Primary Receiver Localization Using Sparsity and Interference Tweets

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***Abstract*—A hierarchical access setup is considered, where secondary users can (re-)use frequency bands allocated to licensed systems, provided ongoing primary communications are not overly disrupted. Since conventional spectrum sensing schemes can detect and localize "active" sources but not "passive" users, the number of primary *receivers* and their locations are generally unknown. Supposing a minimal coordination between primary and secondary systems, a novel method for unveiling areas where primary receivers are located is proposed in this paper. The primary system broadcasts short messages - here refereed to as "interference tweets" - indicating the number of receivers that are interfered. Using these tweets, together with a grid-based discretization of the primary coverage region, the locations where receivers are likely to reside are obtained by solving a sparse linear regression problem. Subsequently, the estimated locations are used to optimize resource allocation of the secondary network operation under interference constraints.**

***Index Terms*—Cognitive radios, underlay access, receiver localization, sparsity.**

I. Introduction

Cognitive radio (CR) technologies hold significant potential to address spectrum scarcity for wireless networks, which is primarily due to rigid and exclusive spectrum licensing policies. Using sophisticated sensing and resource allocation (RA) techniques, secondary users (SUs) can (re-)use spectral resources allocated to licensed systems, provided no disruptive interference is caused (or inflicted) to primary users (PUs) [1].

As in conventional wireless setups, acquiring the (statistics of) SU-to-PU channel gains is key for protecting PUs from co-channel interference [2]–[4]. However, since PUs have generally no incentive to use spectral resources to exchange channel training signals with SUs [1], training-based channel estimation cannot be employed to estimate the SU-to-PU channels. Furthermore, since conventional spectrum sensing techniques can detect and localize active PU *sources*, but not *passive* users, even the locations of PU receivers are generally unknown. Uncertainty in the PU receiver locations translates here into imperfect knowledge of the channel distributions.

A novel method for unveiling areas where PU receivers are located is proposed in this paper, based on a minimal information exchange between PU and SU systems. Specifically, it is assumed that the PU system broadcasts short messages – referred to as *interference tweets* – to indicate the number of interfered PUs. Although most existing CR sensing approaches assume no collaboration from the PUs [1], it has been demonstrated that this minimal SU-PU interaction can yield major improvements in spectrum (re)use efficiency [5]. The interference tweets are exploited to estimate the average rate of interference per SU. With these quantities, and upon employing a grid to discretize the primary coverage region, the localization task is cast as a sparse linear regression problem, where the vector of unknowns collects the probabilities of a PU receiver being located at a grid point. A similar setup was recently considered in [5], where a Bayesian estimator was used to estimate the receiver locations. Compared to [5], the information carried by the tweet is different, and the number of PU receivers is not known a priori. Since the optimum Bayesian recursive estimator designed in [5] is not tractable for the present setup, developing low-complexity receiver-map estimators is well motivated.

The effectiveness of the proposed PU receiver localization scheme is assessed for an underlay CR network where SU transmissions are scheduled based on *i)* SU-to-SU instantaneous channels; *ii)* the received interference tweets; and, *iii)* the PU receiver map obtained through the proposed localization method.[1]

II. Preliminaries and System Model

Consider a secondary network with M nodes $\{U_m\}_{m=1}^{M}$ deployed over an area $\mathcal{A} \subset \mathbb{R}^2$, and assume that SUs share a flat-fading frequency channel with an incumbent PU system in an *underlay* setup [1]. Based on the output of the spectrum sensing stage, SUs implement adaptive RA to maximize network performance, while protecting the PUs from excessive interference; see e.g., [2]–[5]. Secondary transmissions are assumed orthogonal, and a binary scheduling variable $w_m(t)$ is used to indicate whether SU U_m is scheduled to transmit at time t ($w_m(t) = 1$), or not ($w_m(t) = 0$). If active at time t, SU U_m loads a power $p_m(t)$, which, for simplicity, is constrained to belong to a finite set $\mathcal{P}_m := \{p_m^l\}_{l=1}^{L}$ (discrete modes).

