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A new inversion method for NMR signal processing

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***Abstract*—We present a new, semi-analytic inversion method for nuclear magnetic resonance (NMR) log measurements. Our method represents multiwait-time measurements via short sums of exponentials. The resulting sparse T_2 distribution requires fewer T_2 relaxation times than present in linearized inversion methods. The T_1 relaxation times, and corresponding amplitudes are estimated via convex optimization and a semi-analytic algorithm. We obtain an efficient way to represent the NMR data that can be utilized to estimate petrophysical properties and for compression in logging-while-drilling applications.**

I. Introduction

Nuclear magnetic resonance (NMR) logging tools indirectly measure the amount of hydrogen atoms in a geological formation which provides a way to infer about its porosity and permeability. Currently, NMR logging tools are the only available tools that provide information about pore geometry and disposition of fluids. In this regard, NMR tools are invaluable in determining the quality, production planning and development of a reservoir.

While advancements in logging-while-drilling (LWD) tool design and manufacturing improve reliability of real-time NMR measurements, transmission of the raw measured or processed data from downhole to uphole is still limited by the telemetry bandwidth. Compression algorithms are utilized to transmit either raw or processed echo trains or petrophysical measurements derived from the T_2 inversion process [5], [4]. Readers interested in the physics of NMR measurements and related inverse problems are referred to [3].

Motivated by the compression problem for LWD, we have developed a new inversion method for NMR log data and applied it to compute efficient representations of Carr-Purcell-Meiboom-Gill (CPMG) echo decay train measurements. These representations only require a small number of relaxation times T_2 and T_1, and corresponding amplitudes, thus reducing the amount of parameters transmitted uphole.

Linear inversion methods select in advance a fixed set of T_2 and T_1 relaxation times and compute, solving a linear system, the corresponding amplitudes a which are compressed for transmission uphole [3], [5], [4]. These methods yield many more parameters than indicated by physical considerations. In contrast, non-linear optimization-based methods seek to estimate a small set of parameters (a, T_1, T_2)s, albeit at a higher computational cost [7], [9]. Unlike current NMR data inversion methods, our method does not require predefined T_2 and T_1 values, nor does it solve a large non-linear optimization problem. It is a semi-analytic inversion method that computes an approximate representation of the data in terms of a sparse set of parameters (a, T_1, T_2). Using a common set of exponentials to represent the data, we obtain the T_2 values which we use subsequently to compute the amplitudes a via convex optimization. Finally, T_1 values are obtained in an analytic fashion by appropriate averaging. In our preliminary experiments, the proposed method provides a more efficient representation of the data than those generated by linearized methods. We expect that our method will prove computationally less demanding than non-linear optimization methods.

II. NMR Inversion problem

NMR logging tools typically acquire CPMG echo decay trains. Given N multiwait-time measured echo trains, M_n, $n = 1, \ldots, N$, each consisting of K_n echoes, $M_n(k)$, $k = 1, \ldots, K_n$, the NMR inversion problem is typically formulated as follows: find a set of positive parameters $(a_j, T_{1,j}, T_{2,j})$, such that the error sequences ϵ_n in

$$M_n(k) = \sum_{j=1}^{J} a_j \left(1 - \mathrm{e}^{-\frac{T_{W,n}}{T_{1,j}}}\right) \mathrm{e}^{-\frac{k\,T_E}{T_{2,j}}} + \epsilon_n(k), \quad (1)$$

are within the level of noise [3, Section 6.2]. Here $T_{2,j}$ are the T_2 relaxation times, a_j are the T_2 amplitudes (which are the partial porosity of the pores), $T_{1,j}$ are the corresponding T_1 relaxation times (associated with the size of the pores), $T_{W,n}$ is the n^{th} wait-time and T_E is the time sample between consecutive echoes, also referred to as the echo-spacing. The wait times $T_{W,n}$ are positive and distinct and we assume that they are ordered as $T_{W,1} > T_{W,2} > \ldots > T_{W,N}$.

