LOW-COMPLEXITY ROBUST DATA-DEPENDENT DIMENSIONALITY REDUCTION BASED ON JOINT ITERATIVE OPTIMIZATION OF PARAMETERS

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ABSTRACT

This paper presents a low-complexity robust data-dependent dimensionality reduction based on a modified joint iterative optimization (MJIO) algorithm for reduced-rank beamforming and steering vector estimation. The proposed robust optimization procedure jointly adjusts the parameters of a rank-reduction matrix and an adaptive beamformer. The optimized rank-reduction matrix projects the received signal vector onto a subspace with lower dimension. The beamformer/steering vector optimization is then performed in a reduced-dimension subspace. We devise efficient stochastic gradient and recursive least-squares algorithms for implementing the proposed robust MJIO design. The proposed robust MJIO beamforming algorithms result in a faster convergence speed and an improved performance. Simulation results show that the proposed MJIO algorithms outperform some existing full-rank and reduced-rank algorithms with a comparable complexity.

1. INTRODUCTION

Adaptive beamforming algorithms often encounter problems when they operate in dynamic environments with large sensor arrays. These include snapshot deficiency, steering vector mismatches caused by calibration and pointing errors, and a high computational complexity. In terms of complexity, an expensive inverse operation of the covariance matrix of the received data is often required, resulting in a high computational complexity that may prevent the use of adaptive beamforming in important applications like sonar and radar. In order to overcome this computational complexity issue, adaptive versions of the linearly constrained beamforming algorithms such as minimum variance distortionless response (MVDR) with stochastic gradient and recursive least squares [1] have been extensively reported. These adaptive algorithms estimate the data covariance matrix iteratively and the complexity is reduced by recursively computing the weights. However, in a dynamic environment with large sensor arrays such as those found in radar and sonar applications, adaptive beamformers with a large number of array elements may fail in tracking signals embedded in strong interference and noise. The convergence speed and tracking properties of adaptive beamformers depend on the size of the sensor array and the eigen-spread of the received covariance matrix [1]. Regarding the steering vector mismatches often found in practical applications of beamforming, they are responsible for a significant performance degradation of algorithms. Prior work on robust beamforming design [2, 3, 4] has considered different strategies to mitigate the effects of these mismatches. An effective method to deal with mismatches is the Robust Capon Beamforming (RCB) technique of [2]. A key limitation of [2] and other robust techniques [3, 4] is their high cost for large sensor arrays and their suitability to dynamic environments.

Reduced-rank signal processing techniques [4]-[11] provide a way to address some of the problems mentioned above. Reduced-dimension methods are often needed to speed-up the convergence of beamforming algorithms and reduce their computational complexity. They are particularly useful in scenarios in which the interference lies in a low-rank subspace and the number of degrees of freedom required the mitigate the interference through beamforming is significantly lower than that available in the sensor array. In reduced-rank schemes, a rank-reduction matrix is introduced to project the original full-dimension received signal onto a lower dimension. The advantage of reduced-rank methods lie in their superior convergence and tracking performance achieved by exploiting the low-rank nature of the signals. It offers a large reduction in the required number of training samples over full-rank methods [1]. Several reduced-rank strategies for processing data collected from a large number of sensors have been reported in the last few years, which include beamspace methods [4], Krylov subspace techniques [8, 11], and methods based on joint and iterative optimization of parameters [7, 9, 10].

Despite the improved convergence and tracking performance achieved with Krylov methods [8, 11], they are relatively complex and may suffer from numerical problems. On the other hand, the joint optimization technique reported in [9] outperforms the Krylov-based method with efficient adaptive implementations. However, this algorithm suffers from the problem of rank one. In order to address this problem, in this paper, we introduce a low-complexity robust data-dependent dimensionality reduction based on a modified joint iterative optimization (MJIO) algorithm for reduced-rank beamforming and steering vector estimation. The proposed MJIO design strategy jointly optimizes the rank-reduction matrix and a reduced-rank beamformer, which ensures that the rank-reduction matrix has a desired rank. Another contribution of this work is the introduction of a bank of perturbed steering vectors as candidate array steering vectors around the true steering vector. The candidate steering vectors are responsible for performing rank reduction and the reduced-rank beamformer forms the beam in the direction of the signal of interest (SoI). We devise efficient stochastic gradient(SG) and recursive least-squares (RLS) algorithms for implementing the proposed robust MJIO design. Simulation results show that the proposed MJIO algorithms outperform existing full-rank and reduced-rank algorithms with a comparable complexity.

