrays in wide sense stationary interference. Augmentable arrays, such as minimally redundant or co-prime, can exploit additional degrees of freedom beyond the number of sensors in spatially stationary fields in order to increase array gain. However, near-field targets result in spatial non-stationarity due to range dependent wavefront curvature. Additionally, source or array motion limits the number of available snapshots. Adaptive near-field beamforming with far-field interference suppression is considered by nulling in the augmented, or co-array, dimensions but steering in the near-field with sensors in their physical locations. A reduced rank processing scheme is used that requires fewer snapshots than full rank methods. The proposed technique is shown to increase array gain when the number of interferers exceeds the number of sensors in both high and low snapshot support regions.**

I. Introduction

Near-field dynamic targets introduce a non-stationary component both spatially, through non-planewave propagation, and temporally, due to target motion. This paper addresses non-planewave propagation with near-field beamforming and considers target dynamics in the limited snapshot case. Near-field sources in the presence of far-field interference occurs in microphone array processing for speech, guidance systems for homing, and passive sonar. Non-adaptive solutions have focused on weight designs that jointly consider near-field and far-field beampatterns [1]. Adaptive techniques have been developed based on higher-order statistics, which require additional snapshots over second-order methods [2]. This paper addresses interference suppression by exploiting the wide sense stationarity of the interference while using near-field steering vectors. Additionally due to the limited number of snapshots, a variation of dominate mode rejection is proposed, which assumes an interference dominated environment [3]. This allows the suppression of more interferers than sensors while mitigating the impact of snapshot deficiency.

Traditionally, rank-deficient adaptive beamforming improves interference suppression when the number of snapshots is less than the number of sensors but greater than the number of discrete interferers. Typically, twice as many snapshots as interferers are required when the sources are quasi-stationary during the time window [4]. Unlike previous rank-deficient approaches, this paper exploits a non-uniform array where a filled uniform linear array at half-wavelength spacing has been thinned; or equivalently, a given number of sensors are placed non-uniformly at multiples of a half-wavelength. The total degrees of freedom can be increased using a non-uniform array by forming a covariance matrix corresponding to the difference co-array when the observed field is spatially wide sense stationary [5]. This exploits the Toeplitz structure of an augmented received data covariance matrix. Augmentable arrays have been used to maximize array aperture given a limited number of sensors. Minimally redundant linear arrays (MRLA) are fully augmentable such that every multiple of the smallest inter-element spacing, assumed to be at most a half-wavelength, is observed up to the length of the full aperture [6]. The problem considered is given in Figure 1 where an under-sampled array is used to null many far-field interferers in the presence of a near-field source. Non-uniform fully augmentable arrays can be processed as uniformly spaced arrays. The number of times each inter-element spacing is observed forms the co-array, which has no zeros or holes in a fully augmentable array. Other types of arrays, such as co-prime arrays, have recently been considered since designs for MRLAs with a large number of sensors are unknown and require computationally intensive searches [7]. Augmentable arrays can detect and resolve more sources than the number of sensors (see [8] and references therein). Conventional augmentation techniques for MRLAs have been well studied and require approximately twice the number of snapshots to achieve spectrum estimation performance comparable to non-augmented methods [5]. A rank-deficient beamformer is proposed to reduce the number of snapshots required in order to mitigate the impact of augmentation.

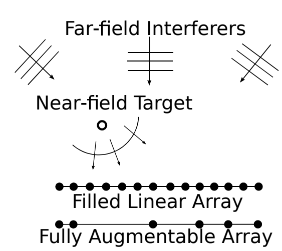


Fig. 1. Example scenario of near-field target in far-field interference

The rest of the paper is organized as follows. The signal model of the received data is described in Section II, and the proposed adaptive beamformer is derived in Section III. The results of array gain analysis are given in Section IV. Monte Carlo simulation with more sources than sensors is given in Section V to demonstrate traditional dominate mode rejection processing compared to the augmented approach with a near-field target in an interference dominate environment.