The inversion problem (1) may be solved using linear [3] or non-linear [11], [7], [8], [9] methods but is always an ill-posed problem with non-unique solutions [3], [8]. To address this issue, the problem is usually approached by fixing specific T_1 and T_2 relaxation time values or imposing artificial bounds on them. Regularization factors that impose smoothness on the solution may be used as well. In contrast, our approach takes advantage of the exponential nature of the model (1). In practice, our method determines a small number of terms J required to represent all echo trains while exploiting physical bounds on the ratios between the relaxation times T_1 and T_2.

III. Proposed new algorithm for inversion

A. Step 1: Estimating $T_{2,j}$

The expected error ϵ of an exponential fit

$$\left| M\left(k\right) - \sum_{j=1}^{J} w_j \gamma_j^k \right| < \epsilon, \quad k = 0, \ldots, K \tag{2}$$

is governed by the decay of the singular values of a rectangular Hankel matrix $\mathbf{M}$ of entries $\left[\mathbf{M}\right]_{m,l} = M\left(m+l\right)$, $0 \leq m \leq K-L$, $0 \leq l \leq L$. Here $L \leq K/2$ is a parameter which overestimates the minimal number of terms J, which, for physical reasons, is a small number. Two solutions to the exponential fit problem via Hankel matrices are presented in [10], [6]. In the recent approach [1], [2], a square Hankel matrix is considered and the minimal number of terms J in (2) is directly related to the index of the singular value of $\mathbf{M}$ closest to ϵ. Here, singular values are sorted in decreasing order and normalized so that $\sigma_1 = 1$.

Even though, for fixed n, the model (1) can be expressed in the form (2), where w_j is replaced by $a_j \left(1 - \mathrm{e}^{-\frac{T_{W,n}}{T_{1,j}}}\right)$ and γ_j by $\mathrm{e}^{-\frac{T_E}{T_{2,j}}}$, the inversion problem requires determination of γ_j's that can simultaneously fit the N echo trains M_n. We achieve this fit by performing a singular value decomposition of the matrix

$$\mathbf{M} = \begin{bmatrix} \frac{1}{\sqrt{K_1 - L + 1}} \mathbf{M}_1 \\ \frac{1}{\sqrt{K_2 - L + 1}} \mathbf{M}_2 \\ \vdots \\ \frac{1}{\sqrt{K_n - L + 1}} \mathbf{M}_N \end{bmatrix},$$

where $\mathbf{M}_n$ are Hankel matrices of entries $\left[\mathbf{M}_n\right]_{m,l} = M_n\left(m+l\right)$, $0 \leq l \leq L \leq \min_n \left\{K_n\right\}/2$, $0 \leq j \leq K_n - L$. We pick a singular value σ of $\mathbf{M}$ close to the standard deviation of the errors ϵ_n, i.e. $E\left[\sum_n \epsilon_n^2\right]^{1/2}$ and form the polynomial of degree $L-1$ whose coefficients are the entries of the right singular vector associated with σ. Due to the real positivity constraint on $T_{2,j}$, we set γ_j to be the roots of this polynomial that lie within $[0,1]$ and estimate $T_{2,j} = T_E / \ln \gamma_j^{-1}$. If the level of noise is too high or it is hard to estimate, we simply compute the roots in $(0,1)$ associated to all the right singular vectors of $\mathbf{M}$ and pick the set that provides the best fit of the model (1). In addition, linear inversion methods could be used as a preliminary step to denoise the echo trains.

B. Step 2: Estimating a_j

To match (1), using the values γ_j of step 1, we solve a constrained non-negative least square problem

$$M_n(k) \approx \sum_{j=1}^{J} w_{n,j} \gamma_j^k$$

for $w_{n,j}$, with constraints $w_{n,j} > 0$ and $w_{n+1,j} < w_{n,j}$, for $n = 1, \ldots, N-1$ which follow from the ordering of wait times. We show next that such a solution may be factor as

