2013 5th IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP)

Adaptive Waveform Design for Target Enumeration in Cognitive Radar

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***Abstract*—In this paper, the problem of sequential waveform design for target enumeration for cognitive multiple-input single-output (MISO) radar is investigated. In the proposed technique, the transmit spatial waveform is adaptively determined at each step based on observations in the previous steps. The waveform is determined to minimize an approximated lower bound on the average sample number (ASN) required to achieve given error rates. The algorithm is tested via simulations and shown to exhibit superior performance compared to orthogonal waveform transmission.**

I. Introduction

Cognitive radar is an emerging technology proposed in [1] and has been investigated in several works. A cognitive radar system adaptively interrogates the radar environment based on previous observations, side information, and task priorities. It adaptively illuminates the environment in a closed loop manner in order to optimize some predefined objective functions. In [2] an adaptive technique for waveform design for target localization was proposed. This technique is based on minimizing performance lower bounds on the target parameters and it was shown to automatically focus the transmit beam towards the targets directions in a very low signal-to-noise ratio (SNR). In [3], two adaptive waveform design techniques using sequential hypothesis testing for target classification was proposed. In [4] the problem of adaptive waveform design for sequential target detection with subspace interference was investigated. In the last two works, the Kullback-Leibler divergence (KLD) was used as a criterion for adaptive waveform design.

Multiple-input multiple-output (MIMO) radar [5]-[7], has attracted the attention of many researchers in the last decade. One of the main directions of research within the topic of MIMO radar is waveform design, which has been intensively investigated in the recent years. Waveform optimization for MIMO radar target localization using the Cramér-Rao bound (CRB), was studied in [8]. In [9], [10] waveform design based on mutual information and minimum mean-square-error (MMSE) was considered and it was shown that by using optimized waveforms, better detection performance can be obtained. In [11] sequential Bayesian inference was investigated using adaptive polarized waveform design for target tracking.

In this paper, an adaptive spatial waveform design technique for target enumeration is proposed. A multiple-input single-output radar is considered. At each step, the posterior probabilities for the different hypotheses which are characterized by the number of targets, are computed. The spatial waveform for the next transmit pulse is designed in order to minimize an approximated lower bound on the average sample number (ASN).

The paper is organized as follows. The next section presents the signal model and the problem statement. In Section III the criterion for waveform optimization is stated and an adaptive waveform design scheme is presented. Section IV derives an algorithm based on Bayesian information criterion (BIC) for target enumeration and computation of the posterior probabilities. In Section V, the performance of the proposed adaptive waveform design technique is evaluated and compared to fixed, orthogonal waveform transmission.

II. Signal Model

Consider a mono-static radar consisting of a transmit array of N_T transmitters and a single receiving element. The received signal model for a given range-Doppler cell in the presence of M targets can be expressed as [5], [6]

$$x_{k,l} = \mathbf{s}_{k,l}^T \sum_{m=1}^{M} \alpha_m \mathbf{a}_T(\theta_m) + v_{k,l}, \quad k = 1, 2, \ldots, \quad l = 1, \ldots, L \tag{1}$$

where $x_{k,l} \in \mathbb{C}$, $v_{k,l} \in \mathbb{C}$, and $\mathbf{s}_{k,l} \in \mathbb{C}^{N_T}$ are the received signal, the additive noise, and the transmit signal vector at the lth snapshot of the kth step, and L denotes the number of snapshots at each step. The parameters α_m and θ_m denote the complex attenuation and direction of the mth target, respectively, and $\mathbf{a}_T(\cdot) \in \mathbb{C}^{N_T}$ is the transmit array steering vector where the array origin is set to the location of the receiving element. We assume that $\{v_{k,l}\}$ are independent and identically distributed (i.i.d.) complex circularly symmetric Gaussian random variables with zero mean and variance σ^2. We will assume that the number of targets at the considered range-Doppler cell, M, is smaller than the number of transmit array elements, N_T. This assumption is required also in non-parametric source enumeration methods, which are based on identification of the noise subspace whose size is given by $M - N_T$.

