

# Multicast Relay Beamforming through Dual Approach

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**Abstract**—We consider physical layer multicasting in an amplify-and-forward multi-antenna relay network. Assuming each relay antenna has individual power budget, our objective is to design the relay processing matrix to minimize the maximum individual antenna power for a given received SNR target at each destination. As the problem is NP-hard, we propose an approximate solution by solving the problem in the Lagrange dual domain. Through this Lagrange dual approach, we reveal a prominent structure, which enables us to derive a semi-closed form expression for the relay processing matrix that depends on a set of dual variables. These dual variables can be determined through an efficient semi-definite programming formulation. Compared with the traditional semi-definite relaxation (SDR) approach, the proposed solution has much lower computational complexity. Furthermore, it produces the optimal solution if such solution can be extracted from the SDR approach. Thus, the proposed solution can serve as a good alternative to the SDR approach, for the performance and complexity trade-off.

**Index Terms**—Relay Beamforming, Multicast, Amplify-and-Forward, Per-Antenna Power Control

## 1. INTRODUCTION

The next generation relay network needs efficient physical layer multicasting design for some important emerging wireless applications such as real-time video broadcasting. We study the design of amplify-and-forward (AF) multi-antenna relaying in a multicasting scenario, where a source sends common information to a few destination users through the assistance of a relay. A processing matrix is used at the relay to process received signals to forward to all users. The focus of the design is to develop efficient algorithms to determine the relay processing matrix so that good performance at each user can be achieved. In addressing this problem, we impose a more practical constraint that each relay antenna has its own individual power budgets. For a relay equipped with multiple antennas, the constraint reflects the individual RF front-end power amplifier at each antenna; For multiple relays equipped with single antenna to form a virtual multi-antenna system for collaborative processing, individual antenna power budget is particularly more realistic. These per-antenna power constraints render the design problem more challenging than that with the traditional sum-power constraint.

For single pair of source and destination, optimally designing the relay processing matrix has been studied under different performance criteria [1]–[4]. The relay processing design for multiple sources and/or destinations has also been studied in [5]–[7], where numerical methods were proposed to obtain approximate solutions, or suboptimal structure was imposed to simplify the problem. Regardless of single or multiple pairs of source and destinations, most existing designs for multi-antenna relay processing rely on a sum-power constraint among relay antennas, which leads to more analytically tractable problems. When per-antenna power constraints are imposed, existing techniques developed under the sum-power constraint are no longer applicable. For the scenario of a single pair of source and destination, the optimal relay processing matrix under per-antenna power budget is obtained recently in [8].

The problem of physical layer multicast transmit beamforming has been well studied in a direct downlink scenario [9]–[11] under the sum-power constraint. It was shown that the multicasting optimization

problem is in general NP-hard. Thus, the focus is on providing computationally efficient approximate solutions for the problem. A semi-definite relaxation (SDR) approach was proposed for an approximate solution [9]. This approach has been most popular so far in solving the multicast transmit beamforming problem due to its good quality of performance with polynomial time [12]. Similarly, for relay multicasting, the SDR approach can be adopted to solve the problem. In this paper, instead of SDR, we propose an alternative approach to find an approximate solution.

In this work, for the AF multi-antenna relay multicasting, we aim at designing the relay processing matrix to minimize the maximum relay per-antenna power consumption, with SNR target requirement at each destination. We develop an approximate solution for the problem in the Lagrange dual domain. Through a sequence of transformations, we derive a semi-closed form expression for the approximate solution of relay processing matrix, of which a set of dual variables are determined numerically through an efficient semi-definite programming (SDP) formulation. Our semi-closed form solution is obtained with much lower computational complexity, as compared with the SDR approach. The computational efficiency comes from the smaller size of SDP and the semi-closed form solution without any iteration which is needed in the SDR approach. In terms of performance, the solution obtained sometimes is in fact optimal. In this case, both proposed approach and SDR approach produce the optimal solution at the same time. However, unlike the proposed approach, obtaining such optimal solution from the SDR approach is not always straightforward. It requires the knowledge on the existence of the optimal solution and methods to extract it (e.g., [13]). When the solution is non-optimal, the SDR approach tends to have better performance. Thus, both approaches should be considered in generating the solution to achieve overall good performance with high computational efficiency.