$$w_{n,j} = a_j p_{n,j}, \tag{3}$$

where $a_j > 0$ and $p_{n,j}$ are the polarization factors

$$p_{n,j} = 1 - \mathrm{e}^{-\frac{T_{W,n}}{T_{1,j}}}. \tag{4}$$

In order to estimate $p_{n,j}$ from $w_{n,j}$, let $\beta_n \in (0,1)$ be

$$\beta_n = T_{W,n+1}/T_{W,n}, \qquad n = 1, \ldots, N-1 \tag{5}$$

and observe that

$$\begin{aligned} \left(1 - \frac{w_{n,j}}{a_j}\right)^{\beta_n} &= \left(1 - p_{n,j}\right)^{\beta_n} = e^{-\frac{\beta_n T_{W,n}}{T_{1,j}}} \\ &= e^{-\frac{T_{W,n+1}}{T_{1,j}}} = 1 - \frac{w_{n+1,j}}{a_j}, \end{aligned} \tag{6}$$

where we have used (3), (4), and (5). We rewrite (6) as

$$0 = y_{n,j}^{\beta_n} - q_{n,j} y_{n,j} + q_{n,j} - 1, \tag{7}$$

where $q_{n,j} = w_{n+1,j}/w_{n,j}$ and $y_{n,j} = 1 - p_{n,j}$ are both in $(0,1)$. Thus, finding the polarization factors $p_{n,j}$ is equivalent to finding zeros of $g(y) = y^{\beta_n} - q_{n,j} y + q_{n,j} - 1$ for $y \in (0,1)$. Note that

$$q_{n,j} = \frac{p_{n+1,j}}{p_{n,j}} = \frac{1 - \mathrm{e}^{-\frac{T_{W,n+1}}{T_{1,j}}}}{1 - \mathrm{e}^{-\frac{T_{W,n}}{T_{1,j}}}} > \frac{T_{W,n+1}}{T_{W,n}} = \beta_n, \tag{8}$$

which, together with $\beta_n \in (0,1)$, implies that g is a strictly concave function on $(0,1)$ which attains its maximum at $Y_{n,j} = \left(\beta_n / q_{n,j}\right)^{\frac{1}{1-\beta_n}} < 1$. Since $g(0) = q_{n,j} - 1 < 0$, $g(1) = 0$, and g is strictly increasing in $(0, Y_{n,j})$, it has exactly one zero in $(0, Y_{n,j})$. Hence, for each n, (7) has a unique solution $y_{n,j}$ and we set $p_{n,j} = 1 - y_{n,j}$. Due to (3), we estimate a_j as a weighted arithmetic mean

$$a_j \approx \sum_{n=1}^{N-1} \frac{w_{n,j}}{p_{n,j}} P_a(n), \tag{9}$$

where the probability measure P_a (see Section III-D) excludes very small values of $p_{n,j}$ generated when $T_{W,n}/T_{1,j}$ is very small. Also, if $T_{W,n}/T_{1,j}$ is large, the polarization factor $p_{n,j}$ is very close to 1 and we can use (3) to directly estimate a_j as $w_{n,j}$.

C. Step 3: Estimating $T_{1,j}$

Using (9), we introduce the new estimate $p_{n,j} = w_{n,j}/a_j$ which, by (4), yields

$$\frac{1}{T_{1,j}} = -\frac{1}{T_{W,n}} \ln\left(1 - \frac{w_{n,j}}{a_j}\right), \tag{10}$$

for each n. Similar to [12], [13], where the expectation of T_1 relaxation times are computed via a harmonic mean based on

TABLE I
MEASUREMENT $(a_j, \chi_j, T_{2,j})$ AND ACQUISITION $(T_E, T_{W,n}, K_n)$ PARAMETERS USED TO GENERATE THE SYNTHETIC DATA.

j	a_j	$\chi_j = T_{1,j}/T_{2,j}$	$T_{2,j}$ (s)
1	0.0411	1.25	0.0224
2	0.0412	1.25	0.0259
3	0.0391	1.25	0.0300
4	0.0011	2	1.1589
5	0.0260	2	1.3413

$T_E = 1$ ms

n	$T_{W,n}$(s)	K_n
1	9.0	1000
2	3.0	1000
3	1.0	1000
4	0.3	300
5	0.1	100
6	0.03	30
7	0.01	10

known distributions of T_2 relaxation times, we estimate the corresponding $T_{1,j}$ as a weighted harmonic mean:

$$T_{1,j} = \left[-\sum_{n=1}^{N} \frac{1}{T_{W,n}} \ln\left(1 - \frac{w_{n,j}}{a_j}\right) P_{T_1}(n) \right]^{-1},$$

for an appropriately chosen probability measure P_{T_1}, which we discuss next.

