A Rao-Blackwellized Random Exchange Diffusion Particle Filter for Distributed Emitter Tracking

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***Abstract*—We introduce in this paper the fully distributed, Rao-Blackwellized Random Exchange Diffusion Particle Filter (RB ReDif-PF) to track a moving emitter using multiple received-signal-strength (RSS) sensors with unknown noise variances. In a simulated scenario with a partially connected network, the proposed RB ReDif-PF outperformed a suboptimal tracker that assimilates local neighboring measurements only. Compared to a broadcast-based filter which exactly mimics the optimal centralized tracker, ReDif-PF showed a degradation in steady-state error performance. However, compared to alternative fully distributed consensus-based trackers in the literature, ReDif-PF is better suited for real-time applications since it does not require iterative inter-node communication between measurements arrivals.**

***Index Terms*—Distributed Particle Filters, RSS Emitter Tracking, Diffusion, Wireless Sensor Networks**

I. Introduction

In an agent-based network, multiple agents at different locations equipped with sensing, processing and communication capabilities of their own cooperate to execute an estimation task without relying on a global data fusion center. Fully distributed versions of the Kalman filter have been proposed e.g. in [1], [2], [3] to track unknown state vectors in linear, Gaussian state-space models. On the other hand, several distributed particle filters (PFs) [4] have been proposed e.g. in [5], [6], [7], [8], [9] to handle nonlinear distributed estimation tasks in which Kalman filters are no longer optimal.

Assuming conditional independence of the different sensor observations given the state vector, the distributed PF requires the computation of a product of likelihood functions that depend on local data only [10]. Previous works in the literature suggest approximating that product using iterative average consensus [10] or selective gossip algorithms [11]. Alternatively, the likelihood product can be computed exactly in a finite number of iterations [9] using either iterative minimum consensus [12] or flooding [13]. However, consensus or flooding-based solutions require iterative inter-node communication between two consecutive sensor measurements and, therefore, are not well-suited for real-time processing.

To circumvent the limitations of the consensus-based trackers, we introduced in [9] the new fully distributed Random Exchange Diffusion Particle Filter (ReDif-PF), which eliminates the need for iterative inter-node communication between measurements and basically uses random information dissemination [1] to build at each network node different Monte Carlo representations of the posterior distribution of the hidden states conditioned on different random sets of measurements coming from the entire network. In this paper, we use Rao-Blackwellization [14] to extend the algorithm in [9] to a distinct scenario where the sensor models have unknown parameters. Specifically, we consider the problem of tracking a moving emitter using multiple received-signal-strength (RSS) sensors with unknown noise variance. A comprehensive discussion regarding the advantages of considering sensor variances as unknown parameters can be found in our long paper [8].

The paper is divided into 5 sections. Sec. I is this Introduction. Sec. II describes the state and sensor models. Sec. III describes the new Rao-Blackwellized ReDif-PF algorithm, whose performance is evaluated in Sec. IV. Finally, we present our conclusions in Sec. V.

II. Emitter Tracking Using Multiple RSS Sensors

Without loss of generality, we assume that the emitter trajectory is described by the white noise acceleration model [15]

$$\mathbf{x}_{n+1} = \mathbf{F}\mathbf{x}_n + \mathbf{w}_n \tag{1}$$

where $\mathbf{x}_n \triangleq \begin{bmatrix} x_n & \dot{x}_n & y_n & \dot{y}_n \end{bmatrix}^T$ is the hidden state vector at time step n consisting of the positions and velocities of the target's centroid respectively in dimensions x and y, $\mathbf{F}$ is the state transition matrix and $\{\mathbf{w}_n\}$ is a sequence of independent, identically distributed (i.i.d.) zero-mean Gaussian vectors with covariance matrix $\mathbf{Q}$. Matrices $\mathbf{F}$ and $\mathbf{Q}$, parameterized by the sampling period T and the acceleration noise σ^2_{accel}, are detailed in [8] and [15].

