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Distributed Spectrum Sensing in the Presence of Selfish Users

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***Abstract*—We study the problem of decentralized spectrum sensing in the presence of selfish secondary users. We employ diffusion strategies to guide the estimation process and a reputation mechanism to encourage secondary users to participate in the sharing of information. Simulation results illustrate the performance of the proposed technique for spectrum sensing over cognitive radios.**

I. Introduction

Cooperative spectrum sensing by secondary users (SUs) in a cognitive radio scenario can help avoid interference with transmissions by the primary user (PU) [1]. Spectrum sensing can be implemented either in a centralized manner [2] or decentralized manner [3] through coordination among the SUs. The latter approach exploits the spatial diversity of the SUs more fully and is scalable and robust, while the centralized approach is vulnerable to failure by the fusion center. The cooperative spectrum sensing problem generally involves a parameter estimation step. Various distributed strategies exist for the decentralized solution of estimation problems, most notably the consensus strategy [4]–[7] and the diffusion strategy [8]–[10]. It has been shown in the prior work [11] that diffusion strategies have superior convergence, stability, and mean-square-error performance. For this reason, we shall employ diffusion adaptation to estimate the parameters of interest.

In collaborative spectrum sensing, it is not difficult to envision situations where some SUs may behave in a selfish manner and would participate in the sharing of information with other SUs only if this activity is beneficial to them. One example of such a scenario is studied in [12] where the SUs operate with the intention of maximizing their own transmission rates under the constraint of limited interference to the PUs. Other scenarios are studied in [13]–[15] using coalitional game formulations. In this paper, we examine the decentralized spectrum sensing problem in the presence of selfish SUs. We assume that the sharing of information among neighboring SUs entails some communication cost. In this way, each SU becomes interested in minimizing the error in estimating the parameter of interest to enable enhanced spectrum sensing (this objective favors cooperation) while reducing the cost of communicating with neighbors (this objective disfavors cooperation). We explained in [16] that under similar scenarios involving information-sharing games, the dominant strategy for each user is not to participate in the sharing of information. In order to address this inefficient behavior, we embedded a reputation mechanism from [17] into the design of an adaptive collaborative process and developed a scheme that encourages users to cooperate. We show in this article how a similar design strategy can be developed for *online* cooperative spectrum sensing and leads to enhanced detection performance. In comparison to the framework in [16], here we to formulate a decentralized detection mechanism to guide the cooperation step.

Notation: We use lowercase letters to denote vectors and scalars, uppercase letters for matrices, plain letters for deterministic variables, and boldface letters for random variables. All vectors in our treatment are column vectors, with the exception of the regression vectors, $u_{k,i}$.

II. System Model

We consider a network with N secondary users (SUs) and one primary user (PU). The frequency spectrum is divided into M sub-bands and the signal powers over these sub-bands are collected into a column vector w^o with nonnegative entries. The channels between the PU and the SUs are assumed to be frequency-selective and time-variant as follows. For each sub-band, and at each time instant i, the channel *power* gains from the PU to the $k-$th SU are represented by a $1 \times M$ vector $\boldsymbol{u}_{k,i} \in \mathbb{R}_+^{1\times M}$ with non-negative entries. We assume that the channel information $u_{k,i}$, which is a realization for the random process $\boldsymbol{u}_{k,i}$ at time i, can be estimated through pilot signals during a training phase [18], [19]. During each i-th time interval, each SU k measures the received power that results from the aggregation of the signal powers in w^o multiplied by the channel power gains in $\boldsymbol{u}_{k,i}$. We denote the received power by $\boldsymbol{s}_k(i)$. This measurement is generally subject to noise and we write in a manner similar to [3]:

$$\boldsymbol{s}_k(i) = \boldsymbol{u}_{k,i} w^o + \boldsymbol{v}'_k(i) \tag{1}$$

where $\boldsymbol{v}'_k(i)$ combines the receiver and measurement noise sources and is assumed to have mean $\bar{v}_k$ and variance $\sigma^2_{v,k}$. We assume the random processes $\boldsymbol{u}_{k,i}$ and $\boldsymbol{v}'_\ell(j)$ are spatially and temporally independent over k, ℓ, i and j. To sense the spectrum, each SU solves a detection problem of the form:

$$\begin{cases} \mathcal{H}_0: & w^o = 0 \\ \mathcal{H}_1: & w^o = w^s \end{cases} \tag{2}$$

where $w^s \in \mathbb{R}_+^{M\times 1}$ represents the spectrum pattern that results from the presence of the PU. We assume that w^o varies slowly over time. We further assume that $\bar{v}_k$ is known by each SU,