**Notations:** The Kronecker product is denoted as  $\otimes$ . Hermitian and transpose are denoted as  $(\cdot)^H$  and  $(\cdot)^T$ , respectively. Conjugate is denoted as  $(\cdot)^*$ .  $\text{vec}(\mathbf{A})$  vectorizes the matrix  $\mathbf{A} = [\mathbf{a}_1, \dots, \mathbf{a}_N]$  to  $[\mathbf{a}_1^T, \dots, \mathbf{a}_N^T]^T$ . The notation  $\mathbf{A} \succeq 0$  means that the matrix  $\mathbf{A}$  is positive semi-definite; and  $\mathbf{a} \succeq 0$  denotes element-wise inequality. The maximum eigenvalue of the matrix  $\mathbf{A}$  is denoted as  $\sigma_{\max}(\mathbf{A})$ . For  $\mathbf{A}$  being positive semi-definite,  $\mathbf{A}^{\frac{1}{2}}$  denotes its square-root with  $\mathbf{A} = \mathbf{A}^{\frac{1}{2}} \mathbf{A}^{\frac{1}{2}}$ .

## 2. PROBLEM FORMULATION

We consider a dual-hop AF multi-antenna relaying system in a multicast scenario where the source transmits common data to  $K$  destination users through a relay equipped with  $N$  antennas. The channel vector between the source and the relay, and the relay and user  $k$  is denoted by  $\mathbf{h}_1 \in \mathbb{C}^{N \times 1}$  and  $\mathbf{h}_{2k} \in \mathbb{C}^{N \times 1}$ , respectively. The signals received at the relay are processed with a relay processing matrix  $\mathbf{W} \in \mathbb{C}^{N \times N}$  and then are forwarded to all users. Let  $\mathbf{y}_r$  be the received signal vector at the relay. The received signal at user  $k$  is given by

$$y_{dk} = \mathbf{h}_{2k}^T (\mathbf{W} \mathbf{y}_r) + n_d = \mathbf{h}_{2k}^T \mathbf{W} \mathbf{h}_1 \sqrt{P_o} s + \mathbf{h}_{2k}^T \mathbf{W} \mathbf{n}_r + n_{dk} \quad (1)$$

where  $s$  is the transmitted signal from the source with unit power  $E|s|^2 = 1$ ,  $P_o$  is the transmit power at the source, and  $n_{dk}$  is the AWGN at user  $k$ 's receiver with variance  $\sigma_d^2$ . The received signal-to-noise ratio (SNR) at user  $k$  is obtained as

$$\text{SNR}_k = \frac{P_o |\mathbf{h}_{2k}^T \mathbf{W} \mathbf{h}_1|^2}{\sigma_r^2 \|\mathbf{h}_{2k}^T \mathbf{W}\|^2 + \sigma_d^2}. \quad (2)$$

With the practical assumption that each transmit antenna at the relay is individual power controlled with its own power budget, the per-antenna power on the output of each transmit antenna at the relay is given by  $E\{|\mathbf{W} \mathbf{y}_r|_i|^2\} = [P_o \mathbf{W} \mathbf{h}_1 \mathbf{h}_1^H \mathbf{W}^H + \sigma_r^2 \mathbf{W} \mathbf{W}^H]_{ii}$ , for  $i = 1, \dots, N$ .