This paper is organized as follows. The system model is described in Section 2. The reduced-rank MVDR beamforming with MJIO is formulated in Section 3. A robust version of MJIO is investigated in Section 4 and simulations are discussed in Section 5.

*This work was supported by UK MOD under contract from the Centre for Defence Enterprise

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2. SYSTEM MODEL AND PROBLEM STATEMENT

Let us consider a uniform linear array (ULA) with M sensor elements, which receive K narrowband signals where $K \leq M$. The DoAs of the K signals are $\theta_0, \ldots \theta_{K-1}$. The received vector $\boldsymbol{x}[i] \in \mathbb{C}^{M\times 1}$ at the i-th snapshot (time instant), can be modelled as

$$\boldsymbol{x}[i] = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s}[i] + \boldsymbol{n}[i], \qquad i = 1, \ldots, N \tag{1}$$

where $\boldsymbol{\theta} = [\theta_0, \ldots, \theta_{K-1}]^T \in \mathbb{R}^{K\times 1}$ convey the DoAs of the K signal sources. $\boldsymbol{A}(\boldsymbol{\theta}) = [\boldsymbol{a}(\theta_0), \ldots, \boldsymbol{a}(\theta_{K-1})] \in \mathbb{C}^{M\times K}$ comprises K steering vectors which are given as

$$\boldsymbol{a}(\theta_k) = [1, e^{-2\pi j \frac{\iota}{\lambda_c}\cos(\theta_k)}, \ldots, e^{-2\pi j (M-1)\frac{\iota}{\lambda_c}\cos(\theta_k)}]^T. \tag{2}$$

where λ_c is the wavelength and ι is the inter-element distance of the ULA. The K steering vectors $\boldsymbol{a}\{\boldsymbol{\theta_k}\} \in \mathbb{C}^{M\times 1}$ are assumed to be linearly independent. The source data are modelled as $\boldsymbol{s} \in \mathbb{C}^{K\times 1}$ and $\boldsymbol{n}[i] \in \mathbb{C}^{M\times 1}$ is the noise vector, which is assumed to be zero-mean, N is assumed to be the observation size and $[i]$ denotes the time instant. For full-rank processing, the adaptive beamformer output for the SoI is written as

$$y_k[i] = \boldsymbol{\omega}_k^H[i]\boldsymbol{x}[i], \tag{3}$$

where the beamformer $\boldsymbol{\omega}_k \in \mathbb{C}^{M\times 1}$ is derived according to a design criterion. The optimal weight vector is obtained by maximizing the signal-to-interference-plus-noise ratio (SINR) and

$$\text{SINR}_{\text{opt}} = \frac{\boldsymbol{\omega}_{\text{opt}}^H \boldsymbol{R}_k \boldsymbol{\omega}_{\text{opt}}}{\boldsymbol{\omega}_{\text{opt}}^H \boldsymbol{R}_{i+n} \boldsymbol{\omega}_{\text{opt}}}, \tag{4}$$

where $\boldsymbol{R}_k$ and $\boldsymbol{R}_{i+n}$ denote the SoI and interference-plus-noise covariance matrices, respectively. Full-rank beamformers usually suffer from high complexity and low convergence speed. In the following, we focus on the design of low-complexity reduced-dimension beamforming algorithms.

3. DIMENSION REDUCTION WITH MODIFIED JIO

In this section we describe reduced-rank algorithms based on the proposed MJIO design of beamformers. The scheme jointly optimizes a rank-reduction matrix and a reduced-rank beamformer that operates at the output of the projection matrix. The bank of adaptive beamformers in the front-end is responsible for performing dimensionality reduction, which is followed by a reduced-rank beamformer which effectively forms the beam in the direction of the SoI. This two-stage scheme allows the adaptation with different update rates, which could lead to a significant reduction in the computational complexity per update. Specifically, this complexity reduction can be obtained as the dimensionality reduction performed by the rank-reduction matrix could be updated less frequently than the reduced-rank beamformer. The design criterion of the proposed MVDR-MJIO beamformer is given by the optimization problem