Suppose that PU transmitters communicate with Q PU *receivers* geolocated at $\{\mathbf{x}^q \in \mathcal{A}\}_{q=1}^{Q}$, and let $h_{m,\mathbf{x}^q}$ denote the *instantaneous* channel gain between U_m and position $\mathbf{x}^q$. Fading processes $\{h_{m,\mathbf{x}^q}\}$ are mutually independent, and

[1] *Notation:* $(\cdot)^{\mathcal{T}}$ stands for transposition; $\mathbb{E}_{\mathbf{g}}[\cdot]$ denotes expectation with respect to the random process $\mathbf{g}$; $\Pr\{A\}$ the probability of event A; x^* the optimal value of x; $\mathbb{I}_{\{\cdot\}}$ the indicator function ($\mathbb{I}_{\{x\}} = 1$ if x is true, and zero otherwise); and, $[x]_a^b$ the projection of the scalar x onto $[a, b]$; that is, $[x]_a^b := \min\{\max\{x, a\}, b\}$.

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their distributions are assumed known to the SU network. Thus, given the maximum instantaneous interference power I tolerable by the PUs, the secondary network can determine the interference probabilities at each location $\mathbf{x}^q$. In case of Rayleigh fading, one has that $\Pr\{p_m(t)\, h_{m,\mathbf{x}^q} > I | w_m(t) = 1\} = e^{-I/(p(t)\gamma_{m,\mathbf{x}^q})}$, where $\gamma_{m,\mathbf{x}^q} := \mathbb{E}_h[h_{m,\mathbf{x}^q}]$ is the *average* path loss between U_m and PU receiver q.

Let $z^q_{\mathbf{x}}$ be a binary variable indicating whether PU receiver q is located at $\mathbf{x} \in \mathcal{A}$, and consider discretizing the PU coverage region to obtain a set of G (possibly regularly spaced) grid points at known locations $\mathcal{G} := \{\mathbf{g}_i\}_{i=1}^G$ [5]. Clearly, if variables $\{z^q_{\mathbf{x}}\}$ were known, and PU q was located at one of the grid points, the probability of SU U_m interfering PU receiver q would be $\sum_{i=1}^G e^{-I/(p_m(t)\gamma_{m,\mathbf{g}_i})} z^q_{\mathbf{g}_i}$. To account for uncertain locations, the probabilities (beliefs) $\Pr\{z^q_{\mathbf{g}_i} = 1\}$ are considered. Those will be also referred to as *receiver maps*. Moreover, to account for the PU receiver's presence off the preselected grid points a "spill over" region collecting grid points around the actual location of PU q [6] is introduced. With $\mathcal{G}^{(r)}$ denoting the grid points in this spill over region, it readily follows that $\Pr\{z^q_{\mathbf{g}_i} = 1\} \geq 0$ for $\mathbf{g}_i \in \mathcal{G}^{(r)}$, and $\Pr\{z^q_{\mathbf{g}_i} = 1\} = 0$ otherwise [6]. Using these notational conventions, we proceed to formulate our linear observation model. With i^q denoting a binary variable taking the value 1 if PU receiver q is interfered, and supposing that SU U_m is scheduled to transmit with power $p_m(t) = p^l_m$, define the coefficient $\phi_{m,l,\mathbf{g}_i} := e^{-\frac{I}{p^l_m \gamma_{m,\mathbf{g}_i}}}$. Then, the probability of interfering PU receiver q is $y^q_{m,l} := \Pr\{i^q = 1 | w_m(t) = 1, p_m(t) = p^l_m\} = \sum_{i=1}^G \phi_{m,l,\mathbf{g}_i} \Pr\{z^q_{\mathbf{g}_i} = 1\}$. Upon marginalizing the transmit power, let us define $y^q_m := \Pr\{i^q = 1 | w_m(t) = 1\}$ and, summing across PUs, $y_m := \sum_q y^q_m$. The former can be viewed as the long-term probability of SU m interfering PU q, while the latter can be viewed as *the average number of PUs the SU m interferes*. Considering now the coefficients $\phi_{m,\mathbf{g}_i} := \sum_{p \in \mathcal{P}_m} \phi_{m,l,\mathbf{g}_i} \Pr\{p_m = p\}$, variables y_m can be written as a function of the beliefs via the *linear* model $y_m = \sum_{i=1}^G \sum_{q=1}^Q \phi_{m,\mathbf{g}_i} \Pr\{z^q_{\mathbf{g}_i} = 1\}$.

