

# Stochastic Programming for Energy Planning in Microgrids with Renewables

Gabriela Martinez

Dept. of Biological & Environmental Eng.  
Cornell University  
Email: gabriela.martinez@cornell.edu

Nikolaos Gatsis

Dept. of Electrical & Computer Eng.  
The University of Texas at San Antonio  
Email: nikolaos.gatsis@utsa.edu

Georgios B. Giannakis

Dept. of Electrical & Computer Eng.  
University of Minnesota  
Email: georgios@umn.edu

**Abstract**—Energy management for a microgrid featuring distributed generation from conventional and renewable energy sources and adjustable loads is the theme of this paper. The microgrid is connected to the main grid, thus enabling energy import from and export to the main grid. A two-stage stochastic programming formulation is developed, where first-stage decisions are the conventional generation schedules and adjustable load set points, while second-stage decisions include energy transactions with the main grid as well as load adjustments made in real time. The two-stage problem is reformulated as an equivalent linear program. Simulated tests demonstrate the interplay between the various energy management decisions.

## I. INTRODUCTION

Microgrids are small-footprint power systems featuring distributed generation (DG), and electricity end-users. DG refers to small-scale power generators that use conventional fuels and generators relying on renewable energy sources (RES), such as wind or solar energy. Typical microgrid loads include critical non-dispatchable types and also adjustable ones.

DG brings power closer to where it is consumed, thereby incurring fewer thermal losses and bypassing transmission network congestion. Microgrids therefore find their place in diverse infrastructure setups such as campuses, rural areas, islands, or distribution networks serving residential and commercial end-users [1]. Microgrids can be operated connected to or disconnected from the main grid.

In this context, the present paper deals with optimal energy management for both supply and demand of a microgrid incorporating renewable energy. Stochastic programming tools are leveraged to cope with the renewable energy uncertainty, while optimal real-time control of the adjustable load consumption is incorporated. The microgrid is connected to the main grid, and energy can be sold to or purchased from the main grid.

Energy management in microgrids has been addressed in [2] and [3] without taking the renewable energy uncertainty into account. Chance-constrained optimization has proved to be a valuable tool in coping with renewable energy uncertainty as demonstrated in e.g., [4]–[6]; albeit in these works, no real-time decisions regarding energy transactions with the main grid or load adjustments are considered. Recently, a robust energy management model for microgrids in connected

mode has been developed [7], but without real-time load adjustments. Two-stage stochastic programming has also been pursued—see e.g., [8]–[10] and references therein.

This paper develops a two-stage stochastic optimization model to minimize the microgrid operating cost. First-stage decisions yield the conventional generation schedules and adjustable load set points, and are made ahead of the scheduling horizon. Second-stage decisions are taken in real time, and these include energy import from or export to the main grid, as well as load adjustments. The microgrid operating cost comprises the cost of conventional DG offset by the adjustable load utilities at the scheduled set points, and the expected real-time transaction and load adjustment cost.

The stochastic programming model developed here considers the expected transaction cost as opposed to the robust formulation of [7], which deals with the worst-case transaction cost. Moreover, the model in [7] does not consider real-time load adjustments. Different than [8]–[10], the intricacies of the transaction mechanism may introduce non-convexity in the problem. Furthermore, typical two-stage stochastic programming approaches require an accurate renewable energy production forecast for the entire horizon. The present model on the other hand entails second-stage decisions made on a slot-by-slot basis, requiring only a forecast for the current slot.

The remainder of this paper is organized as follows. Section II details the two-stage optimization formulation. The problem is transformed to an equivalent linear program in Section III. Numerical tests are presented in Section IV, and Section V includes pointers to future research directions.

## II. PROBLEM STATEMENT

This section details the two-stage stochastic optimization model for microgrid energy management. The ensuing subsections present the microgrid components and the associated decision variables, followed by the optimization formulation. The scheduling horizon is denoted by  $\mathcal{T}$ , while periods (slots) are generically indexed by  $t$ .

Scheduling decisions are made in two stages. The first stage corresponds to a time ahead of the scheduling horizon, and entails conventional generation schedules and adjustable load set points. The second stage pertains to the real-time operation, and includes energy import from or export to the main grid, as well as load adjustments.