$$\begin{aligned} \min_{\boldsymbol{\omega}, \boldsymbol{s_d}} \quad & \boldsymbol{\omega}^H \boldsymbol{S}_D^H \boldsymbol{R} \boldsymbol{S}_D \boldsymbol{\omega}, \\ \text{subject to} \quad & \boldsymbol{\omega}^H \sum_{d=1}^{D} \boldsymbol{q}_d \boldsymbol{s}_d^H \boldsymbol{a}_d = 1, \end{aligned} \tag{5}$$

where $\boldsymbol{R}$ is the covariance matrix obtained from sensors, vector $\boldsymbol{q}_d$ with dimension $D \times 1$ is a zero vector except its d-th element been one. The vector $\boldsymbol{s}_d \in \mathbb{C}^{M\times 1}$ is the d-th column of the projection matrix $\boldsymbol{S}_D \in \mathbb{C}^{M\times D}$. The vectors $\boldsymbol{a}_d, d = 1 \ldots D$ represent the assumed steering vector and $D-1$ small perturbations of the assumed steering vector. Each recursion updates a different column of $\boldsymbol{S}_D$. An increased rank of $\boldsymbol{S}_D$ is required for higher d, and the rank one problem in [9] can be avoided. The constrained optimization problem in (5) can be solved by using the method of Lagrange multipliers [2]. The Lagrangian of the MVDR-MJIO design is expressed by

$$f(\boldsymbol{\omega}, \boldsymbol{s}_d) = E\Big\{\Big|\boldsymbol{\omega}^H \sum_{d=1}^{D} \boldsymbol{q}_d \boldsymbol{s}_d^H \boldsymbol{x}\Big|^2\Big\} + \lambda\Big(\boldsymbol{\omega}^H \sum_{d=1}^{D} \boldsymbol{q}_d \boldsymbol{s}_d^H \boldsymbol{a} - 1\Big). \tag{6}$$

3.1. Stochastic Gradient Adaptation

In this subsection, we present a low-complexity SG [1] adaptive reduced-rank algorithm for efficient implementation of the MJIO algorithm. By computing the instantaneous gradient terms of (6) with respect to $\boldsymbol{\omega}[i]^*$ and $\boldsymbol{s}_d[i]^*$, we obtain

$$\boldsymbol{\omega}[i+1] = \boldsymbol{\omega}[i] - \mu_w \boldsymbol{P}_w[i] \boldsymbol{S}_D^H[i] \boldsymbol{x}[i] z^*[i], \tag{7}$$

$$\boldsymbol{s}_d[i+1] = \boldsymbol{s}_d[i] - \mu_s \boldsymbol{P}_s[i] \boldsymbol{x}[i] z^*[i] w_d^*[i], d = 1, \ldots, D, \tag{8}$$

where w_d is the dth element of the reduced-rank beamformer $\boldsymbol{\omega}[i]$ and the projection matrices that enforce the constraints are

$$\boldsymbol{P}_w[i] = \boldsymbol{I}_D - (\boldsymbol{a}_D^H[i]\boldsymbol{a}_D[i])^{-1} \boldsymbol{a}_D[i] \boldsymbol{a}_D^H[i], \tag{9}$$

and

$$\boldsymbol{P}_s[i] = \boldsymbol{I}_M - (\boldsymbol{a}^H[i]\boldsymbol{a}[i])^{-1} \boldsymbol{a}[i] \boldsymbol{a}^H[i], \tag{10}$$

the scalar $z^*[i] = \boldsymbol{x}^H[i]\boldsymbol{S}_D[i]\boldsymbol{\omega}[i] = \tilde{\boldsymbol{x}}^H[i]\boldsymbol{\omega}$ and

$$\boldsymbol{a}_D[i] = \sum_{d=1}^{D} \boldsymbol{q}_d \boldsymbol{s}_d[i]^H \boldsymbol{a}[i] \in \mathbb{C}^{D\times 1}. \tag{11}$$

is the estimated steering vector in reduced dimension. The calculation of $\boldsymbol{P}_\omega[i]$ requires a number of $D^2 + D + 1$ complex multiplications, the computation of $\boldsymbol{P}_s[i]$ and $z[i]$ requires $D^2 + DM + M + 1$ and $DM + D$ complex multiplications, respectively. Therefore, we can conclude that for each iteration, the SG adaptation requires $4MD + 4D^2 + 3D + M + 6$ complex multiplications.