A. Observation Model

The measurements $z_{r,0:n} = \{z_{r,0}, \ldots, z_{r,n}\}$ in dBm at the r-th node of a network of R RSS sensors are modeled as [16]

$$z_{r,n} = g_r(\mathbf{x}_n) + v_{r,n} \tag{2}$$

where, conditioned on the *unknown* parameters $\{\sigma_1^2, \ldots, \sigma_R^2\}$, $v_{r,n}|\sigma_r^2 \sim \mathcal{N}(v_{r,n}|0, \sigma_r^2)$, $\forall r$, and $\{\mathbf{x}_0, \{\mathbf{w}_n\}, \{v_{r,n}\}\}$ are mutually independent for $n \geq 0$ and $r \in \mathcal{R} \triangleq \{1, \ldots, R\}$. The nonlinear function $g_r(\cdot)$ in (2) is in turn given by

$$g_r(\mathbf{x}) = P_0 - 10\eta_r \log\left(\frac{\|\mathbf{H}\mathbf{x} - \mathbf{x}_r\|}{d_0}\right) \tag{3}$$

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where $\mathbf{x}_r$ represents sensor position, $||.||$ is the Euclidean norm, (P_0, d_0, η_r) are known model parameters (see [16] for details) and $\mathbf{H}$ is a 2×4 projection matrix such that $H(1,1) = H(2,3) = 1$ and $H(i,j) = 0$ otherwise. We also denote by $\mathbf{N}_r$ the set of nodes in the neighborhood of node r.

Using a Bayesian approach, we model the unknown noise variances $\{\sigma_r^2\}$, $r \in \mathcal{R}$, as random variables that are mutually independent for $s \neq r$ and identically distributed *a priori* as $\sigma_r^2 \sim \mathcal{IG}(\sigma_r^2|\alpha, \beta)$ where $\mathcal{IG}$ denotes the inverse Gamma distribution. The real-valued constants $\{\alpha, \beta\}$ are the model's hyper-parameters (see [8] for further details).

III. Random Exchange Diffusion Particle Filter

We derived in [8] the optimal centralized PF for the problem stated in Sec. II and introduced both exact and approximate decentralized implementations of that optimal solution using either fully-distributed iterative consensus algorithms or broadcast algorithms. In this paper, we propose an alternative solution to the same problem using a new Rao-Blackwellized version of the suboptimal ReDif-PF algorithm previously introduced in [9] in a simpler scenario with known noise variances. We refer to that new algorithm as the RB ReDif-PF. We derive first the exact RB ReDif-PF solution to the problem and propose in the sequel an approximate version that further reduces the inter-node communication cost.

ReDif-PF with Known Sensor Variances: Let $\mathcal{Z}_{s,0:n-1}$ denote the set of all network measurements assimilated by node s up to instant $n-1$. Next, let $\left\{\mathbf{x}_{s,0:n-1}^{(q)}\right\}$ with associated weights $\left\{w_{s,n-1}^{(q)}\right\}$, $q \in \mathcal{Q} \triangleq \{1, \cdots, Q\}$, be a properly weighted set that represents the posterior p.d.f. $p(\mathbf{x}_{0:n-1}|\mathcal{Z}_{s,0:n-1})$ at node s. Assume now that, at instant $n-1$, node s sends its particles, weights and hyper-parameters to a remote node r that can assimilate at instant n the measurements $\mathcal{Z}_{r,n} = \{z_{t,n}\}$, $t \in \mathbf{N}_r \cup \{r\}$. At instant n, the new particle set at node r, $\mathbf{x}_{r,0:n}^{(q)} = (\mathbf{x}_{s,0:n-1}^{(q)}, \mathbf{x}_{r,n}^{(q)})$ with updated weights $w_{r,n}^{(q)}$ such that

$$\mathbf{x}_{r,n}^{(q)} \sim p(\mathbf{x}_n|\mathbf{x}_{s,n-1}^{(q)}) \quad (4)$$

$$w_{r,n}^{(q)} \propto w_{s,n-1}^{(q)}\, p(\mathcal{Z}_{r,n}|\mathbf{x}_{r,0:n}^{(q)}, \mathcal{Z}_{s,0:n-1}) \quad (5)$$

is now a properly weighted set, see also [9], to represent the updated posterior $p(\mathbf{x}_{0:n}|\mathcal{Z}_{r,n}, \mathcal{Z}_{s,0:n-1})$, where $\{\mathcal{Z}_{r,n}, \mathcal{Z}_{s,0:n-1}\}$ is redefined as $\mathcal{Z}_{r,0:n}$.