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### A. Conventional distributed generation

Consider a microgrid comprising a set  $\mathcal{M}$  of fuel-consuming DG units generically indexed by  $m$ . Let  $P_{G_m}^t$  be the power generated by the  $m$ th unit at slot  $t$ . The corresponding cost is  $C_m(P_{G_m}^t)$ , and is taken to be convex and strictly increasing. The power output is constrained to be between lower and upper bounds  $P_{G_m}^{\min}$  and  $P_{G_m}^{\max}$ . Ramp-up and ramp-down limits, denoted as  $R_m^{\text{up}}$  and  $R_m^{\text{down}}$ , constrain the power output between successive slots through the inequalities  $P_{G_m}^t - P_{G_m}^{t-1} \leq R_m^{\text{up}}$  and  $P_{G_m}^{t-1} - P_{G_m}^t \leq R_m^{\text{down}}$ . Let vector  $\mathbf{p}_G^t$  collect  $P_{G_m}^t$  for all  $m$ , and  $\mathbf{p}_G$  collect  $\mathbf{p}_G^t$  for all  $t$ .

### B. Renewable distributed generation

The microgrid features a set  $\mathcal{I}$  of renewable DG units. The power output of the  $i$ th such unit at slot  $t$  is random, and is denoted by  $W_i^t$ . With  $\mathbf{w}^t$  collecting  $W_i^t$  for all  $i$ , and  $\mathbf{w}$  collecting  $\mathbf{w}^t$  for all  $t$ , the distribution of  $\mathbf{w}$  is assumed to be known in terms of scenarios, as explained in Section III.

### C. Adjustable and nonadjustable loads

A set  $\mathcal{N}$  of adjustable loads is served by the microgrid energy sources. The load dispatch is taking place in two stages. The first-stage decision is the desirable load set point, which is denoted by  $P_{D_n}^t$  for the  $n$ th load at slot  $t$ . The second-stage decision pertains to the load adjustment  $A_n^t$ , which may be positive or negative. This adjustment depends on the actual renewable energy generated at slot  $t$ , as explained shortly.

The actual consumption at slot  $t$  is given by  $P_{D_n}^t + A_n^t$ , and is constrained to be within lower and upper bounds  $P_{D_n}^{\min}$  and  $P_{D_n}^{\max}$ , respectively. The set point is determined via a utility function  $U_n(P_{D_n}^t)$ , which is selected to be concave and strictly increasing. Negative load adjustments are penalized linearly with a price  $\delta_n^t$ , while positive load adjustments are not penalized, as the latter means more power to the loads.

In addition to the adjustable loads, let  $L^t$  denote a known nonadjustable load at slot  $t$ .

### D. Transaction with the main grid

The difference between the consumed and generated power,  $\sum_n (P_{D_n}^t + A_n^t) + L^t - \sum_m P_{G_m}^t - \sum_i W_i^t$ , signifies the shortage or excess of power at slot  $t$ . The amount of power shortage will be imported from the main grid with known price  $\alpha^t$  at slot  $t$ , while excess power can be exported to the main grid with known price  $\beta^t$ .

### E. Two-stage formulation

Having described the microgrid components, this subsection details the scheduling problem.

1) *Second-stage problem*: The generation schedules  $\mathbf{p}_G$  and load set points  $\mathbf{p}_D$  are considered known in the second stage. At the beginning of the slot  $t$ , an accurate *short-term* prediction of the produced renewable energy  $\mathbf{w}^t$  becomes available. Then, the real-time scheduling problem per  $t$  amounts to minimizing the cost stemming from the load adjustment penalty and the transaction with the main grid:

$$\mathcal{Q}^t(\mathbf{p}_G^t, \mathbf{p}_D^t, \mathbf{w}^t) = \min_{\mathbf{a}^t \in \mathbb{R}^{|\mathcal{N}|}} \delta_n^t [A_n^t]^-$$

$$+ \alpha^t \left[ \sum_{n \in \mathcal{N}} (P_{D_n}^t + A_n^t) + L^t - \sum_{m \in \mathcal{M}} P_{G_m}^t - \sum_{i \in \mathcal{I}} W_i^t \right]^+ - \beta^t \left[ \sum_{n \in \mathcal{N}} (P_{D_n}^t + A_n^t) + L^t - \sum_{m \in \mathcal{M}} P_{G_m}^t - \sum_{i \in \mathcal{I}} W_i^t \right]^- \quad (1a)$$