Our objective is to design an optimal  $\mathbf{W}$  at the relay to minimize the relay per-antenna power usage for data forwarding, subject to received SNR targets at each user. Let the received SNR target at user  $k$  be  $\gamma_k$ . We consider minimizing the maximum per-antenna transmit power at the relay subject to constraints on SNR target of each user, given as

$$\min_{\mathbf{W}} \max_{1 \leq i \leq N} [P_o \mathbf{W} \mathbf{h}_1 \mathbf{h}_1^H \mathbf{W}^H + \sigma_r^2 \mathbf{W} \mathbf{W}^H]_{ii} \quad (3)$$

$$\text{s.t. } \text{SNR}_k \geq \gamma_k, \quad \forall k \quad (4)$$

It is straightforward to see that the above min-max power minimization problem is equivalent to the following problem

$$\min_{\mathbf{W}} P_r \quad (5)$$

$$\text{s.t. } \text{SNR}_k \geq \gamma_k, \quad \forall k$$

$$[P_o \mathbf{W} \mathbf{h}_1 \mathbf{h}_1^H \mathbf{W}^H + \sigma_r^2 \mathbf{W} \mathbf{W}^H]_{ii} \leq P_r, \quad \forall i \quad (6)$$

which also corresponds to a common per-antenna power constraint.

We also impose the following assumption of the channel between the source and relays.

A1): The channel between the source and the relay over each antenna is active, i.e.,  $h_{1i} \neq 0, \forall i$ , where  $h_{1i}$  is the  $i$ th element in  $\mathbf{h}_1$ .

The above assumption is very mild and generally holds in practical scenarios, as for a fading channel  $P(h_{1i} = 0) = 0$ .

### 3. MULTICAST RELAY PROCESSING DESIGN

By vectorizing the relay processing matrix  $\mathbf{W}$ , we first transform the received SNR expression in (2) to the following form.

$$\text{SNR}_k = \frac{P_o |\mathbf{g}_k^H \mathbf{w}|^2}{\|\mathbf{R}_{\mathbf{n}k}^{\frac{1}{2}} \mathbf{w}\|^2 + \sigma_d^2} \quad (7)$$

where  $\mathbf{w} \triangleq \text{vec}(\mathbf{W}^H)$ ,  $\mathbf{g}_k \triangleq \mathbf{h}_{2k} \otimes \mathbf{h}_1$ , and  $\mathbf{R}_{\mathbf{n}k} \triangleq \mathbf{h}_{2k} \mathbf{h}_{2k}^H \otimes \mathbf{I} \sigma_r^2$ , for  $k = 1, \dots, K$ , where  $\mathbf{I}$  is an  $N \times N$  identity matrix. Let  $\mathbf{W}^H = [\mathbf{w}_1, \dots, \mathbf{w}_N]$ . Following (7), the constraint (6) can be re-expressed in terms of  $\mathbf{w}_i$ , where

$$[P_o \mathbf{W} \mathbf{h}_1 \mathbf{h}_1^H \mathbf{W}^H + \sigma_r^2 \mathbf{W} \mathbf{W}^H]_{i,i} = \mathbf{w}_i^H (P_o \mathbf{h}_1 \mathbf{h}_1^H + \sigma_r^2 \mathbf{I}) \mathbf{w}_i.$$

#### A. Lagrange Dual Approach

The optimization problem (5) is non-convex, as SNR constraints in (7) are non-convex w.r.t.  $\mathbf{w}$ . In fact, for the direct-link multicast beamforming, it has been shown that the total power minimization problem is NP-hard [9], and the SDR approach is typically used to find the approximate solution. In our problem, we find the

approximate solution in the Lagrange dual domain. With  $\mathbf{w}$ , the Lagrangian for (5) is given as

$$L(P_r, \mathbf{w}, \mathbf{\Lambda}, \boldsymbol{\nu}) = P_r - \sum_{k=1}^K \nu_k \left[ \frac{P_o}{\gamma_k} |\mathbf{w}^H \mathbf{g}_k|^2 - \|\mathbf{R}_{\mathbf{n}k}^{\frac{1}{2}} \mathbf{w}\|^2 - \sigma_d^2 \right] + \sum_{i=1}^N \lambda_i \left[ \mathbf{w}_i^H (P_o \mathbf{h}_1 \mathbf{h}_1^H + \sigma_r^2 \mathbf{I}) \mathbf{w}_i - P_r \right] \quad (8)$$