The sufficient statistic for estimating the target parameters at the kth step is given by correlating the received signal with transmit signal. Let $\mathbf{A}_T \triangleq [\mathbf{a}_T(\boldsymbol{\theta}_1), \ldots, \mathbf{a}_T(\boldsymbol{\theta}_M)]$, $\boldsymbol{\alpha} \triangleq [\alpha_1, \ldots, \alpha_M]^T$, and $\mathbf{R}_{\mathbf{s}_k} \triangleq \left(\frac{1}{L}\sum_{l=1}^{L} \mathbf{s}_{k,l}\mathbf{s}_{k,l}^H\right)^*$. Then, the data model for the sufficient statistic, $\mathbf{y} \triangleq \mathbf{R}_{\mathbf{s}_k}^{-1/2} \frac{1}{L}\sum_{l=1}^{L} \mathbf{s}_{k,l}^* x_{k,l}$, is given by [6]

$$\mathbf{y}_k = \mathbf{R}_{\mathbf{s}_k}^{1/2} \mathbf{A}_T \boldsymbol{\alpha} + \mathbf{w}_k, \quad k = 1, 2, \ldots. \tag{2}$$

It can be verified that the noise vector $\{\mathbf{w}_k\}$ is an i.i.d. sequence of complex circularly symmetric Gaussian random vectors with zero mean and covariance $\sigma^2\mathbf{I}_{N_T}$. In the model in (1), we assumed that $\boldsymbol{\alpha}$ is constant during the L samples of the signal.

At each step, k, we will perform J statistically independent trials, with the same transmit signal. The model can be stated as

$$\mathbf{y}_{k,j} = \mathbf{R}_{\mathbf{s}_k}^{1/2}\mathbf{A}_T\boldsymbol{\alpha}_j + \mathbf{w}_{k,j}, \quad k = 1, 2, \ldots, \quad j = 1, \ldots, J, \quad (3)$$

where the sequence of coefficients vectors $\{\boldsymbol{\alpha}_j\}$ is assumed to be i.i.d. whose elements are complex circularly symmetric Gaussian vectors with zero mean and non-singular covariance matrix $\mathbf{R}_{\boldsymbol{\alpha}}$.

Let $\mathbf{Y}_k \triangleq [\mathbf{y}_{k,1}, \ldots, \mathbf{y}_{k,J}]$. In this work, we are interested in the design of the transmit signal autocorrelation matrix at the kth step, $\mathbf{R}_{\mathbf{s}_k}$, given observations in previous steps (history), denoted by $\mathbf{Y}^{(k-1)} = [\mathbf{Y}_1, ..., \mathbf{Y}_{k-1}]$, where the objective is to minimize the ASN required to determine the number of targets with given probability of error. Based on the model assumptions described above, the columns of $\mathbf{Y}_k$ are i.i.d. vectors $\mathbf{y}_{k,j} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_{\mathbf{y}_k})$ where

$$\mathbf{R}_{\mathbf{y}_k} = \mathbf{R}_{\mathbf{s}_k}^{1/2}\mathbf{A}_T\mathbf{R}_{\boldsymbol{\alpha}}\mathbf{A}_T^H\mathbf{R}_{\mathbf{s}_k}^{1/2} + \sigma^2\mathbf{I}_{N_T}. \quad (4)$$