subject to

$$P_{D_n}^{\min} \leq P_{D_n}^t + A_n^t \leq P_{D_n}^{\max}, \quad n \in \mathcal{N} \quad (1b)$$

where  $[x]^+ := \max\{x, 0\}$  and  $[x]^- := \max\{-x, 0\}$ . The three terms of the objective in (1a) constitute the load adjustment penalty, the energy import cost, and the opposite of the revenue due to energy export, respectively. The optimal value depends on the schedules  $\mathbf{p}_G^t, \mathbf{p}_D^t$ , and the renewable energy realization  $\mathbf{w}^t$ . Convexity properties of problem (1) are addressed next.<sup>1</sup>

**Lemma 1.** *If  $\alpha^t \geq \beta^t$ , then (1) is convex in  $\mathbf{a}^t$ , and the optimal value  $\mathcal{Q}^t(\mathbf{p}_G^t, \mathbf{p}_D^t, \mathbf{w}^t)$  is convex in  $\mathbf{p}_G^t$  and  $\mathbf{p}_D^t$ .*

Lemma 1 asserts that convexity is ensured when the import price  $\alpha^t$  is no less than the export price  $\beta^t$  for all  $t \in \mathcal{T}$ .

2) *First-stage problem*: The first-stage problem optimizes the generation schedules and load set points as follows:

$$\min_{\mathbf{p}_G, \mathbf{p}_D} \sum_{m \in \mathcal{M}} C_m(P_{G_m}^t) - \sum_{n \in \mathcal{N}} U_n(P_{D_n}^t) + \mathbb{E}_{\mathbf{w}} \left[ \sum_{t \in \mathcal{T}} \mathcal{Q}^t(\mathbf{p}_G^t, \mathbf{p}_D^t, \mathbf{w}^t) \right] \quad (2a)$$

subject to

$$P_{G_m}^{\min} \leq P_{G_m}^t \leq P_{G_m}^{\max}, \quad m \in \mathcal{M}, t \in \mathcal{T} \quad (2b)$$

$$P_{G_m}^t - P_{G_m}^{t-1} \leq R_m^{\text{up}}, \quad m \in \mathcal{M}, t \in \mathcal{T} \quad (2c)$$

$$P_{G_m}^{t-1} - P_{G_m}^t \leq R_m^{\text{down}}, \quad m \in \mathcal{M}, t \in \mathcal{T} \quad (2d)$$

$$P_{D_n}^{\min} \leq P_{D_n}^t \leq P_{D_n}^{\max}, \quad n \in \mathcal{N}, t \in \mathcal{T}. \quad (2e)$$

The objective in (2a) includes the generation cost, load utility, and *expected* cost due to second-stage adjustments. Comparing (1) with (2), it is worth noting that the first-stage decisions are valid for all renewable energy output realizations (scenarios), while the second-stage decisions are adaptive to the actual renewable energy output at every slot.

Under the assumptions of Lemma 1, problem (2) is convex, and can be written in a form amenable to numerical solvers as explained in the ensuing subsection.

## III. LINEAR PROGRAM REFORMULATION

For the remainder of this paper, it will be assumed that the costs  $C_m(P_{G_m}^t)$  and the utilities  $U_n(P_{D_n}^t)$  are piecewise linear; both are typical modeling options. Problem (2) can then be conveniently written as a linear program, upon certain modeling simplifications for the distribution of  $\mathbf{w}$ , and introduction of appropriate auxiliary variables.

Specifically, the distribution of  $\mathbf{w}$  can be described by a set of plausible renewable generation output scenarios (realizations). With  $\mathcal{S}$  denoting the set of scenarios, the renewable

<sup>1</sup>Proofs will be included in the journal version of this paper.

energy output  $\{\mathbf{w}^t(s)\}_{t \in \mathcal{T}}$  across the entire horizon and an associated probability  $\pi(s)$  are specified for each  $s \in \mathcal{S}$ . With this modeling choice, the expectation in (2a) can be substituted by the sum  $\sum_s \pi(s) \sum_t \mathcal{Q}^t(\mathbf{p}_G^t, \mathbf{p}_D^t, \mathbf{w}^t(s))$ . There are different methods to obtain sets of scenarios based on physical modeling considerations—see e.g., [11, Ch. 3], [10] and [6] for wind energy.