where  $\mathbf{\Lambda} \triangleq \text{diag}(\lambda_1, \dots, \lambda_N)$  is the diagonal matrix of Lagrange multipliers corresponding to the per-antenna power constraints, and  $\boldsymbol{\nu} = [\nu_1, \dots, \nu_K]^T$  be the vector of Lagrange multipliers associated with the SNR constraints in (4). By the Lagrangian dual approach, we obtain  $\mathbf{w}$  by solving the dual problem

$$\max_{\mathbf{\Lambda}, \boldsymbol{\nu}} \min_{P_r, \mathbf{w}} L(P_r, \mathbf{w}, \mathbf{\Lambda}, \boldsymbol{\nu}) \quad (9)$$

$$\text{s.t. } \mathbf{\Lambda} \succeq 0, \quad \boldsymbol{\nu} \succeq 0. \quad (10)$$

We now show that the solution to the Lagrange dual problem (9) can be obtained by an equivalent optimization problem given in the following theorem.

*Proposition 1:* The Lagrange dual problem associated with the optimization problem (5) is equivalent to the following problem

$$\max_{\mathbf{\Lambda}} \min_{\boldsymbol{\nu}, \mathbf{w}} \sigma_d^2 \sum_{k=1}^K \nu_k \quad (11)$$

$$\text{s.t. } \frac{P_o \sum_{k=1}^K \frac{\nu_k}{\gamma_k} |\mathbf{w}^H \mathbf{g}_k|^2}{\mathbf{w}^H \boldsymbol{\Sigma} \mathbf{w}} \geq 1 \quad (12)$$

$$\text{tr}(\mathbf{\Lambda}) \leq 1, \mathbf{\Lambda} \text{ is diagonal} \quad (13)$$

$$\mathbf{\Lambda} \succeq 0, \boldsymbol{\nu} \succeq 0 \quad (14)$$

where  $\boldsymbol{\Sigma} \triangleq \mathbf{\Lambda} \otimes (P_o \mathbf{h}_1 \mathbf{h}_1^H + \sigma_r^2 \mathbf{I}) + \sum_{k=1}^K \nu_k \mathbf{R}_{\mathbf{n}k}$ .

*Proof:* The Lagrangian in (8) is given by

$$L(P_r, \mathbf{w}, \mathbf{\Lambda}, \boldsymbol{\nu}) = \sigma_d^2 \sum_{k=1}^K \nu_k + P_r [1 - \text{tr}(\mathbf{\Lambda})] + \mathbf{w}^H \left[ \boldsymbol{\Sigma} - P_o \sum_{k=1}^K \frac{\nu_k}{\gamma_k} \mathbf{g}_k \mathbf{g}_k^H \right] \mathbf{w}. \quad (15)$$

Substituting (15) into (9) and solving the inner minimization of the dual problem (9), we have the following equivalent problem

$$\max_{\mathbf{\Lambda}, \boldsymbol{\nu}} \sigma_d^2 \sum_{k=1}^K \nu_k \quad \text{s.t. } (10), (13), \text{ and } \boldsymbol{\Sigma} \succeq P_o \sum_{k=1}^K \frac{\nu_k}{\gamma_k} \mathbf{g}_k \mathbf{g}_k^H. \quad (16)$$

In order to show the equivalence of the problem (11) and (16), we have the following lemmas. The proofs are omitted due to space limitation.

*Lemma 1:* Let  $\mathbf{A}$  and  $\mathbf{B}$  be  $N \times N$  positive definite and positive semi-definite matrices, respectively. Then,

$$\mathbf{A} \succeq \mathbf{B} \Leftrightarrow 1 - \sigma_{\max}(\mathbf{A}^{-\frac{1}{2}} \mathbf{B} \mathbf{A}^{-\frac{1}{2}}) \geq 0 \quad (17)$$

*Lemma 2:* Under the assumption A1, at the optimality of the problem (16), we have the optimal  $\lambda_i^o > 0$ , for all  $i = 1, \dots, N$ .

Using Lemma 2, we have the following.