In non-parametric source enumeration [12], the hypotheses are described by the rank of the signal covariance matrix, $\mathbf{A}_T\mathbf{R}_{\boldsymbol{\alpha}}\mathbf{A}_T^H$. In this approach, it is assumed that the columns of $\mathbf{A}_T$ are linearly independent and the matrix $\mathbf{R}_{\boldsymbol{\alpha}}$ is non-singular. Accordingly, under the assumption that $\mathbf{R}_{\mathbf{s}_k}$ is non-singular, hypothesis M, is characterized by

$$H_M: \ \text{rank}\left(\mathbf{R}_{\mathbf{s}_k}^{1/2}\mathbf{A}_T\mathbf{R}_{\boldsymbol{\alpha}}\mathbf{A}_T^H\mathbf{R}_{\mathbf{s}_k}^{1/2}\right) = M, \quad M = 0, \ldots, N_T - 1 \quad (5)$$

III. Sequential Waveform Design for Target Enumeration

For non-singular transmit auto-correlation matrix, $\mathbf{R}_{\mathbf{s}_k}$, model order selection can be easily performed using well known techniques, such as, non-parametric [12], or parametric [13] approaches. In this work, the cognitive radar system interrogates the radar environment in order to minimize a lower bound on the ASN for target enumeration. We will adopt a Bayesian approach, where at each step the conditional probability of each hypothesis given prior observations is computed. These conditional probabilities (posterior probabilities at step $k-1$) are used to design the transmit auto-correlation matrix at step k, $\mathbf{R}_{\mathbf{s}_k}$.

We will adopt a sequential hypothesis testing (SHT), where at each step, a decision is made either on one of the hypotheses or on whether to continue the experiment and collect additional observations. The criterion for decision to continue or stop the experiment is the probability of error. In [14] a multi-hypothesis sequential test was proposed and was employed also in [3]. According to this approach, at the kth step, the likelihood ratio between every two pairs of hypotheses, m and n, is computed, and denoted by $\Psi_{m,n}^{(k)}$. Let $p_{mn}, \ (m \neq n) = 0, \ldots, N_T - 1$ denote the desired probability of incorrectly deciding H_n given true hypothesis H_m. Then the experiment is terminated and H_m is selected if $\Psi_{m,n}^{(k)} > (1 - p_{mn})/p_{mn}, \ \forall n \neq m$.

In [15] it is shown that if $\lim_{k\to\infty} \frac{1}{k}\log\Psi_{m,n}^{(k)} = q_{m,n}$, where $q_{m,n}$ is a positive finite constant, then

$$ASN \geq \max_{m\neq n} \frac{\log(1/p_{m,n})}{q_{m,n}}. \quad (6)$$

For i.i.d. observations, the constants $q_{m,n}$ are given by $q_{m,n} = KLD(H_m||H_n)$ where $KLD(H_m||H_n)$ is the Kullback-Leibler divergence (KLD) of the probability density function (pdf) under hypothesis H_n from the pdf under hypothesis H_m. In our case, the observations are not identically distributed since we modify the transmit signal auto-correlation matrix at each step. However, as explained in [3] we expect that as k goes to infinity, the i.i.d. assumption will almost be satisfied. Accordingly, similar to the approach presented in [3], we will use the weighted KLD as a criterion for waveform design.

The problem of target enumeration, as stated in the previous section, is a nested hypothesis problem. Hence, the main contribution to the probability of error is from errors between adjacent hypotheses. Therefore, the criterion for waveform design is composed of weighted KLD's between adjacent hypotheses:

$$Q = \sum_{M=1}^{N_T-1} c_M KLD(H_M||H_{M-1}), \quad (7)$$

where $c_M, \ M = 1, \ldots, N_T - 1$ are the weighting coefficients. For k statistically independent observations, the total KLD between the pdf's is given by the sum of KLD's of the corresponding pdf of each observation, and therefore the criterion can be stated as

$$Q_k = \sum_{M=1}^{N_T-1} c_{k,M} \sum_{l=1}^{k} KLD_l(H_M||H_{M-1}). \quad (8)$$