3.2. Recursive Least Squares Adaptation

Here we derive an adaptive reduced-rank RLS [1] type algorithm for efficient implementation of the MVDR-MJIO method. The reduced-rank beamformer $\boldsymbol{\omega}[i]$ is updated as follows:

$$\boldsymbol{\omega}[i] = \frac{\boldsymbol{R}_D^{-1}[i]\boldsymbol{a}_D[i]}{\boldsymbol{a}_D^H[i]\boldsymbol{R}_D^{-1}[i]\boldsymbol{a}_D[i]}, \tag{12}$$

where

$$\tilde{\boldsymbol{k}}[i+1] = \frac{\alpha^{-1}\boldsymbol{R}_D^{-1}[i]\tilde{\boldsymbol{x}}[i+1]}{1 + \alpha^{-1}\tilde{\boldsymbol{x}}^H[i+1]\boldsymbol{R}_D^{-1}[i]\tilde{\boldsymbol{x}}[i]}, \tag{13}$$

$$\boldsymbol{R}_D^{-1}[i+1] = \alpha^{-1}\boldsymbol{R}_D^{-1}[i] - \alpha^{-1}\tilde{\boldsymbol{k}}[i+1]\tilde{\boldsymbol{x}}^H[i+1]\boldsymbol{R}_D^{-1}[i], \tag{14}$$

The columns $\boldsymbol{s}_d[i]$ of the rank-reduction matrix are updated by

$$\boldsymbol{s}_d[i] = \frac{\boldsymbol{R}^{-1}[i]\boldsymbol{a}_d[i]\boldsymbol{a}_d^H[i]\boldsymbol{\beta}_d[i]}{\boldsymbol{a}_d^H[i]\boldsymbol{R}^{-1}[i]\boldsymbol{a}_d[i]w_d[i]}, d = 1, \ldots, D, \tag{15}$$

where $\boldsymbol{\beta}_d[i] = \sum_{d=1}^{D} \boldsymbol{s}_d[i]w_d[i] - \sum_{l=1, l\neq d}^{D} \boldsymbol{s}_l[i]w_l[i]$ and

$$\boldsymbol{k}[i+1] = \frac{\alpha^{-1}\boldsymbol{R}^{-1}[i]\boldsymbol{x}[i+1]}{1 + \alpha^{-1}\boldsymbol{x}^H[i+1]\boldsymbol{R}^{-1}[i]\boldsymbol{x}[i]}, \tag{16}$$

$$\boldsymbol{R}^{-1}[i+1] = \alpha^{-1}\boldsymbol{R}^{-1}[i] - \alpha^{-1}\boldsymbol{k}[i+1]\boldsymbol{x}^H[i+1]\boldsymbol{R}^{-1}[i], \quad (17)$$

where $0 \ll \alpha < 1$ is the forgetting factor. The inverse of the covariance matrix $\boldsymbol{R}^{-1}$ is obtained recursively. Equation (17) is initialized by using an identity matrix $\boldsymbol{R}^{-1}[0] = \delta\boldsymbol{I}$ where δ is a positive constant. The computational complexity of the proposed adaptive reduced-rank RLS type MVDR-MJIO method requires $4M^2 + 3D^2 + 3D + 2$ complex multiplications. The MVDR-MJIO algorithm has a complexity significantly lower than a full-rank scheme if a low rank ($D \ll M$) is selected.

4. PROPOSED ROBUST CAPON MJIO BEAMFORMING

In this section, we present a robust beamforming method based on the Robust Capon Beamforming (RCB) technique reported in [2] and the MJIO detailed in the previous section for robust beamforming applications with large sensor arrays. The proposed technique, denoted Robust Capon Beamforming MJIO (RCB-MJIO), gathers the robustness of the RCB approach [2] against uncertainties and the low-complexity of MJIO techniques. Assuming that the DoA mismatch is within a spherical uncertainty set, the proposed RCB-MJIO technique solves the following optimization problem:

$$\begin{aligned} \min_{\boldsymbol{a_d},\boldsymbol{s_d}} \quad & \boldsymbol{a}_d^H \boldsymbol{S}_D^H \boldsymbol{R}^{-1} \boldsymbol{S}_D \boldsymbol{a}_d, \\ \text{subject to} \quad & \left\| \boldsymbol{S}_D^H \boldsymbol{a}_d - \boldsymbol{S}_D^H \bar{\boldsymbol{a}} \right\|^2 = \epsilon, \end{aligned} \quad (18)$$