*Lemma 3:* At optimality of the problem (16),  $\text{rank}(\boldsymbol{\Sigma}) = N^2$ , i.e.,  $\boldsymbol{\Sigma}$  is a positive definite matrix.

Combining Lemma 1 and Lemma 3, the dual problem (16) is now equivalent to

$$\begin{aligned} \max_{\mathbf{\Lambda}, \boldsymbol{\nu}} \quad & \sigma_d^2 \sum_{k=1}^K \nu_k \\ \text{s.t.} \quad & (13), (14), \text{ and } \sigma_{\max} \left( \boldsymbol{\Sigma}^{-\frac{1}{2}} \left[ \sum_{k=1}^K \frac{\nu_k}{\gamma_k} \mathbf{g}_k \mathbf{g}_k^H \right] \boldsymbol{\Sigma}^{-\frac{1}{2}} \right) \leq \frac{1}{P_o}. \end{aligned} \quad (18)$$

To see how the optimization problem (18) and (11) are equivalent, we note that the inner minimization of (11) w.r.t.  $\mathbf{w}$  can be interpreted as the following problem

$$\min_{\mathbf{w}} \sum_{k=1}^K \tilde{P}_k \quad \text{s.t.} \quad \frac{\sum_{k=1}^K \alpha_k \tilde{P}_k |\tilde{\mathbf{w}}^H \mathbf{g}_k|^2}{\tilde{\mathbf{w}}^H \tilde{\boldsymbol{\Sigma}} \tilde{\mathbf{w}}} \geq \kappa \quad (19)$$

where we set  $\tilde{P}_k = \nu_k \sigma_d^2$ ,  $\tilde{\boldsymbol{\Sigma}} = \frac{\sigma_d^2}{P_o} \boldsymbol{\Sigma}$ ,  $\alpha_k = 1/\gamma_k$ , and  $\kappa = 1$ . We can solve (19) through a generalized eigenvalue problem, where  $\tilde{\mathbf{w}}$  has the form

$$\tilde{\mathbf{w}} = \tilde{\boldsymbol{\Sigma}}^{-\frac{1}{2}} \cdot \mathcal{P} \left( \tilde{\boldsymbol{\Sigma}}^{-\frac{1}{2}} \left[ \sum_{k=1}^K \alpha_k \tilde{P}_k \mathbf{g}_k \mathbf{g}_k^H \right] \tilde{\boldsymbol{\Sigma}}^{-\frac{1}{2}} \right) \quad (20)$$

where  $\mathcal{P}(\cdot)$  denotes the principle eigenvector of a matrix<sup>1</sup>. Substituting the above into (11), we have

$$\begin{aligned} \max_{\mathbf{\Lambda}} \min_{\boldsymbol{\nu}} \quad & \sigma_d^2 \sum_{k=1}^K \nu_k \\ \text{s.t.} \quad & (13), (14), \text{ and } \sigma_{\max} \left( \boldsymbol{\Sigma}^{-\frac{1}{2}} \left[ \sum_{k=1}^K \frac{\nu_k}{\gamma_k} \mathbf{g}_k \mathbf{g}_k^H \right] \boldsymbol{\Sigma}^{-\frac{1}{2}} \right) \geq \frac{1}{P_o}. \end{aligned} \quad (21)$$

For any given  $\mathbf{\Lambda}$ , the two optimization problems (18) and (21) are equivalent, because at optimality, they both require the constraints to be met with equality  $\sigma_{\max} \left( \boldsymbol{\Sigma}^{-\frac{1}{2}} \left[ \sum_{k=1}^K \frac{\nu_k}{\gamma_k} \mathbf{g}_k \mathbf{g}_k^H \right] \boldsymbol{\Sigma}^{-\frac{1}{2}} \right) = \frac{1}{P_o}$ , where the optimal  $\boldsymbol{\nu}$  is the root of this equation. ■