The coefficients $c_{k,M}, \ M = 1, \ldots, N_T - 1$ can be set based on the prior probabilities of the different hypotheses. Since Q_k depends on $\mathbf{R}_{\mathbf{s}_k}$ only through $KLD_k(\cdot||\cdot)$, then the criterion for determining $\mathbf{R}_{\mathbf{s}_k}$ is given by maximizing

$$Q'_k = \sum_{M=1}^{N_T-1} c_{k,M} KLD_k(H_M||H_{M-1}). \quad (9)$$

Let $\mathbf{R}_M \triangleq \mathbf{U}\boldsymbol{\Lambda}_M\mathbf{U}^H$ denote the singular value decomposition of $\mathbf{A}_T\mathbf{R}_{\boldsymbol{\alpha}}\mathbf{A}_T^H$ under hypothesis H_M, where $\Lambda_M = \text{diag}(\lambda_1, \ldots, \lambda_M, 0, \ldots, 0)$ and $\mathbf{U} = [\mathbf{u}_1, \ldots, \mathbf{u}_{N_T}]$ are the eigenvalues and eigenvectors matrices of $\mathbf{A}_T\mathbf{R}_{\boldsymbol{\alpha}}\mathbf{A}_T^H$, respectively. Then, (4) can be rewritten as

$$\mathbf{R}_{\mathbf{y}_{k,M}} = \mathbf{R}_{\mathbf{s}_k}^{1/2}\mathbf{U}\boldsymbol{\Lambda}_M\mathbf{U}^H\mathbf{R}_{\mathbf{s}_k}^{1/2} + \sigma^2\mathbf{I}_{N_T}. \quad (10)$$

The KLD of the pdf under hypothesis H_{M-1} from the pdf under hypothesis H_M is given by the KLD for two zero-mean complex Gaussian distributions with invertible covariance matrices $\mathbf{R}_{\mathbf{y}_{k,M}}$ and $\mathbf{R}_{\mathbf{y}_{k,M-1}}$:

$$KLD_k(H_M||H_{M-1}) = \text{tr}(\mathbf{C}_k(M, M-1)) - N_T - \log\det(\mathbf{C}_k(M, M-1)). \quad (11)$$

where $\mathrm{tr}(\cdot)$ and $\det(\cdot)$ denote the trace and determinant operators, and $\mathbf{C}_k(M, M-1) \triangleq \mathbf{R}_{\mathbf{y}_{k,M-1}}^{-1/2}\mathbf{R}_{\mathbf{y}_{k,M}}\mathbf{R}_{\mathbf{y}_{k,M-1}}^{-1/2}$. From (10), we can express the following relation between $\mathbf{R}_{\mathbf{y}_{k,M}}$ and $\mathbf{R}_{\mathbf{y}_{k,M-1}}$

$$\mathbf{R}_{\mathbf{y}_{k,M}} = \mathbf{R}_{\mathbf{y}_{k,M-1}} + \lambda_M \mathbf{R}_{\mathbf{s}_k}^{1/2}\mathbf{u}_M\mathbf{u}_M^H\mathbf{R}_{\mathbf{s}_k}^{1/2}. \quad (12)$$

Using (12), the matrix $\mathbf{C}_k(M, M-1)$ can be described as

$$\mathbf{C}_k(M, M-1) = \mathbf{I}_{N_T} + \lambda_M \mathbf{R}_{\mathbf{y}_{k,M-1}}^{-1/2}\mathbf{R}_{\mathbf{s}_k}^{1/2}\mathbf{u}_M\mathbf{u}_M^H\mathbf{R}_{\mathbf{s}_k}^{1/2}\mathbf{R}_{\mathbf{y}_{k,M-1}}^{-1/2}. \quad (13)$$

Using Sylvester's determinant identity and trace property, the KLD from (11) can be expressed as

$$KLD_k\left(H_M \| H_{M-1}\right) = z_{k,M} - \log\left(1 + z_{k,M}\right). \quad (14)$$

where

$$z_{k,M} \triangleq \lambda_M \mathbf{u}_M^H \mathbf{R}_{\mathbf{s}_k}^{1/2}\mathbf{R}_{\mathbf{y}_{k,M-1}}^{-1}\mathbf{R}_{\mathbf{s}_k}^{1/2}\mathbf{u}_M. \quad (15)$$