In order to build, at each instant n and at each node r, different Monte Carlo representations of the posterior distribution conditioned on different sets of observations $\mathcal{Z}_{r,0:n}$ coming from random locations in the entire network, it suffices to implement a protocol where each node r, starting from instant zero, exchanges its weighted particles $\left\{\mathbf{x}_{r,n-1}^{(q)}, w_{r,n-1}^{(q)}\right\}$, $q \in \mathcal{Q}$, from time instant $n-1$ with a randomly chosen neighboring node s, propagates the received particles $\left\{\mathbf{x}_{s,n-1}^{(q)}, w_{s,n-1}^{(q)}\right\}$ using the blind importance function as in Eq. (4), and then updates their weights as in Eq. (5).

Unlike randomized gossip algorithms [17], this procedure diffuses information by randomly propagating posterior statistics across the network. More specifically, as the initial posterior statistics provided by a given node r_0 at time 0 follows a path $\mathcal{P} \triangleq \{r_0, r_1, \ldots, r_n\}$ along the network, it assimilates the available measurements $\mathcal{Z}_{r,n}$ in the neighborhood of each visited node $r \in \mathcal{P}$. Since the initial posteriors at each node follow different paths, the posterior available at node r_n at time n will be different from those in the remaining nodes.

A. Rao-Blackwellized ReDif-PF

In the scenario with *unknown* sensor variances, it can be shown after long algebraic calculations similar to those in Appendix B of the reference [8] that, if, at instant $n-1$,

$$p(\sigma_{1:R}^2|\mathbf{x}_{s,0:n-1}^{(q)}, \mathcal{Z}_{s,0:n-1}) = \prod_{i=1}^{R} \mathcal{IG}(\sigma_i^2|\alpha_{s,i,n-1}, \beta_{s,i,n-1}^{(q)})$$

then

$$p(\mathcal{Z}_{r,n}|\mathbf{x}_{r,0:n}^{(q)}, \mathcal{Z}_{s,0:n-1}) = \prod_{t} p(z_{t,n}|\mathbf{x}_{r,0:n}^{(q)}, \mathcal{Z}_{s,0:n-1}) \quad (6)$$

where $t \in \mathbf{N}_r \cup \{r\}$ and each factor in the product on the right-hand side of (6) is computed independently by solving the integral

$$\begin{aligned}&\int_0^{\infty} p(z_{t,n}|\mathbf{x}_{r,n}^{(q)}, \sigma_t^2)p(\sigma_t^2|\mathbf{x}_{s,0:n-1}^{(q)}, \mathcal{Z}_{s,0:n-1})\mathrm{d}\sigma_t^2 \\ &= \int_0^{\infty} \mathcal{N}(z_{t,n}|g_t(\mathbf{x}_{r,n}^{(q)}), \sigma_t^2)\mathcal{IG}(\sigma_t^2|\alpha_{s,t,n-1}, \beta_{s,t,n-1}^{(q)})\mathrm{d}\sigma_t^2 \\ &\propto \frac{\left[\beta_{s,t,n-1}^{(q)}\right]^{\alpha_{s,t,n-1}}}{\Gamma(\alpha_{s,t,n-1})} \frac{\Gamma(\alpha_{r,t,n})}{\left[\beta_{r,t,n}^{(q)}\right]^{\alpha_{r,t,n}}}, \end{aligned} \quad (7)$$