The next step is to introduce appropriate auxiliary variables. Specifically, variables  $R^{t+}(s)$  and  $R^{t-}(s)$  for  $t \in \mathcal{T}$  and  $s \in \mathcal{S}$  are introduced to represent the energy shortage or excess, as will be seen shortly. Likewise, variables  $A_n^{t+}(s)$  and  $A_n^{t-}(s)$  for  $n \in \mathcal{N}$ ,  $t \in \mathcal{T}$ , and  $s \in \mathcal{S}$  representing the positive and negative load adjustments are also introduced.

Let vectors  $\mathbf{r}^+$ ,  $\mathbf{r}^-$ ,  $\mathbf{a}^+$ ,  $\mathbf{a}^-$  correspondingly collect the previously mentioned auxiliary variables. With the aim of solving (2), the following problem can be formulated:

$$\begin{aligned} \min_{\mathbf{p}_G, \mathbf{p}_D, \mathbf{r}^+, \mathbf{r}^-, \mathbf{a}^+, \mathbf{a}^-} & \sum_{m \in \mathcal{M}} C_m(P_{G_m}^t) - \sum_{n \in \mathcal{N}} U_n(P_{D_n}^t) \\ & + \sum_{s \in \mathcal{S}} \pi(s) \sum_{t \in \mathcal{T}} \left[ \sum_{n \in \mathcal{N}} \delta_n^t A_n^{t-}(s) + \alpha^t R^{t+}(s) - \beta^t R^{t-}(s) \right] \end{aligned} \quad (3a)$$

subj. to

$$P_{G_m}^{\min} \leq P_{G_m}^t \leq P_{G_m}^{\max}, \quad m \in \mathcal{M}, t \in \mathcal{T} \quad (3b)$$

$$P_{G_m}^t - P_{G_m}^{t-1} \leq R_m^{\text{up}}, \quad m \in \mathcal{M}, t \in \mathcal{T} \quad (3c)$$

$$P_{G_m}^{t-1} - P_{G_m}^t \leq R_m^{\text{down}}, \quad m \in \mathcal{M}, t \in \mathcal{T} \quad (3d)$$

$$P_{D_n}^{\min} \leq P_{D_n}^t \leq P_{D_n}^{\max}, \quad n \in \mathcal{N}, t \in \mathcal{T} \quad (3e)$$

$$P_{D_n}^{\min} \leq P_{D_n}^t + A_n^{t+}(s) - A_n^{t-}(s) \leq P_{D_n}^{\max}, \quad n \in \mathcal{N}, t \in \mathcal{T}, s \in \mathcal{S} \quad (3f)$$

$$\begin{aligned} \sum_{n \in \mathcal{N}} (P_{D_n}^t + A_n^{t+}(s) - A_n^{t-}(s)) + L^t - \sum_{m \in \mathcal{M}} P_{G_m}^t - \sum_{i \in \mathcal{I}} W_i^t \\ = R^{t+}(s) - R^{t-}(s), \quad t \in \mathcal{T}, s \in \mathcal{S} \end{aligned} \quad (3g)$$

$$A_n^{t+}(s) \geq 0, A_n^{t-}(s) \geq 0, R^{t+}(s) \geq 0, R^{t-}(s) \geq 0, \quad n \in \mathcal{N}, t \in \mathcal{T}, s \in \mathcal{S}. \quad (3h)$$

The previous formulation includes the first-stage decisions  $\mathbf{p}_G$  and  $\mathbf{p}_D$ , as well as the second-stage decisions for every possible renewable energy production scenario, through the auxiliary variables  $A_n^{t+}(s)$ ,  $A_n^{t-}(s)$ ,  $R^{t+}(s)$ , and  $R^{t-}(s)$ . Specifically, it is apparent from (3f)–(3h) that the load adjustment is written as  $A_n^{t+}(s) - A_n^{t-}(s)$ , while the energy shortage/excess as  $R^{t+}(s) - R^{t-}(s)$ . Notice further that the expectation in (2a) is substituted by an explicit expression based on the objective in (1a).

In order for problem (3) to be equivalent to (2), only one among the  $A_n^{t+}(s)$  and  $A_n^{t-}(s)$  should be allowed to be nonzero, and likewise for  $R^{t+}(s)$ , and  $R^{t-}(s)$ . This property is established in the following lemma for the optimal solution of (3).