The next section will leverage the linear relationship and the values (estimates) of y_m and $\phi_{m,\mathbf{g}_i}$ to obtain the receiver maps. A simple alternative to acquire $\phi_{m,\mathbf{g}_i}$, is to run the following *online* averaging

$$\hat{\phi}_{m,\mathbf{g}_i}(t) := \left(\sum_{\tau=1}^t w_m(\tau)\right)^{-1} \sum_{\tau=1}^t w_m(\tau) e^{-\frac{I}{p_m(\tau)\gamma_{m,\mathbf{g}_i}}}. \quad (1)$$

Similarly, the value of y_m can be acquired by averaging the interference tweets (details will be given in the next section). In both cases, an exponentially weighted moving average (EWMA) strategy could be employed to track slow time-varying channel and power statistics.

III. Sparsity-aware receiver localization

The objective is to develop an estimator for the receiver maps (beliefs), based on a *minimal* interplay between PU and SU systems. Towards this end, the grid $\mathcal{G}$ will be partitioned into R clusters $\{\mathcal{G}^{(r)}\}_{r=1}^R$, satisfying the following conditions: *i)* $\bigcup_{r=1}^R \mathcal{G}^{(r)} = \mathcal{G}$; and, *ii)* $\mathcal{G}^{(r)} \bigcap \mathcal{G}^{(s)} = \emptyset$ for all $r \neq s$. Two main assumptions are considered for designing the estimator:

(as1) At each time instant t, the PU system broadcasts the message $o(t) \in \mathbb{N}\backslash\{0\}$ indicating the number of PU receivers that were interfered (if any).

Different from [5], neither the number of PUs nor the specific PU receivers that were interfered are assumed known.

(as2) At most one PU receiver is located within area $\mathcal{G}^{(r)}$.

In practice, *(as2)* can be easily satisfied by selecting a dense grid, and sufficiently small clusters $\{\mathcal{G}^{(r)}\}_{r=1}^R$ [6], [7]. The premise for this partition is that $Q \leq R$, with the clusters $\{\mathcal{G}^{(r)}\}_{r=1}^R$ representing the "spill over" regions mentioned in Section II (see also [6]). Notice further that clusters were supposed non-overlapping. A scenario with overlapping clusters does not require major changes in the formulation, but it substantially increases the computational complexity of the localization algorithms; see Section IV. This issue will be the subject of future research.

Under these assumptions, the problem of acquiring $\Pr\{z^q(\mathbf{g}_i) = 1\}$ is not identifiable (the user index is never revealed). For this reason, the alternative receiver maps $\Pr\{z^{(r)}(\mathbf{g}_i) = 1\}$ are considered. Those represent the probability that a PU receiver is located at location $\mathbf{g}_i$, with $\mathbf{g}_i \in \mathcal{G}^{(r)}$. Note that if a PU receiver is actually located within the area $\mathcal{G}^{(r)}$, then $\Pr\{z^{(r)}(\mathbf{g}_i) = 1\} > 0$ for (some of) the points $\mathbf{g}_i \in \mathcal{G}^{(r)}$. If no PUs are located within $\mathcal{G}^{(r)}$, then $\Pr\{z^{(r)}(\mathbf{g}_i) = 1\} = 0$, for all $\mathbf{g}_i \in \mathcal{G}^{(r)}$.

Using (as2), y_m can be written as a linear function of the new probabilities $\Pr\{z^{(r)}(\mathbf{g}_i) = 1\}$ via

$$y_m = \sum_{r=1}^R \sum_{i | \mathbf{g}_i \in \mathcal{G}^{(r)}} \phi_{m,\mathbf{g}_i} \Pr\{z^{(r)}(\mathbf{g}_i) = 1\} \quad (2)$$

The values of $\phi_{m,\mathbf{g}_i}$ can be acquired using (1). Similarly, leveraging (as1), SU U_m can utilize the accumulated interference tweets $\{o(\tau)\}_{\tau=1}^t$ to quantify the average number of PU users that were interfered by its transmissions; that is,

$$\hat{y}_m(t) := \left(\sum_{\tau=1}^t w_m(\tau)\right)^{-1} \sum_{\tau=1}^t o(\tau) w_m(\tau). \quad (3)$$