where $\Gamma(\cdot)$ denotes the Gamma function and

$$\alpha_{r,t,n} = \alpha_{s,t,n-1} + \frac{1}{2} \quad (8)$$

$$\beta_{r,t,n}^{(q)} = \beta_{s,t,n-1}^{(q)} + \frac{1}{2}\left[z_{t,n} - g_t(\mathbf{x}_{r,n}^{(q)})\right]^2, \quad (9)$$

with $g_t(\cdot)$ calculated as in (3). Thus, in this scenario, node r also needs to exchange its hyper-parameters $\left\{\alpha_{r,i,n-1}, \beta_{r,i,n-1}^{(q)}\right\}$, $q \in \mathcal{Q}$ and $i \in \mathcal{R}$, with the chosen node s. Furthermore, at node r and instant n, the updated parameter posterior p.d.f.

$$p(\sigma_{1:R}^2|\mathbf{x}_{r,0:n}^{(q)}, \mathcal{Z}_{r,0:n}) = \prod_{i=1}^{R} \mathcal{IG}(\sigma_i^2|\alpha_{r,i,n}, \beta_{r,i,n}^{(q)}) \quad (10)$$

where $\alpha_{r,i,n}$ and $\beta_{r,i,n}^{(q)}$ are updated as in Eqs. (8) and (9) if $i \in \mathbf{N}_r \cup \{r\}$, or, otherwise, are kept equal respectively to $\alpha_{s,i,n-1}$ and $\beta_{s,i,n-1}^{(q)}$.

B. Approximate RB ReDif-PF

To circumvent the inconvenience of having to transmit Q particles and weights per node at each time step, we follow the lead in [18] and build a Gaussian Mixture Model (GMM) representation of the marginal posterior $p(\mathbf{x}_n|\mathcal{Z}_{s,0:n-1})$ of the form

$$p(\mathbf{x}_n|\mathcal{Z}_{s,0:n-1}) \approx \sum_{k \in \boldsymbol{\mathcal{K}}} \eta_{s,n}^{(k)} \mathcal{N}(\mathbf{x}_n|\mu_{s,n-1}^{(k)}, \Sigma_{s,n-1}^{(k)}) \quad (11)$$

where the parameters $\eta^{(k)}_{s,n-1}$, $\mu^{(k)}_{s,n-1}$, and $\Sigma^{(k)}_{s,n-1}$ are obtained from the weighted particle set $\left\{\mathbf{x}^{(q)}_{s,n-1}, w^{(q)}_{s,n-1}\right\}$, $q \in \mathcal{Q}$, at node s using the Expectation-Maximization (EM) [19] algorithm. Node s now transmits only the parameters of the GMM model to node r. Then, node r locally resamples Q new particles according to the distribution in (11) from the received GMM parameters and resets its importance weights to $1/Q$.

To further reduce the communication burden, we follow the lead in [8], [20], [21], and, for each $t \in \mathcal{R}$, approximate the marginal posteriors $p(\sigma^2_t|\mathbf{x}^{(q)}_{0:n-1}, \mathcal{Z}_{s,0:n-1})$ for all particle labels q and all sequences $\mathbf{x}^{(q)}_{0:n-1}$ by a new inverse Gamma p.d.f. with parameters $\tilde{\alpha}_{s,t,n-1}$ and $\tilde{\beta}_{s,t,n-1}$, *independent* of q and chosen such that the approximated distribution $\mathcal{IG}(\sigma^2_t|\tilde{\alpha}_{s,t,n-1}, \tilde{\beta}_{s,t,n-1})$ matches the first and second moments of

$$E\left\{p(\sigma^2_t|\mathbf{x}_{0:n-1}, \mathcal{Z}_{s,0:n-1})\right\} \approx \sum_{q=1}^{Q} w^{(q)}_{s,n-1}\mathcal{IG}(\sigma^2_t| \alpha_{s,t,n-1}, \beta^{(q)}_{s,t,n-1}) \quad (12)$$

where the expectation in (12) is taken over all realizations of $\mathbf{x}_{0:n-1}$ conditioned on $\mathcal{Z}_{s,0:n-1}$. For all q, we then replace $\alpha_{s,t,n-1}$ and $\beta^{(q)}_{s,t,n-1}$ in Eqs. (8) and (9) with $\tilde{\alpha}_{s,t,n-1}$ and $\tilde{\beta}_{s,t,n-1}$ respectively, such that it suffices for node s to transmit just $2R$ hyper-parameters to node r as opposed to $R \times (Q+1)$ where Q is the number of particles.