D. A choice for the probability measures P_a and P_{T_1}

As already pointed out, for long wait times, (4) implies $p_{n,j} \approx 1$ and, hence, $w_{n,j}$ is already a good estimate for a_j. On the other hand, short wait times provide better estimates for $T_{1,j}$. In our numerical examples, we choose a uniform distribution for P_α (α is either a or T_1) defined as

$$P_\alpha(n) = \frac{1}{|I_\alpha|} \sum_{m \in I_\alpha} \delta_{n,m}$$

for some index set $I_\alpha \subset \{1, \ldots, N\}$ having $|I_\alpha|$ number of elements, where $\delta_{m,n}$ is the Kronecker delta function, equal to one when $m = n$ and zero otherwise. I_a contains indices corresponding to long wait times and I_{T_1} indices corresponding to short wait times. In this way we avoid numerical errors that direct use of (10) could cause.

IV. NUMERICAL EXAMPLES

A. Noise-free case

We test the proposed method on noise-free synthetic data generated using (1) with $\epsilon_n(k) = 0$, for all n, k. The acquisition and measurement parameters are listed in Table I. The synthetic data, its approximation, and the logarithm of the absolute error (which is less than 10^{-8}) are displayed in Figure 1. The relative errors of the estimated parameters are listed in Table II.

B. Noisy case

We also test the proposed algorithm on simulated noisy measurements by adding zero-mean Gaussian white noise with a standard deviation of 0.005 to the noise-free synthetic data shown in Figure 1. The noisy measurements, our denoised approximation (with $J = 2$ terms), and their difference are displayed in Figure 2. This difference lies within the noise level.

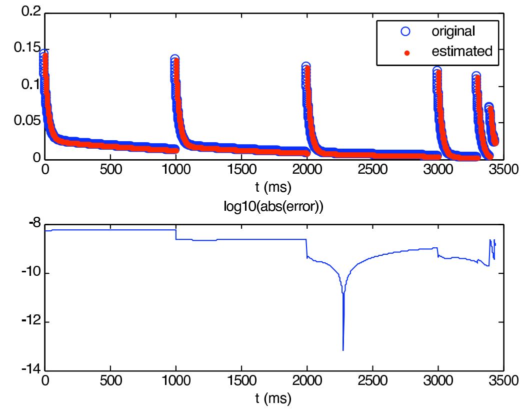

Fig. 1. [Top] Synthetic data (blue) generated using the parameters in Table I and the corresponding estimate using the proposed method (red). [Bottom] Logarithm of the approximation error.

TABLE II
RELATIVE ERROR OF ESTIMATED PARAMETERS

j	$\lvert\tilde{a}_j - a_j\rvert/a_j$	$\lvert\tilde{\chi}_j - \chi_j\rvert/\chi_j$	$\lvert\tilde{T}_{2,j} - T_{2,j}\rvert/T_{2,j}$
1	6.9042×10^{-7}	1.937×10^{-6}	5.0104×10^{-8}
2	1.4627×10^{-6}	2.9483×10^{-6}	2.7475×10^{-7}
3	2.2767×10^{-6}	1.2760×10^{-6}	9.3993×10^{-8}
4	9.3651×10^{-3}	1.2843×10^{-4}	6.4953×10^{-4}
5	4.1239×10^{-4}	6.3996×10^{-6}	3.2651×10^{-5}

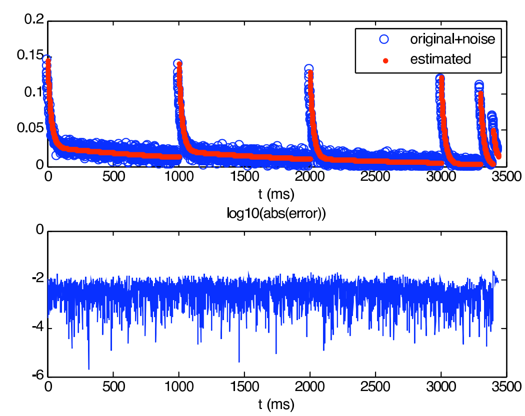

Fig. 2. [Top] Noisy measurement (blue) and denoised approximation (red). [Bottom] The logarithm of the absolute value of the difference between them.