Assuming the optimization problem (5) is feasible, we obtain  $\mathbf{w}$  following (20) in Proposition 1, up to an arbitrary scale factor  $\beta$ , i.e.,

$$\mathbf{w} = \beta \boldsymbol{\Sigma}^{\circ-\frac{1}{2}} \mathbf{u}^o \quad (22)$$

where  $\mathbf{u}^o \triangleq \mathcal{P} \left( \boldsymbol{\Sigma}^{\circ-\frac{1}{2}} \left[ \sum_{k=1}^K \frac{\nu_k^o}{\gamma_k} \mathbf{g}_k \mathbf{g}_k^H \right] \boldsymbol{\Sigma}^{\circ-\frac{1}{2}} \right)$ , and  $\boldsymbol{\Sigma}^o$  being under the optimal  $\mathbf{\Lambda}^o, \boldsymbol{\nu}^o$  of the problem (16). The value of  $\beta$  is determined to ensure that all the SNR target constraints (4) are met. It follows that

$$|\beta| = \sigma_d / \sqrt{\min_{1 \leq k \leq K} \left( \frac{P_o}{\gamma_k} \left| \mathbf{u}^{oH} \boldsymbol{\Sigma}^{\circ-\frac{1}{2}} \mathbf{g}_k \right|^2 - \|\mathbf{R}_{nk}^{-\frac{1}{2}} \boldsymbol{\Sigma}^{\circ-\frac{1}{2}} \mathbf{u}^o\|^2 \right)}.$$

Note that an arbitrary phase rotation in  $\mathbf{w}$  does not affect the SNR value. Thus, without loss of generality, we simply set  $\beta = |\beta|$ .

Finally, we need to obtain the optimal solutions  $(\mathbf{\Lambda}^o, \boldsymbol{\nu}^o)$  to determine  $\mathbf{w}$  in (22). They are obtained by solving the optimization problem (16), which is a dual SDP problem. To see this, we reformulate (16) as the following SDP problem

$$\min_{\mathbf{x}} \quad \boldsymbol{\sigma}^T \mathbf{x} \quad \text{s.t.} \quad \mathbf{b}^T \mathbf{x} - 1 \leq 0, \mathbf{x} \succeq 0, \sum_{i=1}^{N+K} x_i \mathbf{G}_i \succeq 0 \quad (23)$$

where  $\boldsymbol{\sigma} \triangleq [\mathbf{0}_{N \times 1}^T, -\sigma_d^2 \mathbf{1}_{K \times 1}^T]^T$ ,  $\mathbf{b} \triangleq [\mathbf{1}_{N \times 1}^T, \mathbf{0}_{K \times 1}^T]^T$ ,  $\mathbf{x} = [x_1, \dots, x_{N+K}]^T \triangleq [\lambda_1, \dots, \lambda_N, \nu_1, \dots, \nu_K]^T$ , and  $\mathbf{G}_i$  is an  $N \times N$  block diagonal matrix, for  $i = 1, \dots, N$ , with the  $i$ th diagonal block

<sup>1</sup>Although there exist other optimal solutions of  $\tilde{\mathbf{w}}^o$ , the solution in (20) leads to the maximum eigenvalue of a Hermitian matrix

being  $(P_o \mathbf{h}_1 \mathbf{h}_1^H + \sigma_r^2 \mathbf{I})$  and all other  $(N-1)$  diagonal blocks being  $\mathbf{0}_{N \times N}$ , and  $\mathbf{G}_{N+k} \triangleq \mathbf{R}_{nk} - \frac{P_o}{\gamma_k} \mathbf{g}_k \mathbf{g}_k^H$ , for  $k = 1, \dots, K$ .

It is known that the SDP algorithm has a polynomial worst-case complexity, and performs very well in practice [14]. It can be efficiently implemented through the interior point method [15].