The idea now is to use the values of $\hat{y}_m(t)$ and $\hat{\phi}_{m,\mathbf{g}_i}(t)$ to estimate, at time t, the receiver maps. To this end, let $\boldsymbol{\beta}^{(r)}$ be a $|\mathcal{G}^{(r)}| \times 1$ vector collecting the beliefs $\Pr\{z^{(r)}(\mathbf{g}_i) = 1\}$ for $\mathbf{g}_i \in \mathcal{G}^{(r)}$; $\hat{\boldsymbol{\phi}}^{(r)}_m(t)$ a vector collecting the regressors $\hat{\phi}_{m,\mathbf{g}_i}(t)$ for $\mathbf{g}_i \in \mathcal{G}^{(r)}$ [cf. (1)]; and define vectors $\boldsymbol{\beta} := [(\boldsymbol{\beta}^{(1)})^{\mathcal{T}}, \ldots, (\boldsymbol{\beta}^{(R)})^{\mathcal{T}}]^{\mathcal{T}}$ and $\hat{\boldsymbol{\phi}}_m(t) := [(\hat{\boldsymbol{\phi}}^{(1)}_m(t))^{\mathcal{T}}, \ldots, (\boldsymbol{\phi}^{(R)}_m(t))^{\mathcal{T}}]^{\mathcal{T}}$. With these notational conventions, (2) can be written as $\hat{y}_m(t) = \sum_{r=1}^R (\hat{\boldsymbol{\phi}}^{(r)}_m(t))^{\mathcal{T}} \boldsymbol{\beta}^{(r)} = (\hat{\boldsymbol{\phi}}_m(t))^{\mathcal{T}} \boldsymbol{\beta} + e_m(t)$, where $e_m(t)$ stands for the errors in the estimate of the average interference rates $\{\hat{y}_m(t)\}$. Upon defining vectors $\hat{\mathbf{y}}(t) := [\hat{y}_1(t), \ldots, \hat{y}_M(t)]^{\mathcal{T}}$ and $\mathbf{e}(t) := [e_1(t), \ldots, e_M(t)]^{\mathcal{T}}$ and matrix $\hat{\boldsymbol{\Phi}}(t) := [\boldsymbol{\phi}_1(t), \ldots, \boldsymbol{\phi}_M(t)]^{\mathcal{T}}$, the following linear model holds[2]

$$\hat{\mathbf{y}}(t) = \hat{\mathbf{\Phi}}(t)\boldsymbol{\beta} + \mathbf{e}(t). \quad (4)$$

For dense grids, one has that $M \ll G$, and thus (4) is an under-determined system of linear equations. Nevertheless, since the number of grid points G can be much higher than the PU receivers (i.e., $R \ll G$), the vector of unknowns $\boldsymbol{\beta}$ is inherently *sparse*. Specifically, *group* sparsity [8] emerges from the employed grid-based model, since the entries of a sub-vector $\boldsymbol{\beta}^{(r)}$ are all zeros if no PU receivers are located within area $\mathcal{G}^{(r)}$; and, $\boldsymbol{\beta}^{(r)} \neq \mathbf{0}$ otherwise. Though, using a sufficiently dense grid, a PU receiver may be located in proximity to a point $\mathbf{g}_i \subset \mathcal{G}^{(r)}$; this implies that sparsity emerges also at an entry-level, since only a few of the entries in $\boldsymbol{\beta}^{(r)}$ may be nonzero.

The so-called sparse group Lasso (SG-Lasso) [9], [10] provides a parsimonious model estimate, where sparsity is accounted for both at the group- and at the single-coefficient levels. To this end, the conventional least-squares (LS) cost is regularized with the sparsity-promoting terms $\lambda_1\|\boldsymbol{\beta}\|_1$ and $g_{\lambda_2}(\boldsymbol{\beta}) := \lambda_2 \sum_{r=1}^{R} \|\boldsymbol{\beta}^{(r)}\|_2$, where λ_1 and λ_2 are tuning parameters. Thus, taking also into account the non-negativity of $\boldsymbol{\beta}$, the vector of beliefs can be estimated by solving the following sparse linear regression problem[3]:

$$\hat{\boldsymbol{\beta}}(t) := \arg\min_{\boldsymbol{\beta}\succeq\mathbf{0}} \frac{1}{2}\left\|\hat{\mathbf{y}}(t) - \hat{\mathbf{\Phi}}(t)\boldsymbol{\beta}\right\|_2^2 + g_{\lambda_2}(\boldsymbol{\beta}) + \lambda_1 \mathbf{1}_G^{\mathcal{T}}\boldsymbol{\beta}. \quad (5)$$