TABLE III
PARAMETERS ESTIMATED FROM NOISY VERSION OF SYNTHETIC DATA PRESENTED IN FIGURE 1.

j	a_j	$\chi_j = T_{1,j}/T_{2,j}$	$T_{2,j}$
1	0.1236	2.2184	0.0255×10^3
2	0.0275	1.3973	1.3685×10^3

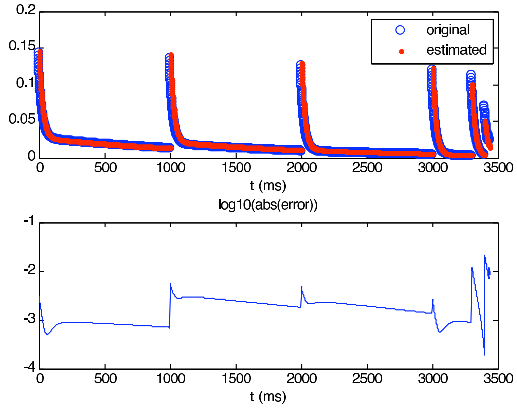

Fig. 3. [Top] Noise-free data (blue) and denoised approximation of the noisy data (red). [Bottom] The logarithm of the absolute value of the difference between them.

TABLE IV
ESTIMATE OF POROSITY, $\Phi = \sum_{j=1}^{J} a_j$, OBTAINED FROM NOISY DATA COMPARED WITH THE EXACT POROSITY OF THE SYNTHETIC NOISE-FREE DATA GENERATED USING THE PARAMETERS IN TABLE 3.

Porosity estimated from noise-free data (Table I)	Porosity estimated from noisy data (Table III)	Relative error
$\Phi_0 = 0.1485$	$\Phi_E = 0.1511$	$\frac{\lvert\Phi_E - \Phi_0\rvert}{\Phi_0} = 0.0175$

V. DISCUSSION

In Figure 3 (top) we superimposed the denoised approximation of the data in Figure 2 with the approximation of the noise-free data (see Figure 1) generated by the parameters listed in Table I. The denoised approximation requires only $J = 2$ terms for the error to stay within the noise level. Furthermore, when we compare the porosity computed using the amplitudes of the noise-free data and the denoised approximation, the relative error is less than 2% (see Table IV).

In practice, linearized inversion methods use between 16 and 32 T_2 relaxation times and between 1 and 5 T_1 relaxation times, hence, the need to compress and transmit up-hole in the range of 16 and 160 amplitudes. In our numerical experiments, we observed that at most 4, if not less, triples (a, T_1, T_2) were sufficient to achieve an approximation within the noise level. Therefore, only 12 values are compressed and transmitted up-hole. Compared to the best linearized inversion scenario, the proposed method provides, at least, a 25% reduction in the number of parameters compressed and transmitted up-hole.

VI. CONCLUSION

We have presented a new, semi-analytic inversion method for nuclear magnetic resonance log data. This method assumes sparsity on the T_2 relaxation times and, consequently, finds a sparse model to represent the data within the noise level. The sparsity assumption eliminates the need for processing parameters present in linearized inversion methods. Because our method is a semi-analytic method, it is potentially more efficient than non-linear optimization-based inversion methods.

The resulting T_1 and T_2 relaxation times and corresponding amplitudes are useful for the estimation of petrophysical properties and for data compression in LWD applications. For LWD applications, our method produces fewer values to be compressed and transmitted up-hole than linearized inversion methods.

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