II. Received Signal Model

Consider a linear passive array of M sensors with distances, d_m, and first element located at $d_1 = 0$. For passive array processing, narrowband data received by the array, $\mathbf{x}$, is a summation of the interference, signal and noise, $\mathbf{x} = \mathbf{A}_I\mathbf{s}_I + \nu\mathbf{a}(\theta_s, r_s) + \mathbf{n}$ where the interference steering vectors, $[\mathbf{a}(\theta_q)]_m = \exp(-jkd_m\cos(\theta_q))$, define $\mathbf{A}_I = [\mathbf{a}(\theta_1) \ \cdots \ \mathbf{a}(\theta_Q)]$ for wavenumber k. The near-field target source steering vector is a function of bearing and range respectively, $\mathbf{a}(\theta_s, r_s)$, from the first element as defined in (1), and the target source is circularly symmetric complex normally distributed $\nu \sim \mathcal{CN}(0, \sigma_s^2)$. Signal amplitude as a function of range for near-field propagation is not considered, resulting in the phase only steering vector given by

$$[\mathbf{a}(\theta, r)]_m = \exp\left(-jk\left(d_m^2 + r^2 - 2d_m r\cos(\theta)\right)^{\frac{1}{2}}\right). \quad (1)$$

The interference is distributed according to $\mathbf{s}_I \sim \mathcal{CN}(0, \boldsymbol{\Sigma})$, where interferers are assumed uncorrelated such that $\boldsymbol{\Sigma}$ is a diagonal matrix. The noise is assumed to be wide sense stationary distributed according to $\mathbf{n} \sim \mathcal{CN}(0, \mathbf{Q})$, where $\mathbf{Q}$ is unknown. The noise covariance is not assumed to be diagonal, but the interference is assumed to dominate the noise. The received data is distributed according to $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$ where

$$\mathbf{R} = \mathbf{A}_I\boldsymbol{\Sigma}\mathbf{A}_I^{\mathrm{H}} + \sigma_s^2\mathbf{a}(\theta_s, r_s)\mathbf{a}^{\mathrm{H}}(\theta_s, r_s) + \mathbf{Q}. \quad (2)$$

III. Augmented Adaptive Beamforming

The objective of this paper is to enhance signal to interference and noise gain for a near-field source in the presence of far-field interferers. Note that near-field sources may change bearing or range quickly, especially at close ranges, and severely limit the number of snapshots available. The received data covariance matrix estimate is given by $\hat{\mathbf{R}} = \frac{1}{L}\sum_{l=1}^{L}\mathbf{x}(l)\mathbf{x}(l)$, using L i.i.d. snapshots. Assuming wide sense stationarity, the autocorrelation is a function of the relative distance between sensors, which results in a Toeplitz covariance matrix where each diagonal is a constant term [5]. Additionally, all inter-element spacings are assumed to be a half-wavelength, such that $d = \lambda/2$. An augmented covariance matrix is formed by averaging over each observation of the relative distance, $d\tau$, given by

$$\mathbf{T}_\tau = \frac{\sum_{a,b=1}^{M}[\hat{\mathbf{R}}]_{ab}\delta(d\tau, d_a - d_b)}{\sum_{a,b=1}^{M}\delta(d\tau, d_a - d_b)}, \quad (3)$$

where $\mathbf{T}_\tau$ refers to the τth diagonal of $\mathbf{T}$ $\delta(a, b)$ is the Kronecker delta function, and d_a, d_b are used to index different sensor locations, d_m. Let the non-negative contiguous τ's be indexed from 0 to M_α, which is assumed to be greater than the number of sensors, M. The increased size of the covariance matrix results in increased variance, on the order of twice the variance for conventional non-adaptive beamforming [5]. Note that augmented covariances formed using the direct approach of (3) may have negative eigenvalues, but positive definite matrix completion can be used to ensures all eigenvalues are positive [9].