Given $\hat{\boldsymbol{\beta}}(t)$, the number of PU receivers is given by $\hat{Q} = |\{r : \hat{\beta}_i^{(r)} > 0\}|$, whereas their locations are estimated as [6]:

$$\hat{\mathbf{x}}^{(r)}(t) = \frac{\sum_{i|\mathbf{g}_i\in\mathcal{G}^{(r)}} \mathbf{g}_i \hat{\beta}_i^{(r)}(t)}{\sum_{i|\mathbf{g}_i\in\mathcal{G}^{(r)}} \hat{\beta}_i^{(r)}(t)}. \quad (6)$$

In principle, a constraint $\mathbf{1}_{|\mathcal{G}^{(r)}|}^{\mathcal{T}}\boldsymbol{\beta}^{(r)} \leq 1$ could be added per cluster r, to ensure that the probability of a PU receiver being within $\mathcal{G}^{(r)}$ does not exceed 1. However, the very same effect can be obtained by properly adjusting λ_1, or by replacing $\lambda_1\mathbf{1}_G^{\mathcal{T}}\boldsymbol{\beta}$ with its weighted counterpart $\sum_r \lambda_{1,r}\mathbf{1}_{|\mathcal{G}^{(r)}|}^{\mathcal{T}}\boldsymbol{\beta}^{(r)}$ for a set of properly chosen coefficients $\{\lambda_{1,r}\}$ [12, pp. 241–249].

Problem (5) can be conveniently re-formulated as a second order cone program (SOCP), and thus efficiently solved via standard interior point methods. However, a reduced-complexity algorithm attaining the optimal solution of SG-Lasso problem will be developed in the ensuing section, using the Alternating Direction Method of Multipliers (ADMM) [13, Sec. 3.4]. But first, a remark is in order.

[2]Although originally y_m and $\phi_{m,\mathbf{g}_i}$ were power dependent (i.e., $y_{m,l}$ and $\phi_{m,l,\mathbf{g}_i}$), power was marginalized and, hence, the power level index l was dropped. Clearly, the model in (4) also holds if the power dependence is restored. This would imply that the number of rows of $\hat{\mathbf{y}}(t)$ and $\hat{\mathbf{\Phi}}(t)$ would be L times larger, increasing the computational complexity.

[3]Matrix $\hat{\mathbf{\Phi}}(t)$ is in general uncertain, since it collects estimates of the probabilities $\{\phi_{m,\mathbf{g}_i}\}$ [cf. (1)]. To account for possible non-negligible model mismatches, the sparse total least-squares (TLS) framework proposed in [11], and further extended in [10] to the case of hierarchical sparsity, can be naturally employed here too.

A. ADMM-based solver

Consider introducing an auxiliary vector variable $\boldsymbol{\gamma}$, and reformulate (5) as follows [9], [10]:

$$\min_{\boldsymbol{\gamma},\boldsymbol{\beta}\succeq\mathbf{0}} \frac{1}{2}\left\|\hat{\mathbf{y}}(t) - \hat{\mathbf{\Phi}}(t)\boldsymbol{\beta}\right\|_2^2 + g_{\lambda_2}(\boldsymbol{\gamma}) + \lambda_1\mathbf{1}^{\mathcal{T}}\boldsymbol{\gamma} \quad (7a)$$

$$\text{subject to} \;\; \boldsymbol{\beta} = \boldsymbol{\gamma}. \quad (7b)$$

Indeed, constraint (7b) renders problems (5) and (7) equivalent. Letting $\mathbf{z}$ denote the Lagrange multipliers associated with the equality constraint (7b), the quadratically augmented Lagrangian function of (7) is