**Lemma 2.** *If  $\alpha^t \geq \beta^t$  and  $\alpha^t > 0$  for all  $t \in \mathcal{T}$ , then only one among the optimal  $R^{t+*}(s)$  and  $R^{t-*}(s)$  may be positive, for all  $t \in \mathcal{T}$  and  $s \in \mathcal{S}$ . Likewise, if  $\delta_n^t > 0$  for all  $t \in \mathcal{T}$*

TABLE I  
PARAMETERS OF CONVENTIONAL DG UNITS.

$m$	1	2	3
$P_{G_m}^{\min}$ (kWh)	10	8	15
$P_{G_m}^{\max}$ (kWh)	50	45	70
$a_m$ (cents/kWh)	40	60	20

TABLE II  
PARAMETERS OF ADJUSTABLE LOADS.

$n$	1	2	3	4	5	6
$P_{D_n}^{\min}$ (kWh)	0.5	4	2	5.5	1	7
$P_{D_n}^{\max}$ (kWh)	10	16	15	20	27	32
$b_m$ (cents/kWh)	30	40	50	60	70	80

and  $n \in \mathcal{N}$ , then only one among the optimal  $A_n^{t+*}(s)$  and  $A_n^{t-*}(s)$  may be positive, for all  $n \in \mathcal{N}$ ,  $t \in \mathcal{T}$ , and  $s \in \mathcal{S}$ . If  $\delta_n^t = 0$  for some  $t$  and  $n$ , then from any optimal pair  $(A_n^{t+*}(s), A_n^{t-*}(s))$ , it is possible to recover a pair so that only one of  $A_n^{t+*}(s)$  and  $A_n^{t-*}(s)$  may be positive.

Lemma 2 establishes that the differences  $A_n^{t+}(s) - A_n^{t-}(s)$  and  $R^{t+}(s) - R^{t-}(s)$  yield the optimal load adjustments and energy shortage/excess amounts, rendering problems (3) and (2) equivalent. The advantage is that (3) is a linear program, and can be solved very efficiently. It is also worth emphasizing that (3) not only returns the generation schedules and load set points, but also the real-time decisions for load adjustments and energy import/export as a function of the plausible renewable energy production scenarios.

#### IV. NUMERICAL TESTS

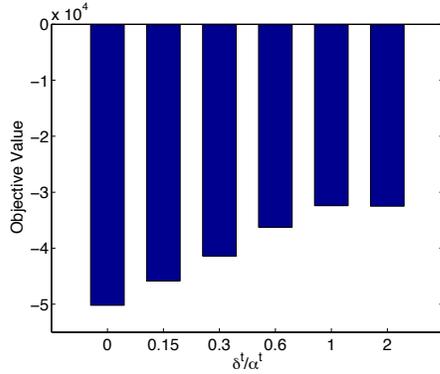
The design is tested on a microgrid with  $|\mathcal{M}| = 3$  conventional DG units,  $|\mathcal{I}| = 4$  wind energy plants, and  $|\mathcal{N}| = 6$  adjustable loads, while the horizon length is  $\mathcal{T} = 8$  hours. The parameters of the conventional DG units are listed in Table I, where the cost has the linear expression  $C_m(P_{G_m}^t) = a_m P_{G_m}^t$ , and the ramp limits for all units are  $R_m^{\text{up}} = R_m^{\text{down}} = 40 \text{ kWh}$ . Table II lists the adjustable load parameters, where the utility function has the form  $U_n(P_{D_n}^t) = b_n P_{D_n}^t$ . All wind plants are rated at 30kWh, and the scenarios  $\{W_i^t(s)\}$  are generated based on the procedure and the parameters detailed in [6].

The import prices and the system base load are chosen similar to [7], and are listed in Table III. The export prices are set to  $\beta^t = 0.3\alpha^t$ , while the adjustment penalties  $\delta_n^t$  will be varied throughout the numerical tests.