where $\bar{\boldsymbol{a}}$ is the assumed steering vector and $\boldsymbol{a}_d$ is the updated steering vector for each iteration. The constant ϵ is related to the radius of the uncertainty sphere. The Lagrangian of the RCB-MJIO constrained optimization problem is expressed by

$$\begin{aligned} f_{\text{RCB}}(\boldsymbol{a}_d, \boldsymbol{s}_d) = & \left(\sum_{d=1}^{D} \boldsymbol{q}_d \boldsymbol{s}_d^H \boldsymbol{a}_d\right)^H \boldsymbol{R}_D^{-1} \left(\sum_{d=1}^{D} \boldsymbol{q}_d \boldsymbol{s}_d^H \boldsymbol{a}_d\right) + \\ & \lambda_{\text{RCB}} \left(\left\| \sum_{d=1}^{D} \boldsymbol{q}_d \boldsymbol{s}_d^H \boldsymbol{a}_d - \sum_{d=1}^{D} \boldsymbol{q}_d \boldsymbol{s}_d^H \bar{\boldsymbol{a}} \right\|^2 - \epsilon \right), \end{aligned} \quad (19)$$

where $\boldsymbol{R}_D^{-1} = \boldsymbol{S}_D^H \boldsymbol{R}^{-1} \boldsymbol{S}_D$ is the reduced rank covariance matrix. From the above Lagrangian, we will devise efficient adaptive beamforming algorithms in what follows.

4.1. Stochastic Gradient Adaptation

We devise an SG adaptation strategy based on the alternating minimization of the Lagrangian in (19), which yields

$$\begin{aligned} \tilde{\boldsymbol{a}}_d[i+1] &= \tilde{\boldsymbol{a}}_d[i] - \mu_a[i]\boldsymbol{g}_a[i], \\ \boldsymbol{s}_d[i+1] &= \boldsymbol{s}_d[i] - \mu_s[i]\boldsymbol{g}_s[i], \end{aligned} \quad (20)$$

where $\mu_a[i]$ and $\mu_s[i]$ are the step-sizes of the SG algorithms, the parameter vectors $\boldsymbol{g}_a[i]$ and $\boldsymbol{g}_s[i]$ are the partial derivatives of the Lagrangian in (19) with respect to $\tilde{\boldsymbol{a}}_d^*[i]$ and $\boldsymbol{s}_d^*[i]$, respectively. The recursion for $\boldsymbol{g}_a[i]$ is given by

$$\boldsymbol{g}_a[i] = \left(\frac{1}{\lambda_{\text{RCB}[i]}} \boldsymbol{S}_D^H[i]\boldsymbol{R}^{-1}[i]\boldsymbol{S}_D[i] + \boldsymbol{I}_D\right)^{-1} \boldsymbol{S}_D^H[i]\tilde{\boldsymbol{a}}_d[i], \quad (21)$$

where

$$\begin{aligned} \boldsymbol{g}_s[i] = & \boldsymbol{a}_d[i]\check{\boldsymbol{a}}_d^H[i]\boldsymbol{r}_d[i] + \tau_d[i]\boldsymbol{a}_d[i]\boldsymbol{a}_d^H[i]\boldsymbol{s}_d[i], \\ & + \lambda_{\text{RCB}}[i]\boldsymbol{\alpha}_d[i]\boldsymbol{\alpha}_d^H[i]\boldsymbol{s}_d[i], \end{aligned} \quad (22)$$

and

$$\tilde{\boldsymbol{a}}_d = \sum_{d=1}^{D} \boldsymbol{q}_d \boldsymbol{s}_d^H \boldsymbol{a}_d = \boldsymbol{S}_D^H \boldsymbol{a}_d \in \mathbb{C}^{D\times 1}, \quad (23)$$

$$\check{\boldsymbol{a}}_d = \sum_{l=1, l\neq d}^{D} \boldsymbol{q}_l \boldsymbol{s}_l^H \boldsymbol{a}_l \in \mathbb{C}^{D\times 1}. \quad (24)$$