$$\mathcal{L}(\boldsymbol{\beta},\boldsymbol{\gamma},\mathbf{z}) := \frac{1}{2}\left\|\hat{\mathbf{y}}(t) - \hat{\mathbf{\Phi}}(t)\boldsymbol{\beta}\right\|_2^2 + \lambda_2\sum_{r=1}^{R}\|\boldsymbol{\gamma}^{(r)}\|_2 + \lambda_1\mathbf{1}^{\mathcal{T}}\boldsymbol{\gamma} + \mathbf{z}^{\mathcal{T}}(\boldsymbol{\beta}-\boldsymbol{\gamma}) + \frac{c}{2}\|\boldsymbol{\beta}-\boldsymbol{\gamma}\|_2^2 \quad (8)$$

where $c > 0$ is an arbitrary constant [13]. Then, with $k \in \mathbb{N}$ denoting the iteration index, the ADMM cyclically computes: *i)* $\boldsymbol{\beta}(k+1) = \arg\min_{\boldsymbol{\beta}\succeq\mathbf{0}} \mathcal{L}(\boldsymbol{\beta},\boldsymbol{\gamma}(k),\mathbf{z}(k))$; *ii)* $\boldsymbol{\gamma}(k+1) = \arg\min_{\boldsymbol{\gamma}} \mathcal{L}(\boldsymbol{\beta}(k+1),\boldsymbol{\gamma},\mathbf{z}(k))$; and, *iii)* the Lagrange multiplier vector $\mathbf{z}$ updates as $\mathbf{z}(k+1) = \mathbf{z}(k) + c\,(\boldsymbol{\beta}(k+1) - \boldsymbol{\gamma}(k+1))$.

Function $\mathcal{L}(\boldsymbol{\beta},\boldsymbol{\gamma},\mathbf{z})$ is quadratic in $\boldsymbol{\beta}$, and thus the first step of the ADMM boils down to the following update.

[S1] $$\boldsymbol{\beta}(k+1) = \Big[\left(\hat{\mathbf{\Phi}}^{\mathcal{T}}(t)\hat{\mathbf{\Phi}}(t) + c\mathbf{I}_G\right)^{-1} \times \left(\hat{\mathbf{\Phi}}^{\mathcal{T}}(t)\hat{\mathbf{y}}(t) - \mathbf{z}(k) + c\boldsymbol{\gamma}(k)\right)\Big]_0^{\infty}. \quad (9)$$

Next, notice that minimization of the Lagrangian (8) with respect to (w.r.t.) $\boldsymbol{\gamma}$ can be split into R SOCPs, one per sub-vector $\gamma^{(r)}$. Thus, by computing the subdifferential of (8) w.r.t. $\gamma^{(r)}$, it can be shown that the second step of the ADMM amounts to the following nested soft-thresholding operations [9]:

[S2] $$\boldsymbol{\gamma}^{(r)}(k+1) = \begin{cases} \mathbf{0}_{N_b}, & \|\boldsymbol{\mu}^{(r)}\|_2 = 0 \\ \frac{\boldsymbol{\mu}^{(r)}}{c\|\boldsymbol{\mu}^{(r)}\|_2}\left[\|\boldsymbol{\mu}^{(r)}\|_2 - \lambda_2\right]_0^{\infty}, & \|\boldsymbol{\mu}^{(r)}\|_2 > 0 \end{cases} \quad (10)$$

with

$$\boldsymbol{\mu}^{(r)} = \mathcal{T}_{\lambda_1}\left(\mathbf{z}^{(r)}(k) + c\boldsymbol{\beta}^{(r)}(k+1)\right) \quad (11)$$

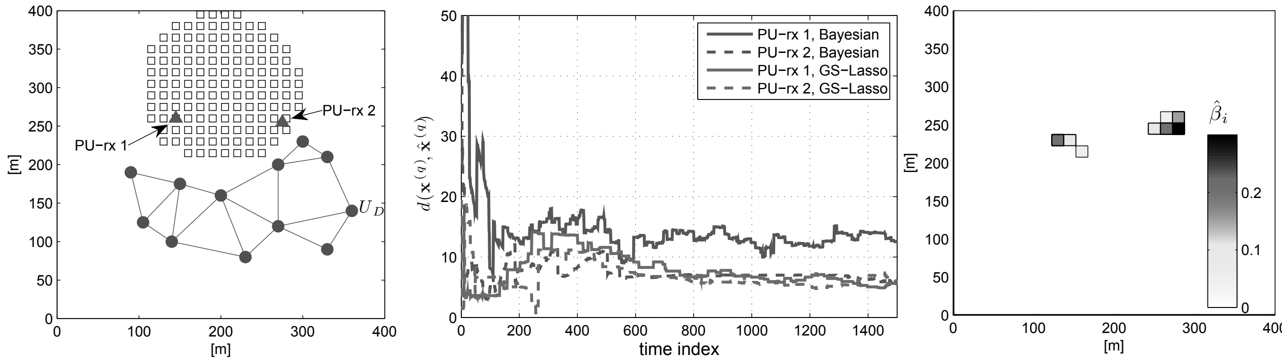
and $\mathcal{T}_{\lambda_1}(\mathbf{z}) := [\text{sgn}(z_1)[|z_1| - \lambda_1]_+, \ldots, \text{sgn}(z_N)[|z_N| - \lambda_1]_+]^T$. The soft-thresholding operation (11) accounts for the sparsity at a single-coefficient level, whereas (10) enforces group sparsity in $\boldsymbol{\gamma}(k+1)$.