The optimization problem can now be stated as

$$\mathbf{R}_{\mathbf{s}_k}^{(opt)} = \arg\max_{\mathbf{R}_{\mathbf{s}_k}} \sum_{M=1}^{N_T-1} c_{k,M}\left(z_{k,M} - \log\left(1 + z_{k,M}\right)\right)$$
$$s.t.\ \ \mathrm{tr}\left(\mathbf{R}_{\mathbf{s}_k}\right) = P,\ \ \mathbf{R}_{\mathbf{s}_k} \succeq 0 \quad (16)$$

where P denotes the total power to be transmitted by the transmit elements. This optimization involves knowledge of the matrices of eigenvectors and eigenvalues, $\mathbf{U}$ and Λ_M under each hypothesis. These values can be substituted with estimates from the previous step. Still, this optimization cannot be performed analytically. Accordingly, we propose to set the matrix of eigenvectors of $\mathbf{R}_{\mathbf{s}_k}$ to be identical to the matrix of eigenvectors of $\mathbf{R}_M$. That is,

$$\mathbf{R}_{\mathbf{s}_k} = \mathbf{U}\Lambda_{\mathbf{s}_k}\mathbf{U}^H. \quad (17)$$

Thus, we control only the eigenvalues of the signal auto-correlation matrix.

Using (17) , $\mathbf{R}_{\mathbf{y}_{k,M}}$ from (10) can be expressed as

$$\mathbf{R}_{\mathbf{y}_{k,M}} = \mathbf{U}\left(\mathbf{\Lambda}_M\Lambda_{\mathbf{s}_k} + \sigma^2\mathbf{I}_{N_T}\right)\mathbf{U}^H. \quad (18)$$

After a few lines of simple algebra and using (17), equation (15) can be rewritten as

$$z_{k,M} = \lambda_M \mathbf{e}_M^T\left(\mathbf{\Lambda}_{M-1} + \sigma^2\Lambda_{\mathbf{s}_k}^{-1}\right)^{-1}\mathbf{e}_M, \quad (19)$$

where $\mathbf{e}_M$ is the Mth column of the identity matrix of size N_T. Since the Mth element on the diagonal of $\mathbf{\Lambda}_{M-1}$ is equal to zero, $z_{k,M}$ can be rewritten as

$$z_{k,M} = \lambda_M\lambda_{\mathbf{s}_{k,M}}/\sigma^2, \quad (20)$$

in which $\lambda_{\mathbf{s}_{k,M}}$ is the Mth element on the diagonal of $\Lambda_{\mathbf{s}_k}$.

In order to understand this result, consider the case in which the coefficients $c_{k,M}$ are equal to zero, except for $M = M_0$, with $c_{k,M_0} = 1$. This setting means that the main contribution to the criterion is due to possible confusion between hypotheses M_0 and $M_0 - 1$. In this case, Q'_k from (9) becomes

$$Q'_k = \lambda_M\lambda_{\mathbf{s}_{k,M}}/\sigma^2 - \log\left(1 + \lambda_M\lambda_{\mathbf{s}_{k,M}}/\sigma^2\right), \quad (21)$$

which is monotonically increasing in $\lambda_{\mathbf{s}_{k,M}}$. Incorporation of the power constraint implies, that the entire signal energy should be transmitted toward $\mathbf{u}_M$. This result could be *a priori* expected, since the system illuminates the subspace of interest that distinguishes between the two hypotheses.