Figure 1 depicts the objective value for different values of  $\delta_n^t = \delta^t$  for all  $n$ , and for  $|\mathcal{S}| = 500$  realizations. Clearly, the case  $\delta^t = 0$  allows for adjustment at no cost in real time, and leads to the smallest objective value. As  $\delta^t$  increases, the adjustment becomes costlier, causing the net cost to become larger. The objective takes its highest value—and remains constant at this value—when  $\delta^t \geq \alpha^t$ . The reason is that importing energy is cheaper than real-time load adjustment when  $\delta^t \geq \alpha^t$ . In other words, the case  $\delta^t \geq \alpha^t$  essentially amounts to allowing no adjustment, that is, setting  $A_n^{t+}(s) = A_n^{t-}(s) = 0$  in (3). This intuition is confirmed by solving (3) with the aforementioned additional

TABLE III  
 IMPORT PRICES AND MICROGRID BASE LOAD

Slot $t$	1	2	3	4	5	6	7	8
$\alpha^t$ (cents/kWh)	20.1	22.0	36.2	66.0	58.3	39.9	25.3	23.4
$L^t$ (kWh)	57.8	58.4	64	65.1	61.5	58.8	55.5	51


 Fig. 1. Objective value for different values of  $\delta_n^t$ .

constraint, which yields approximately the same objective value as depicted in the last two bars of Fig. 1.

Table IV lists the objective value achieved when the number of scenarios varies. The first 6 rows correspond to the same adjustment penalty values as in Fig. 1. The 7th row is obtained for random values of  $\delta_n^t / \alpha^t$ , uniform in the interval (0.3,1), and different for all  $n$ . The last row corresponds to the model without adjustment. The objective values for different number of scenarios do not differ by more than 1%.

Fig. 2 shows the individual expected costs comprising the last summand in (3a), obtained with  $|S| = 500$  scenarios. Several observations can be made. The adjustment cost is clearly zero when  $\delta^t = 0$ , and it increases as  $\delta^t$  increases taking values greater than zero. When  $\delta^t$  reaches the value of the import price  $\alpha^t$ , then the adjustment cost drops to nearly zero. This is due to the fact that no adjustment is typically taking place when  $\delta^t \geq \alpha^t$ , as it is cheaper to import energy in this case. Note by the same token that the import cost is close to zero when  $\delta^t < \alpha^t$ . It is also worth noting that significant export is expected to take place when  $\delta^t < \beta^t = 0.3\alpha^t$ . The premise is that in this case the microgrid cost can be lowered by reducing the adjustable loads below their scheduled set points and by exporting the excess energy. The situation is reversed when  $\delta^t \geq \beta^t$ , whereby there is generally no gain from exporting energy resulting from load reduction.

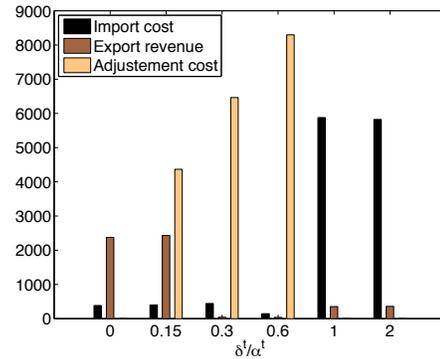
## V. FUTURE DIRECTIONS

This paper developed a two-stage stochastic programming model for energy management in microgrids with DG and controllable loads. First-stage decisions are the conventional DG schedules and load set points, while load adjustments and energy transactions with the main grid take place at the second place in a fashion adaptive to the produced renewable energy.

Distributed storage is a key element of microgrids that adds extra flexibility in the energy management decisions, and will be included in the two-stage stochastic programming

 TABLE IV  
 OBJECTIVE VALUE FOR DIFFERENT NUMBER OF SCENARIOS.

$\delta^t / \alpha^t$	$ S  = 100$	$ S  = 300$	$ S  = 500$
0	-50254.39	-50148.96	-50193.89
0.15	-45987.34	-45662.25	-45836.78
0.3	-41476.50	-41416.10	-41387.92
0.6	-36429.47	-36358.78	-36257.41
1	-32298.52	-32381.26	-32405.08
2	-32779.85	-32355.79	-32498.45
Unif.(0.3, 1)	-37227.35	-37112.69	-37306.73
No adj.	-32484.13	-32235.54	-32473.02


 Fig. 2. Expected costs for different values of  $\delta^t$ .

framework in future work. In addition, developing decentralized solvers that run across the local computation and control modules of the microgrid components—that is, the DG, storage, and adjustable loads—is another future direction.

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