IV. Simulation Results

We assessed the performance of the proposed filter using 100 Monte Carlo runs with simulated data. As a performance benchmark, we used the DcPF algorithm from [21], which is a broadcast-based algorithm that reproduces exactly the optimal centralized filter. The simulated scenario has $R = 25$ RSS sensors with parameters $P_0 = 1dBm$, $d_0 = 1m$, $\eta_r = 3$ and σ^2_r independently sampled at each node according to an $\mathcal{IG}$ distribution with mean 16. The nodes were deployed on a jittered grid within a square of size $100m \times 100m$. In contrast with the DcPF algorithm, ReDif-PF assumes a partially connected network in which each node communicates with other nodes within a range of $40m$.

Fig. 1 shows the sensor positions and one realization of the emitter trajectory generated for $T = 1s$, $\sigma_{accel} = 0.05m/s^2$ and $\mathbf{x}_0 = \begin{bmatrix}25m & 0.5m/s & 35m & 0.5m/s\end{bmatrix}^T$. It also presents the available network connections. The network diameter of the evaluated scenario is $D = 5$ hops and the minimum number of neighbors for any possible node is three.

All filters used $Q = 500$ particles. Particles were initialized considering Gaussian priors with mean $\begin{bmatrix}x_0 & y_0\end{bmatrix}^T$ and covariance matrix $diag(20^2, 20^2)$ for the emitter's position and mean $\begin{bmatrix}\sqrt{\dot{x}^2_0 + \dot{y}^2_0} & \arctan(\dot{y}_0/\dot{x}_0)\end{bmatrix}^T$ and covariance matrix $diag(0.3^2, (5\pi/180)^2)$ for the emitter's velocity.

Fig. 2 shows the evolution of the root-mean-square (RMS) error norm – averaged over all network nodes and Monte Carlo runs – of the emitter position estimates for the DcPF and ReDif-PF algorithms. It also presents the RMS error norm

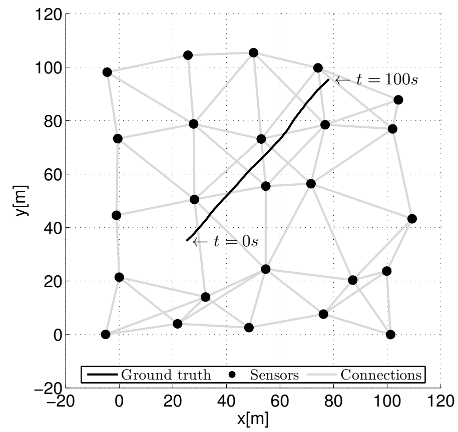

Fig. 1. Evaluated scenario.

for the isolated nodes and for a local cooperation scheme. In the former, each node runs a regularized PF tracker, see [8], which assimilates local measurements only, while in the latter, a node r incorporates all measurements $\mathcal{Z}_{r,n}$ in its vicinity in the same way as in the ReDif-PF tracker, but it does not exchange its updated posterior with its neighbors. The bars shown in Fig. 2 represent the standard deviation of the error norm across all nodes in the network. There are no bars for the DcPF algorithm since it provides the same state estimate at all nodes. The RMS error norm at time step 0 for all algorithms was calculated after the measurements $z_{1:R,0}$ were assimilated.