In order to mitigate the increased variance from covariance matrix augmentation and the limited number of snapshots, a form of dominate mode rejection is proposed. The method and analysis given by Cox [4] is extended to the augmented domain. Eigendecomposition of the augmented covariance matrix is given by

$$\mathbf{T} = \sum_{n=1}^{K}\lambda_n\mathbf{u}_n\mathbf{u}_n^{\mathrm{H}} + \sum_{n=K+1}^{M_\alpha}\lambda_n\mathbf{u}_n\mathbf{u}_n^{\mathrm{H}} \quad (4)$$

where the subspace of the K strongest interferers is approximated by the first term of (4). The estimate of the interference subspace with additive white Gaussian noise is given by $\tilde{\mathbf{T}} = \sum_{m=1}^{K}\lambda_m\mathbf{u}_m\mathbf{u}_m^{\mathrm{H}} + \varepsilon\mathbf{I}$. Additive white noise, ε, provides robustness to steering vector errors as a regularization term. The inverse of the interference subspace with noise is given by $\mathbf{P} = \tilde{\mathbf{T}}^{-1}$ such that

$$\mathbf{P} = \varepsilon^{-1}\left(\mathbf{I} - \sum_{n=1}^{K}\frac{\lambda_n}{\lambda_n + \varepsilon}\mathbf{u}_n\mathbf{u}_n^{\mathrm{H}}\right) \quad (5)$$

where the modified eigenvalues are $\tilde{\lambda}_n = \frac{\lambda_n}{\lambda_n+\varepsilon}$. The inverse of the dominate subspace is used to null the K strongest sources. However, augmentation is inappropriate for near-field steering; therefore, a modified steering vector is proposed as $\mathbf{a_s} = \mathbf{J}\mathbf{a}_\alpha(\theta, r)$, where $\mathbf{a}_\alpha(\theta, r)$ is an augmented steering vector of the form from (1) except $d_m = m{*}d \ \forall \ m \in [1, M_\alpha]$, and $\mathbf{J}$ is a square selection matrix with ones on the diagonal locations where sensors exist and zeros elsewhere. The analytic form of the selection matrix is $\mathbf{J} = \sum_{m=1}^{M}\mathbf{e}_{\tau_m}\mathbf{e}_{\tau_m}^{\mathrm{T}}$ where $\mathbf{e}_\tau$ is a $M_\alpha \times 1$ unit vector with a one at the τth element and zeros elsewhere and $\tau_m = d_m/(\lambda/2) + 1$. The minimum variance distortionless response (MVDR) beamforming weights using the estimated interference covariance matrix, (5), and modified steering vector are given by

$$\mathbf{w} = \frac{\mathbf{P}\mathbf{a}_s}{\mathbf{a}_s^{\mathrm{H}}\mathbf{P}\mathbf{a}_s} \quad (6)$$

where the denominator provides a unity gain constraint. The weights are applied to the augmented covariance matrix with beamformer power output given by $y = \mathbf{w}^{\mathrm{H}}\mathbf{T}\mathbf{w}$. Note that the augmentation operation, (3), causes signal loss for a near-field target. However, the unobserved steering vector elements are ignored via the selection matrix, $\mathbf{J}$, to reduce mismatch. The signal loss will be quantified through array gain analysis.

IV. Array Gain Performance

The metric of interest for quiet source detection in an interference dominated environment is the signal to interference plus noise ratio (SINR). Array gain is the ratio of SINR at the output of a beamforming method compared to the SINR of data received by a single sensor, given by $A = \mathrm{SINR_{out}}/\mathrm{SINR_{in}}$, assumed to be constant for all sensors. First, array gain without diagonal loading is considered. Note this is not equivalent to setting $\varepsilon = 0$ in (5) but using $\mathbf{P} = \mathbf{I} - \sum_{n=1}^{K}\mathbf{u}_n\mathbf{u}_n^{\mathrm{H}}$. Second, array gain with diagonal loading, as in (5), is considered. For the purpose of this work, the true target steering vector and size of the interference subspace are assumed to be known. Due to limited space, the following results are given without derivations assuming $\mathbf{Q} = \sigma_w^2\mathbf{I}$.