#### B. Comparison with the SDR Approach

As mentioned earlier, we can also use the SDR approach to find the approximate solution to the optimization problem (5). In this approach, the min-max power minimization problem can be first formulated as

$$\begin{aligned} \min_{\mathbf{X}} \quad & P_r \\ \text{s.t.} \quad & \text{tr}(\mathbf{G}_i \mathbf{X}) \leq P_r, \forall i, \text{tr} \left( \left( \frac{P_o}{\gamma_k} \mathbf{g}_k \mathbf{g}_k^H - \mathbf{R}_{nk} \right) \mathbf{X} \right) \geq \sigma_d^2, \forall k \\ & \text{rank}(\mathbf{X}) = 1, \quad \mathbf{X} \succeq 0 \end{aligned} \quad (24)$$

where  $\mathbf{X} = \mathbf{w} \mathbf{w}^H$ . The above problem can be relaxed to an SDP problem by removing the rank-1 constraint on  $\mathbf{X}$ . The problem is solved using a bi-section search as an outer loop over an SDP feasibility problem. Then,  $\mathbf{w}$  is extracted from  $\mathbf{X}$  through some randomization method.

Note that each approach formulates the original problem into an SDP. However, the computational complexity of the proposed dual approach is much lower. This is reflected in two aspects:

- 1) The SDP problem in the SDR approach has  $N^2$  variables and  $N + K$  constraints with complexity per iteration of  $\mathcal{O}((N^2)^2 N^2 (N + K))$ , while the SDP problem in the dual approach only has  $N + K$  variables and three constraints with complexity per iteration of  $\mathcal{O}((N + K)^2 (N^2)^2)$ . Therefore, the SDP problem in the dual approach has smaller size and lower complexity to compute when  $N^2 > (N + K)$ .
- 2) In the dual approach,  $\mathbf{w}$  is directly obtained through the semi-closed form solution (22) and only needs to solve a single SDP problem (23). In the SDR approach, a bi-section search over the SDP feasibility test is conducted which needs multiple SDP feasibility tests due to iterations. In addition, if  $\mathbf{X}$  is not rank-1, a randomization procedure is required to obtain  $\mathbf{w}$ .

Note that the dual problem (9) and the relaxed SDP problem of (24) produce the same lower bound of the original optimization problem (5)<sup>2</sup>. There are literature works discussing when the rank-1 solution for  $\mathbf{X}$  exists in a relaxed SDP problem (i.e., generating the optimal solution for the original problem) under certain problem settings, and how to obtain it. Due to the relation of the two approaches, when a rank-1 solution can be obtained, the strong duality holds for the original optimization problem, and both approaches will obtain an optimal solution. Thus, the advantage of the dual approach is that the optimal solution can be directly obtained via the semi-closed form with significantly lower computational complexity.

#### 4. NUMERICAL RESULTS

To study the multicast relay beamforming performance using the proposed approach, we assume the noise powers at the relay and at the destination are set as  $\sigma_r^2 = \sigma_d^2 = 0.1W$ . The source transmitted power  $P_0$  is set to be 10dB above  $\sigma_r^2$ . The entries of  $\mathbf{h}_1$  and  $\mathbf{h}_{2k}$  are assumed i.i.d. zero-mean Gaussian with variance 1, and  $\{\mathbf{h}_{2k}\}$  are i.i.d.. All users have the same SNR target  $\gamma_k = \gamma_o$ , for  $k = 1, \dots, K$ . The results are generated over 1000 Monte Carlo runs.

<sup>2</sup>It is known that the two approaches have the same lower bound because the SDP relaxation problem is essentially the bi-dual of the original optimization problem [12]