As theoretically expected, the ReDif-PF tracker has a performance degradation compared to DcPF since the posterior at each node assimilates just a subset of the available measurements $z_{1:R,n}$ in the whole network at each time step n. However, the ReDif-PF offers an improvement in error performance compared to the local cooperation scheme by better diffusing the information across the network. We also note from Fig. 2 that the standard deviation of the state estimate across the different network nodes is much lower in the ReDif-PF algorithm than in the local cooperation scheme. The ReDif-PF tracker was evaluated with GMM posterior approximations using just one Gaussian mode. Finally, as shown by Fig. 2, isolated nodes were not able to properly track the emitter in the evaluated scenario.

Considering a four-byte and a one-byte network representation respectively for real and Boolean values, the total amount of bytes transmitted and received by all nodes over the network was recorded while running each tracker. Table I summarizes the communication cost for each algorithm in our particular simulation in terms of average transmission (TX) and average reception (RX) rates per node and also quantifies the processing cost for each algorithm in terms of average duty cycle per node, measured in a Intel Core i5 machine with 4GB RAM. The duty cycle of a given node is defined as the ratio between the total node processing time and the simulation period $100s$.

As expected, the ReDif-PF has a communication cost that is one order of magnitude lower than the DcPF's communication requirements at the expense of an increase in processing cost. Furthermore, by comparing the duty cycle of the local

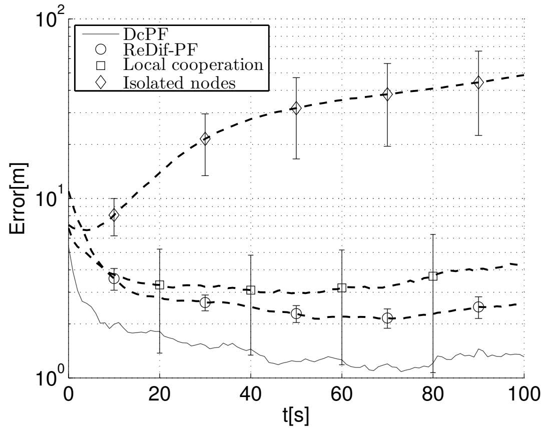

Fig. 2. Evolution of the estimated position RMS error norm.

TABLE I
AVERAGE COMMUNICATION AND PROCESSING COST PER NODE

	RX Rate	TX Rate	Duty Cycle
DcPF	46.89KB/s	1.95KB/s	35.8%
ReDif-PF	525.2B/s	515.7B/s	7.9%
Local Cooperation	4B/s	19.8B/s	9.3%
Isolated Nodes	–	–	2.0%

cooperation scheme and ReDif-PF, we conclude that the regularization step employed by the former to avoid particle degeneration (refer to [8]) is slightly more expensive than computing the GMM and the $\mathcal{IG}$ approximations used by the latter.

V. CONCLUSIONS

We introduced in this paper a Rao-Blackwellized version of the Random Exchange Diffusion Particle Filter, which enables fully distributed tracking of hidden state vectors in cooperative sensor networks with unknown sensor parameters. In particular, we specified the algorithm in an application where we track a moving emitter using multiple RSS sensors with unknown noise variances. The ReDif-PF tracker, introduced originally in a simpler version in [9], is based on random information dissemination and is well suited for real-time applications since, unlike consensus-based approaches, it does not require iterative inter-node communication between measurement arrivals.

The new Rao-Blackwellized version of the ReDif-PF was compared in this paper to an exact broadcast implementation of the optimal centralized PF solution to the problem under investigation, referred to as the DcPF algorithm. As expected, due to its sub-optimality, the ReDif-PF tracker showed a degradation in RMS error performance compared to DcPF in our simulations, but required much lower communication bandwidth than the fully connected broadcast-based algorithm.

The ReDif-PF algorithm RMS error performance was also compared to a local cooperation scheme in which each node assimilates all available measurements in its neighborhood, but does not exchange its posterior statistics with other nodes. By diffusing information over the network, the ReDif-PF tracker showed better error performance than the local cooperation scheme that uses local information only. Additionally, the standard deviation of the error norm considering all nodes in the network is much lower for ReDif-PF than in the local cooperation scheme. As future work, we plan to carry out a theoretical analysis of how efficiently the ReDif-PF algorithm disseminates information across the network.

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