Finally, by using (16), (17), and (20) one obtains

$$\mathbf{\Lambda}_{\mathbf{s}_k}^{(opt)} = \arg\max_{\mathbf{\Lambda}_{\mathbf{s}_k}} \sum_{M=1}^{N_T-1} c_{k,M}\left(\frac{\lambda_M\lambda_{\mathbf{s}_{k,M}}}{\sigma^2} - \log\left(1 + \frac{\lambda_M\lambda_{\mathbf{s}_{k,M}}}{\sigma^2}\right)\right)$$
$$s.t.\ \ \sum_{M=1}^{N_T}\lambda_{\mathbf{s}_{k,M}} = P,\ \ \lambda_{\mathbf{s}_{k,M}} \geq 0,\ M = 1, \ldots, N_T \quad (22)$$

and $\mathbf{R}_{\mathbf{s}_k}$ is constructed using (17), where $\mathbf{U}$ is substituted by its estimate from previous observations. It can be seen that the optimization problem in (22) is a convex set, since the objective function is a weighted sum of convex functions with non-negative coefficients.

IV. Sequential Target Enumeration

In this section, we derive the posterior probabilities of the hypotheses at each step. The posterior probability of hypothesis H_M at step k is given by

$$P(H_M|\mathbf{Y}^k) = \frac{f_{\mathbf{Y}^{(k)}}(\mathbf{Y}^{(k)}|H_M)P(H_M)}{f_{\mathbf{Y}^{(k)}}(\mathbf{Y}^{(k)})} \quad (23)$$

in which $P(H_M)$ and $f_{\mathbf{Y}^{(k)}}(\mathbf{Y}^{(k)}|H_M)$ denote the prior probability and the likelihood function under hypothesis H_M, respectively. For computation of $f_{\mathbf{Y}^{(k)}}(\mathbf{Y}^{(k)}|H_M)$ we use the BIC approximations [16]:

$$f_{\mathbf{Y}^{(k)}}(\mathbf{Y}^{(k)}|H_M) \approx \frac{C}{\det(FIM(\hat{\boldsymbol{\theta}}_M))} f_{\mathbf{Y}^{(k)}}(\mathbf{Y}^{(k)}|H_M; \hat{\boldsymbol{\theta}}_M) \quad (24)$$

where $\hat{\boldsymbol{\theta}}_M$ denotes the ML estimate of the unknown parameters under hypothesis H_M, $FIM(\hat{\boldsymbol{\theta}}_M)$ is the Fisher information matrix for estimating $\boldsymbol{\theta}_M$ by $\mathbf{Y}^{(k)}$, and C is a constant w.r.t. M. In our case, the unknown parameters are given by the eigenvectors and eigenvalues of $\mathbf{R}_M$. In the presence of i.i.d. observations, the logarithm of the term $\det(FIM(\hat{\boldsymbol{\theta}}_M))$ is asymptotically given by the penalty of the minimum description length (MDL) plus some constant. The posterior pdf can now be computed by substituting (24) into (23) where C and the denominator of (23) can be obtained by normalization. In our problem, the observations are not identically distributed since the transmit auto-correlation matrix, $\mathbf{R}_{\mathbf{s}_k}$ is time-varying.

Finally, we need to compute the likelihood function under H_M and compute the ML estimate θ_M. The observations $\mathbf{y}_{k,j}$ modeled in (3), are statistically independent zero-mean complex Gaussian with covariance matrices $\mathbf{R}_{\mathbf{y}_{l,M}}$ given in (10). Thus, the log-likelihood based on the observations from the last k steps under hypothesis M is

$$L(\mathbf{R}_M) = -J\sum_{l=1}^{k}\left(\log\det\left(\mathbf{R}_{\mathbf{y}_{l,M}}\right) + \mathrm{tr}\left(\mathbf{R}_{\mathbf{y}_{l,M}}^{-1}\mathbf{S}_l\right)\right) \quad (25)$$

where $\mathbf{S}_l$ is the sample covariance matrix of the data at the lth step and $\mathbf{R}_M$ is defined immediately after (9). For a single step $k = 1$, derivation of the ML estimates of