Finally, the following dual update is performed.

[S3] $$\mathbf{z}(k+1) = \mathbf{z}(k) + c\,(\boldsymbol{\beta}(k+1) - \boldsymbol{\gamma}(k+1)). \quad (12)$$

B. Map-cognizant RA

The estimated beliefs $\hat{\boldsymbol{\beta}}(t)$ and locations $\{\hat{\mathbf{x}}^{(r)}(t)\}$ are used as inputs to optimize the SU network operation. Since the focus of this paper is on the receiver map estimation task, the considered RA scheme is just outlined next; details can be found in e.g., [4], [5]. Under fairly general conditions, the optimal resource allocation at time t amounts to maximizing



(a) (b) (c)

Fig. 1. (a) Considered scenario. (b) Evolution of the localization error $d(\mathbf{x}^{(r)}, \hat{\mathbf{x}}^{(r)})$. (c) Receiver map at $t = 1000$.

a functional $U_{CR}(t) := U_{SU}(t) + U_{PU}(t)$, where $U_{SU}(t)$ and $U_{PU}(t)$ are the utilities for the SU and PU networks, respectively. With an orthogonal access, utilities can be re-written as $U_{CR}(t) = \sum_m w_m(t)\big(U_{SU,m}(t\ |p_m(t)) + U_{PU,m}(t\ |p_m(t))\big)$. Then, it can be shown that, at time t, the optimal $p_m^*(t)$ for SU m is $p_m^*(t) = \arg\max_{p_m(t)\in\mathcal{P}_m}\big(U_{SU,m}(t\ |p_m(t)) + U_{PU,m}(t\ |p_m(t))\big)$, while the optimum scheduling is $w_m^*(t) = 1$ if $m = \arg\max_l \big(U_{SU,l}(t\ |p_l^*(t)) + U_{PU,l}(t\ |p_l^*(t))\big)$ and $w_m^*(t) = 0$ otherwise. A widely used alternative is to set $U_{SU,m}(t\ |p_m(t)) = \rho_m r_m(t) + \mu_m p_m(t)$, where the rate $r_m(t)$ and power $p_m(t)$ are the resources and ρ_m and π_m their corresponding prices. When probability-of-interference constraints must be guaranteed [2], $U_{SU,m}(t\ |p_m(t))$ takes the form $\lambda \sum_q \Pr\{i^q = 1|w_m(t) = 1, p_m(t)\}$, where λ denotes the Lagrange multiplier associated with the interference constraint [4], while the dependence of $\Pr\{i^q = 1|w_m(t) = 1, p_m(t)\}$ on $\{\Pr\{z^q_{\mathbf{g}_i} = 1\}\}$ was explicitly written when defining $y_{m,l,\mathbf{g}_i}$.

IV. Preliminary results

Consider the scenario depicted in Fig. 1(a), where $M = 12$ SU transceivers (marked with green circles) are deployed over an area of 400×400 m. A PU transmitter communicates with $Q = 2$ PU receivers (marked with blue circles). Notice that both receivers are off the grid. The PU system is protected by setting $I = -70$ dB and $i^{\max} = 0.05$. The path loss obeys the model $\gamma_{m,\mathbf{x}} = \|\mathbf{x}_m - \mathbf{x}\|_2^{-3.5}$, while a Rayleigh-distributed small-scale fading is simulated. The SU network setup, as well as the RA parameters are the the ones considered in [5]. The PU coverage region is discretized using 138 uniformly spaced grid points (marked with squares), each one covering an area of 15×15 m. Further, the grid is partitioned in $R = 10$ clusters. Parameters λ_1 and λ_2 in (5) are set to 0.08 and 0.01, respectively.