We denote $\boldsymbol{\alpha}_d \in \mathbb{C}^{M\times 1}$ as the difference between the updated steering vectors and the assumed one. The scalar τ_d is the d-th diagonal element of $\boldsymbol{R}_D^{-1}$. The term $\boldsymbol{r}_d$ denotes the d-th column vector of $\boldsymbol{R}_D^{-1}$. The Lagrange multiplier obtained is expressed as

$$\lambda_{\text{RCB}}[i] = -\left(\boldsymbol{S}_D[i]^H \boldsymbol{\alpha}_d[i]\boldsymbol{\alpha}_d^H[i]\boldsymbol{s}_d[i]\right)^{\dagger} \boldsymbol{R}_D^{-1}[i]\tilde{\boldsymbol{a}}_d[i]\boldsymbol{a}_d^H[i]\boldsymbol{s}_d[i], \quad (25)$$

The proposed RCB-MJIO SG algorithm corresponds to (7)-(9) and (20)-(25). The calculation of λ_{RCB} requires $MD + D^2 + 4M + D$ complex multiplications, and the computation of $\boldsymbol{g}_a[i]$ and $\boldsymbol{g}_s[i]$ needs $D^3 + MD + D$ and $5M + D + 2$ multiplications, respectively.

4.2. Recursive Least Squares Adaptation

We derive an RLS version of the RCB-MJIO method. The steering vector and the columns of rank-reduction matrix are updated as

$$\tilde{\boldsymbol{a}}_d[i] = \left[\tilde{\boldsymbol{a}}_d[i] - \left(\boldsymbol{I}_D + \lambda_{\text{RCB}}[i]\boldsymbol{R}_D^{-1}[i]\right)^{-1}\tilde{\boldsymbol{a}}_d[i]\right], \quad (26)$$

$$\boldsymbol{s}_d = -\left(\tau_d[i]\boldsymbol{a}_d[i]\boldsymbol{a}_d^H[i] + \lambda_{\text{RCB}}[i]\boldsymbol{\alpha}_d[i]\boldsymbol{\alpha}_d^H[i]\right)^{-1} \boldsymbol{a}_d[i]\check{\boldsymbol{a}}_d^H[i]\boldsymbol{r}_d[i], \quad (27)$$

$$\tilde{\boldsymbol{k}}[i+1] = \frac{\alpha^{-1}\boldsymbol{R}_D^{-1}[i]\tilde{\boldsymbol{x}}[i+1]}{1 + \alpha^{-1}\tilde{\boldsymbol{x}}^H[i+1]\boldsymbol{R}_D^{-1}[i]\tilde{\boldsymbol{x}}[i]}, \quad (28)$$

$$\boldsymbol{R}_D^{-1}[i+1] = \alpha^{-1}\boldsymbol{R}_D^{-1}[i] - \alpha^{-1}\tilde{\boldsymbol{k}}[i+1]\tilde{\boldsymbol{x}}^H[i+1]\boldsymbol{R}_D^{-1}[i], \quad (29)$$

where (26)-(29) need $2D^3 + 7D^2 + 4D + 3$ complex multiplications, and the projection operations need a complexity of MD complex multiplications. It is obvious that the complexity is significantly decreased if the selected rank $D \ll M$. The proposed RCB-MJIO RLS algorithm employs (12) and (26)-(29). The key of the RCB-MJIO RLS algorithm is to update the assumed steering vector $\tilde{\boldsymbol{a}}_d[i]$ with RLS iterations, and the updated beamformer $\boldsymbol{\omega}[i]$ is obtained by plugging (26) into (12) without significant extra complexity.

Note that the complexity introduced by the pseudo-inverse operation can be removed if $\boldsymbol{S}_D$ has orthogonal column vectors, this can be achieved by incorporating the Gram-Schmidt procedure in the calculation of $\boldsymbol{S}_D$. Furthermore, an alternative recursive realization of the robust adaptive linear constrained beamforming method introduced by [12] can be used to further reduce the computational complexity requirement to obtain the diagonal loading terms.

5. SIMULATIONS

In this section, we consider simulations for a ULA with $\lambda_c/2$ spacing between the sensor elements and arrays with 64 and 320 sensor elements. The covariance matrix $\hat{\boldsymbol{R}}$ is obtained by time-averaging recursions with $N = 1, \ldots, 120$ snapshots, we use the spherical uncertainty set and the upper bound is set to $\epsilon = 140$ for 64 sensor elements and $\epsilon = 800$ for 320 sensor elements. There are 4 incident signals while the first is the SoI, the other 3 signals' relative power with respect to the SoI and their DoAs in degrees are detailed in Table I. The algorithms are trained with 120 snapshots and the Signal-to-Noise Ratio (SNR) is set to 10 dB for all the simulations.