A. No Diagonal Loading

The augmented signal covariance is given by $\mathbf{T}_s$ using (2)-(3) assuming $\boldsymbol{\Sigma}, \mathbf{Q} = \mathbf{0}$ of sizes $Q \times Q$ and $M \times M$

respectively. Assuming no steering mismatch, i.e. $\mathbf{a}_s$ and $\mathbf{a}(\theta_s, r_s)$ are equal for all non-zero terms, the signal output is

$$\mathbf{w}^{\mathrm{H}}\mathbf{T}_s\mathbf{w} = \frac{M^2\beta(1-\rho)\sigma_s^2}{|\mathbf{a}_s^{\mathrm{H}}\mathbf{P}\mathbf{a}_s|^2} \tag{7}$$

where the loss due to augmentation is given by

$$\beta = \frac{\mathbf{a}_s^{\mathrm{H}}\mathbf{T}_s\mathbf{a}_s}{\mathbf{a}_s^{\mathrm{H}}\mathbf{R}_s\mathbf{a}_s}, \tag{8}$$

and the nulls in the steering direction are function of the inner product of the modified steering vector with a combination of signal and interference subspaces as expressed by

$$\rho = M^{-2}(\sigma_s^2\beta)^{-1}\mathbf{a}_s^{\mathrm{H}}\left[2\mathcal{R}\left\{\sum_{n=1}^{K}\mathbf{u}_n\mathbf{u}_n^{\mathrm{H}}\mathbf{T}_s\right\} - \left(\sum_{n=1}^{K}\mathbf{u}_n\mathbf{u}_n^{\mathrm{H}}\right)\mathbf{T}_s\sum_{n=1}^{K}\mathbf{u}_n\mathbf{u}_n^{\mathrm{H}}\right]\mathbf{a}_s.$$

The augmented interference and noise covariance matrices are formed using (2)-(3) assuming $\sigma_s^2 = 0$. Eigenvector decomposition is assumed to separate the interference and noise subspace such the largest K eigenvalues and corresponding eigenspace of the interference subspace while the smallest $M_\alpha - K$ eigenvalues correspond to the eigenvectors spanning the noise subspace. The power output of the interference plus noise is written as

$$\mathbf{w}^{\mathrm{H}}(\mathbf{T}_I + \mathbf{T}_w)\mathbf{w} = \frac{\sum_{n=K+1}^{M_\alpha}\lambda_n|\mathbf{a}_s^{\mathrm{H}}\mathbf{u}_n|^2}{(\mathbf{a}_s^{\mathrm{H}}\mathbf{P}\mathbf{a}_s)^2}. \tag{9}$$

The total input interference and noise is defined as $\sigma_w^2 + \sum_{n=1}^{K}\sigma_n^2$, the diagonal $\mathbf{Q}$ and trace of $\mathbf{\Sigma}$ respectively. Thus the array gain is given by

$$A = \frac{M^2\beta(1-\rho)\left(\sigma_w^2 + \sum_{n=1}^{K}\sigma_n^2\right)}{\sum_{n=K+1}^{M_\alpha}\lambda_n|\mathbf{a}_s^{\mathrm{H}}\mathbf{u}_n|^2}. \tag{10}$$

Note that array gain increases with the number of interferers, K, when the interferers are outside of the near-field steered location; however, the number of interferers is limited by M_α instead of M. In this case where the number of interferers is larger than the number of sensors, large gains are possible due to the increasing denominator and decreasing numerator.

B. With Diagonal Loading

Consider the case with diagonal loading as shown in (5). In the same fashion as the previous section, the beamformer power output of the signal is given by

$$\mathbf{w}^{\mathrm{H}}\mathbf{T}_s\mathbf{w} = \frac{M^2\beta(1-\tilde{\rho})\sigma_s^2}{\varepsilon^2|\mathbf{a}_s^{\mathrm{H}}\mathbf{P}\mathbf{a}_s|^2} \tag{11}$$

where the modified loss factor due to the inner product of the signal in the interference space is given by

$$\tilde{\rho} = M^{-2}(\sigma_s^2\beta)^{-1}\mathbf{a}_s^{\mathrm{H}}\left[2\mathcal{R}\left\{\sum_{n=1}^{K}\tilde{\lambda}_n\mathbf{u}_n\mathbf{u}_n^{\mathrm{H}}\mathbf{T}_s\right\} - \left(\sum_{n=1}^{K}\tilde{\lambda}_n\mathbf{u}_n\mathbf{u}_n^{\mathrm{H}}\right)\mathbf{T}_s\sum_{n=1}^{K}\tilde{\lambda}_n\mathbf{u}_n\mathbf{u}_n^{\mathrm{H}}\right]\mathbf{a}_s$$

and the interference output is given by

$$\mathbf{w}^{\mathrm{H}}\mathbf{T}_I\mathbf{w} = \frac{\sum_{n=1}^{K}\lambda_n(1-\tilde{\lambda}_n)^2|\mathbf{a}_s^{\mathrm{H}}\mathbf{u}_n|^2}{\varepsilon^2|\mathbf{a}_s^{\mathrm{H}}\mathbf{P}\mathbf{a}_s|^2}. \tag{12}$$