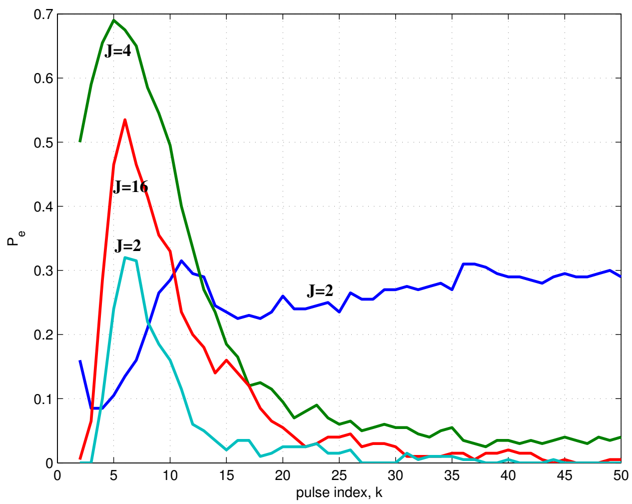

Figure 1. Probability of error for target enumeration versus pulse index k, using a MISO radar with a circular array of 16 elements.

the eigenvectors and eigenvalues of $\mathbf{R}_M$ is straight-forward, but for $k > 1$, it is not tractable. Alternatively, we minimize $\sum_{l=1}^{k} \left\| \mathbf{S}_l - \mathbf{R}_{\mathbf{y}_{l,M}} \right\|_F^2$ where $\|\cdot\|_F$ denotes the Frobenious norm. The solution under hypothesis M is given by the eigenvectors are eigenvalues of $\hat{\mathbf{R}}_M$ which satisfies $\sum_{l=1}^{k} \mathbf{R}_{\mathbf{s}_l}^{1/2} \mathbf{S}_l \mathbf{R}_{\mathbf{s}_l}^{1/2} = \sum_{l=1}^{l} \mathbf{R}_{\mathbf{s}_k}^{1/2} \hat{\mathbf{R}}_M \mathbf{R}_{\mathbf{s}_l}^{1/2}$. This equation can be easily solved, since the right hand side of the equation is linear combinations of the elements of $\mathbf{R}_M$.

V. Example

In the simulations, we considered a circular transmit array of $N_T = 16$ elements, with radius of half a wavelength. A single receiving element was located at the center of the transmit array. $M = 4$ far-field targets were located at directions $60°, 120°, 180°, 240°$. The SNR for all the 4 targets was set to 5dB, where the SNR for the mth target is defined as $SNR_m = \frac{|\alpha_m|^2}{\sigma^2}$. The transmit signal covariance matrix at the first step was set to identity matrix, and the total transmit power was set to $P = 16$. The probability of error in estimating the number of targets was computed using 200 independent trials.

Fig. 1 presents the probability of error of the proposed cognitive method as a function of the pulse step index, k, for different numbers of snapshots. It can be seen that the probability of error for $J \geq 4$ approaches zero as the pulse index increases. Simulations shows that fixed orthogonal waveform provide very poor performance for this scenario, especially in cases where the number of snapshots per pulse, J is lower than the number of transmit array elements.

VI. Conclusion

In this paper, a new technique for adaptive waveform design was proposed for target enumeration by MISO radar. Instead of transmission of identical waveforms, in the proposed techniques, the waveform is determined at each step, in order to minimize the ASN required to obtain predefined probabilities of error in estimation of the number of targets. It is shown that the proposed algorithm concentrates the transmit energy toward the subspace representing the competing hypotheses with highest probabilities. The proposed technique was tested via simulations for adaptive spatial transmit waveform design. The simulations show that the proposed technique results in significantly better results compared to non-adaptive, orthogonal waveform transmission. Further research is required to extend these results to MIMO radar with multiple receivers.

Acknowledgment

This research was supported by THE ISRAEL SCIENCE FOUNDATION (grant No. 1392/11).

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