We first compare the multicast relay beamforming performance under the dual approach and SDR approach. We investigate the gap between the power obtained from the primal approximate solution and that of the dual problem. Let  $g(\mathbf{w})$  be the optimal objective of the dual problem (9) where  $\mathbf{w}$  is obtained by (22). It is a lower bound for the primal problem (5). Let  $J(\mathbf{w})$  be the objective of the primal problem (5) under  $\mathbf{w}$ . Define the gap ratio  $G_{\text{gap}} = \frac{J(\mathbf{w})}{g(\mathbf{w})}$ . When  $G_{\text{gap}}[\text{dB}] = 0$ , the solution is optimal, i.e.,  $\mathbf{w}^{\text{opt}} = \mathbf{w}$ . Similarly, in the SDR approach, we can compute  $G_{\text{gap}}$  between the the approximate solution and the lower bound obtained from the relaxed SDP problem. Fig. 1 shows the CDF of  $G_{\text{gap}}$  under both dual and SDR approaches. As we see, the cases for  $G_{\text{gap}} = 0\text{dB}$  in both approaches is identical, verifying that the optimal solutions are produced by both approaches. When there is a gap, the SDR approach provides smaller gap than that of the dual approach. Table I shows the average processing time in each approach for  $K = 2, 4, 8$  and  $N = 2$  in simulations. As we see, the average computation time for the dual approach remains roughly unchanged, and that for the SDR approach increases with  $K$  more noticeably. The average computation time of the SDR approach is 15-26 times more than that of the dual approach for  $K$  ranges from 2 to 8. Thus, to achieve better performance while being computationally efficient, we should consider both approaches as candidates. Since the computational complexity in the dual approach is much lower than that of the SDR approach, we can use it as the first candidate to produce the approximate solution, and only use the SDR approach when the gap is significant.

We also show the performance under different  $N$  and  $K$ . Fig. 2:Left shows the average per-antenna power usage (normalized against noise  $\frac{1}{N} \sum_i P_i / \sigma_d^2$ ) for different required SNR  $\gamma_o$  with  $N = 2, 4, 6$ , for  $K = 2$ . Substantial reduction of power usage at each antenna is observed as the number of antennas increases. Fig. 2:Right shows the average per-antenna power usage vs. SNR target  $\gamma_o$  with  $K = 1, 2, 4$  for  $N = 2$ . As the number of users  $K$  increases, the average per-antenna power required to meet the SNR target also increases significantly.

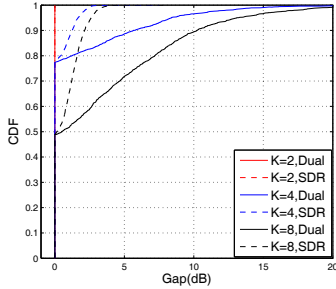


Fig. 1: CDF of  $G_{\text{gap}}$  ( $\gamma_o = 4\text{dB}$ ,  $N = 2$ ).

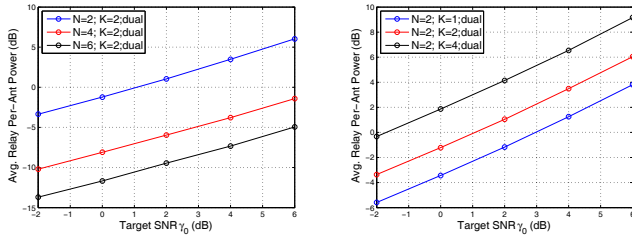


Fig. 2: Left: Average relay antenna power vs.  $\gamma_o$  ( $N = 2, 4, 6$ ,  $K = 2$ ); Right: Average relay antenna power vs.  $\gamma_o$  ( $K = 1, 2, 4$ ,  $N = 2$ ).

$K$	Dual Approach (sec)	SDR Approach (sec)
2	0.13	2.0
4	0.15	3.6
8	0.17	4.5

TABLE I: Average Processing Time ( $\gamma_o = 4\text{dB}$ ,  $N = 2$ )

## 5. CONCLUSION

We have considered a multicast AF relaying scenario with multiple destination users, where we designed the multi-antenna relay processing matrix under the per-antenna power budget. Since the optimization problem is NP-hard, we have obtained an approximate solution for the problem in the Lagrange dual domain. The Lagrange dual approach enables us to obtain a semi-closed form solution of the relay processing matrix, where an efficient SDP formulation is formed to determine the parameters in the semi-closed form solution. Compared with the traditional SDR approach, the proposed method has much lower computational complexity. When the optimal solution can be obtained in either approach, both approaches obtain such solution at the same time. Simulation cases show that the produced solution is optimal for a high percentage of time, and the dual approach can be a good alternative approach considered together with the SDR approach to produce the approximate solution.

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