The localization error $d(\mathbf{x}^{(r)}, \hat{\mathbf{x}}^{(r)}) := \sqrt{(\mathbf{x}^{(r)})^2 - (\hat{\mathbf{x}}^{(r)})^2}$ per PU receiver is quantified in Fig. 1(b), and it is compared to the one achieved when using the Bayesian estimator of [5]. Notice that the Bayesian scheme assumes that the PU system broadcasts a binary message indicating whether at least one receiver is interfered. Furthermore, since the Bayesian scheme requires an estimate of the number of PU receivers, it is assumed that 2 receivers are present. From Fig. 1(b), it can be seen that the proposed localization scheme outperforms the Bayesian method; in fact, the localization error incurred by (5) is around 6 meters for both receivers, while the Bayesian method yields an error of 15 meters for PU $q = 1$.

Pictorially, performance of the receiver localization scheme can be assessed through the maps shown in Fig. 1(c). The value (color) of a point in the map represents the belief $\hat{\beta}_i(t)$ at the corresponding grid point $\mathbf{g}_i \in \mathcal{G}$. Fig. 1(c) shows that, through interference tweets, it is possible to unveil the areas where PU receivers are likely to reside.

Future research will deal with mobile PU receivers and time-varying PU activities.

References

[1] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Sig. Proc. Mag.*, vol. 24, no. 3, pp. 79–89, May 2007.
[2] R. Zhang, "On peak versus average interference power constraints for protecting primary users in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 2112–2120, Apr. 2009.
[3] R. Zhang, Y.-C. Liang, and S. Cui, "Dynamic resource allocation in cognitive radio networks," *IEEE Sig. Proc. Mag.*, vol. 27, no. 3, pp. 102–114, May 2010.
[4] A. G. Marques, L. M. Lopez-Ramos, G. B. Giannakis, and J. Ramos, "Resource allocation for interweave and underlay cognitive radios under probability-of-interference constraints," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 10, pp. 1922–1933, Nov. 2012.
[5] A. G. Marques, E. Dall'Anese, and G. B. Giannakis, "Cross-layer optimization and receiver localization for cognitive networks using interference tweets," submitted Mar. 2013. [Online]: http://arxiv.org/.
[6] S. Farahmand, G. B. Giannakis, and G. Leus, "Sparsity-aware Kalman tracking of target signal strengths on a grid," in *14th Intl. Conf. on Info. Fusion*, Chicago, IL, Jul. 2011.
[7] C. Coue, C. Pradalier, C. Laugier, T. Fraichard, and P. Bessiere, "Bayesian occupancy filtering for multitarget tracking: An automative application," *Intl. J. of Robotics Research*, vol. 25, no. 1, pp. 19–30, Jan. 2006.
[8] M. Yuan and Y. Lin, "Model selection and estimation in regression with grouped variables," *J. of the Royal Stat. Soc.*, vol. 68, pp. 49–67, 2006.
[9] P. Sprechmann, I. Ramírez, G. Sapiro, and Y. C. Eldar, "C-HiLasso: A collaborative hierarchical sparse modeling framework," *IEEE Trans. Sig. Proc.*, vol. 9, no. 59, pp. 4183–4198, Sep. 2011.
[10] E. Dall'Anese, J. A. Bazerque, and G. B. Giannakis, "Group sparse Lasso for cognitive network sensing robust to model uncertainties and outliers," *Elsevier Phy. Commun.*, vol. 5, no. 2, pp. 161–172, Jun. 2012.
[11] H. Zhu, G. Leus, and G. B. Giannakis, "Sparsity-cognizant total least-squares for perturbed compressive sampling," *IEEE Trans. Sig. Proc.*, vol. 59, pp. 2002–2016, May 2011.
[12] T. Hastie, R. Tibshirani, and J. Friedman, *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*. Springer, 2009.
[13] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and Distributed Computation: Numerical Methods*. Englewood Cliffs, NJ: Prentice-Hall, 1989.