The array gain using (11)-(12) is given by

$$A = \frac{M^2\beta(1-\tilde{\rho})\left(\sigma_w^2 + \sum_{n=1}^{K}\sigma_n^2\right)}{\sum_{n=1}^{K}\lambda_n(1-\tilde{\lambda}_n)^2|\mathbf{a}_s^{\mathrm{H}}\mathbf{u}_n|^2 + \sum_{n=K+1}^{M_\alpha}\lambda_n|\mathbf{a}_s^{\mathrm{H}}\mathbf{u}_n|^2} \tag{13}$$

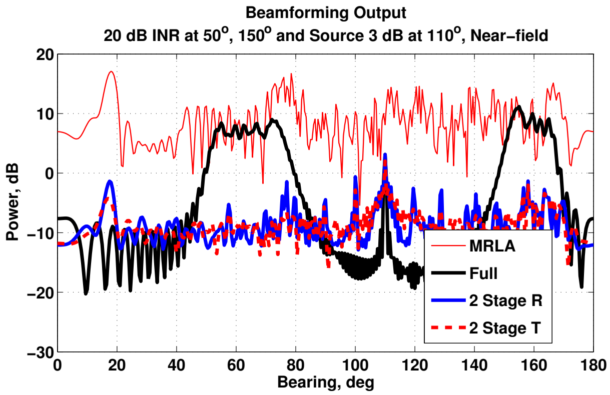
where the effect of diagonal loading on array gain is predominantly shown by the first term on the denominator. Diagonal loading trades interference suppression for gain against white noise; thus the interference will not be perfectly nulled. Consideration of mismatched steering vectors and imperfect subspace knowledge is left for future work.

V. Simulation Results

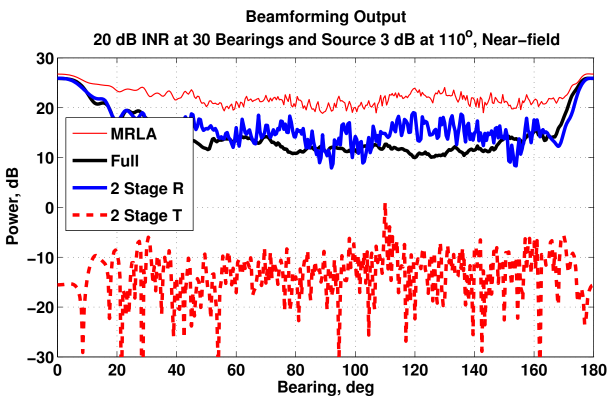
An interference dominated environment is simulated to demonstrate the performance of the proposed beamforming method. Consider a MRLA with 20 elements and total length of 125 half-wavelength spacings (see [10]). The first environment considered is two far-field interferers at 50° and 150° from endfire. The interference to noise ratio, $[\mathbf{\Sigma}]_{q,q}/\sigma_w^2$, is assumed to be 20dB for each interferer where $\mathbf{Q} = \sigma_w^2\mathbf{I}$. The source is located at 110° with range of 100 wavelengths from the first element of the array, which is a distance of less than twice the array length. The signal to noise ratio, σ_s^2/σ_w^2, is 3 dB. Beamforming is performed assuming infinite snapshots; sensitivity to snapshot deficiency is studied below in terms of array gain. The resulting beamformer power is shown in Figure 2a with weights steered to a range of 100 wavelengths. Conventional delay and sum beamforming with the MRLA has incorrectly high levels at all angles, but a filled array with an additional 106 elements is able resolve the 3 sources due to the increased array gain. Note the large plateaus in the filled array where the far-field interference leaks into the near-field beamformer output. The rank-reduced MVDR techniques are referred to as 2 Stage methods, each assuming $K = 2$. For comparison, weights given by Cox [4] are labeled with $\mathbf{R}$ to denote the operation in the received data space. The weights proposed in (6) are labeled with $\mathbf{T}$ to denote operation in the augmented data space. Both of the reduced rank approaches suppress the far-field interferers using the MRLA. A diagonal loading factor of $\varepsilon = 1$ equal to the noise, $\varepsilon = \sigma_w^2$, is used for all results.