Table 1. Interference and DoA Scenario, P(dB) relative to desired user1 / DoA (degree)

Snapshots	signal1 (SoI)	signal 2	signal 3	signal 4
1-120	10/90	20/35	20/135	20/165

In Fig. 1, we select $D = 2$ for rank reduction, the proposed RCB-MJIO method with the RLS algorithm is used to obtain the inverse of the covariance matrix $\hat{\boldsymbol{R}}^{-1}[i]$ for each snapshot. We introduce a maximum of 2 degrees of DOA mismatch which is independently generated by a uniform random generator in each simulation run. A non-orthogonal Krylov projection matrix $\boldsymbol{S_D}[i] \in \mathbb{C}^{64\times 2}$ and a non-orthogonal MJIO rank-reduction matrix is also generated for rank reduction. $\boldsymbol{S_D}[i]$ is initialized as $\boldsymbol{S}_D[0] = [\boldsymbol{I}_D^T, \boldsymbol{0}_{D\times(M-D)}^T]$. In Fig.2, we choose a similar scenario but without DOA mismatch. We can see from the plots that the the MJIO and Krylov algorithms have a superior SINR performance to other existing methods and this is particularly noticeable for a reduced number of snapshots.

In Fig. 3 we compare the output SINRs of the Krylov and the proposed MJIO rank reduction technique using a spherical constraint in the presence of steering vector errors with 320 sensor elements. We assume a DOA mismatch with 2 degrees and 4 interferences with the profile listed in Table I. With Krylov and MJIO rank-reduction, the MVDR-Krylov, MVDR-MJIO, RCB-Krylov and RCB-MJIO have superior SINR performance and a faster convergence compared with their full-rank rivals.

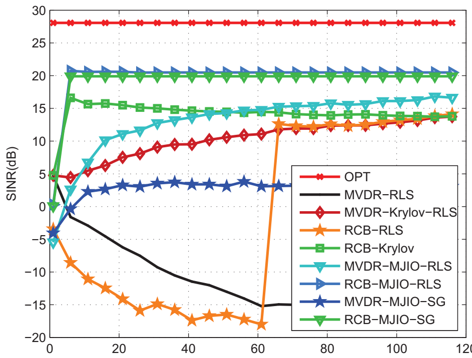

Fig. 1. SINR performance vs. number of snapshots, with steering vector mismatch due to 2 °DoA mismatch. Spherical uncertainty set is assumed for robust beamformers $\epsilon = 140$ (RLS indicates the value $\hat{\boldsymbol{R}}^{-1}$ is obtained by using RLS adaptation), non-orthogonal $\boldsymbol{S_D}[i] \in \mathbb{C}^{64\times 2}$ projection matrix.

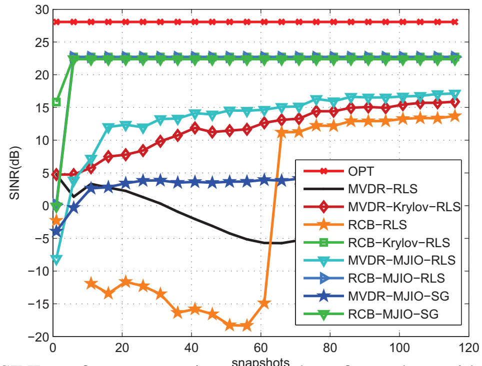

Fig. 2. SINR performance against the number of snapshots without steering vector mismatch.

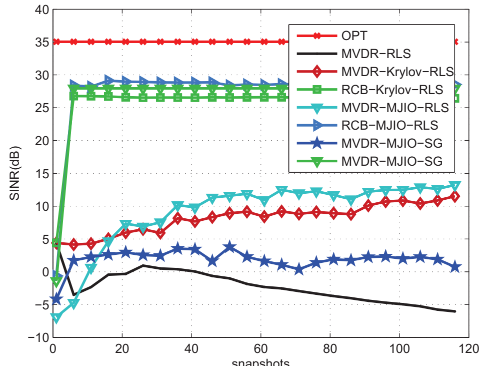

Fig. 3. SINR performance against the number of snapshots with steering vector mismatch due to 2 °DoA mismatch. The spherical uncertainty set is assumed for robust beamformers with $\epsilon = 800$, non-orthogonal $\boldsymbol{S_D}[i] \in \mathbb{C}^{320\times 2}$ rank-reduction matrix.

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