The interference is increased to 30 sources at bearings uniformly (stochastically) distributed in bearing over 180°. Thus the total INR in this case is $20\text{dB}\times 30 = 34.8\text{dB}$, which is much larger than the maximum array gain of conventional beamforming with 126 elements. The outputs are shown in Figure 2b, where a conventional MRLA and filled array fail to show the target. Since the number of interferers exceeds the number of sensors, the adaptive $\mathbf{R}$ based method also fails. However, the number of augmented degrees of freedom is larger than the number of interferences, and the proposed technique using $\mathbf{T}$ shows the target peak at 110°.

Array gain as a function of snapshots is evaluated using a Monte Carlo simulation in an interference dominated environment. The target range of 100 wavelengths is held constant



(a) Beamformer output with two far-field interferers



(b) Beamformer output with 30 far-field interferers

Fig. 2. Simulation results for loud far-field interference with a weak near-field target

as well as INR and SNR levels, but the target and interferer bearings are uniformly distributed over 180°. Array gain as a function of number of snapshots is numerically computed using simulated data and evaluated using the definition given by

$$A = \frac{\mathbf{w}^{\mathrm{H}}\mathbf{T}_s\mathbf{w}}{\mathbf{w}^{\mathrm{H}}(\mathbf{T}_I + \mathbf{T}_w)\mathbf{w}} \frac{\sigma_w^2 + \sum_{n=1}^{K}\sigma_n^2}{\sigma_s^2}. \tag{14}$$

where the weights are calculated using L snapshots and (3)–(6). The ideal covariance matrices, $\mathbf{T}$, are formed assuming infinite snapshots. Using 1000 realizations, the random variables are target/interference bearings, target/interference signals and additive white noise. Array gain is shown for the adaptive methods in Figure 3, where the thick lines denote the mean values with thin surrounding lines denoting the 25% and 75% quantiles. The proposed method, using the augmented covariance matrix, consistently outperforms the non-augmented approach. Due to the additional degrees of freedom, the augmented approach continues to increase with additional snapshots. Note that the augmented approach has higher array gain in the low snapshot region. The rule of thumb provided by Cox is met when the number of snapshots is at least twice the number of interferers [4], which corresponds to 60 snapshots. This results in average array gain increase of 4.4dB using the augmented approach but can be increased with more snapshots unlike previous approaches.

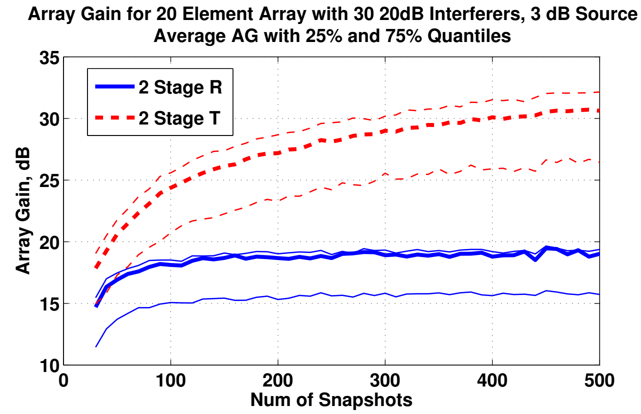


Fig. 3. Monte Carlo simulation of array gain with mean and 25%-75% quantiles in interference dominated environment and near-field source

VI. Conclusion

Fully augmentable arrays can null more interferers than the number of sensors, which can be used to suppress far-field interferences. A method was introduced to exploit the increased degrees of freedom while reducing snapshot dependency. Additionally, quiet near-field targets were considered and the resulting beamforming technique allows for increased array gain in interference dominated environments.

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