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so that the data model can be centered as:

$$\boldsymbol{d}_k(i) \triangleq \boldsymbol{s}_k(i) - \bar{v}_k = \boldsymbol{u}_{k,i} w^o + \boldsymbol{v}_k(i) \tag{3}$$

where $\boldsymbol{v}_k(i) = \boldsymbol{v}'_k(i) - \bar{v}_k$ represents the centered zero-mean noise process.

We shall adopt a simple collaborative strategy for estimating w^o from the streaming data $\{\boldsymbol{d}_k(i), \boldsymbol{u}_{k,i}\}$. In particular, we shall assume that the SUs are randomly paired. For example, when SUs k and ℓ are paired and SU ℓ agrees to collaborate with SU k, then SU k will update its estimate of the parameter vector w^o according to the following Adapt-then-Combine (ATC) diffusion strategy [8]–[10]:

$$\boldsymbol{\psi}_{k,i} = \boldsymbol{w}_{k,i-1} + \mu \boldsymbol{u}_{k,i}^T [\boldsymbol{d}_k(i) - \boldsymbol{u}_{k,i} \boldsymbol{w}_{k,i-1}] \tag{4}$$

$$\boldsymbol{w}_{k,i} = \alpha_k \boldsymbol{\psi}_{k,i} + (1-\alpha_k)\boldsymbol{\psi}_{\ell,i} \tag{5}$$

where μ is a positive step-size factor, which is assumed to be sufficiently small to ensure mean-square stability. The second step (5) uses a coefficient $0 \le \alpha_k \le 1$ to combine the intermediate estimates of SUs k and ℓ. Using results from [10], it can be verified that a sufficiently small step-size μ ensures asymptotic mean stability of $\boldsymbol{w}_{k,i}$ in (4)–(5), i.e.,

$$\mathbb{E}\widetilde{\boldsymbol{w}}_{k,i} \to 0 \quad \text{as} \quad i \to \infty \tag{6}$$

in terms of the error vector $\widetilde{\boldsymbol{w}}_{k,i} \triangleq w^o - \boldsymbol{w}_{k,i}$. We could consider incorporating an additional projection step following (5) to ensure that all entries of $\boldsymbol{w}_{k,i}$ are non-negative. However, such a step generally leads to biased estimates for w^o. In this article, we continue with the unbiased solution that results from (4)–(5). The simulation results in the last section illustrate how this construction leads to good performance.

When SUs k and ℓ are paired together, we assume that they share the noise variances $\sigma_{v,k}^2$ and $\sigma_{v,\ell}^2$, and the channel realizations $u_{k,i}$ and $u_{\ell,i}$. Using this reference knowledge, SU ℓ will decide, according to the procedure described further ahead in (28)–(31), on whether to share its information $\boldsymbol{\psi}_{\ell,i}$ with SU k at time i, and vice-versa. The decision to cooperate by either SU is based on each one of them evaluating a certain performance metric, described in the next section, and which reflects how well cooperation may enhance their detection accuracy against the communication cost. If SU ℓ decides not to share estimates, α_k in (5) is set to 1. For each SU ℓ, sharing the estimates $\boldsymbol{\psi}_{\ell,i}$ bears a known positive transmission cost c.

III. Performance Metric

A. Detection Performance

Let us denote by $\text{EMSE}_{k,i}$ the instantaneous excess-mean-square-error of SU k at time i conditioned on the known realization $u_{k,i}$. This quantity is defined as

$$\text{EMSE}_{k,i} \triangleq \mathbb{E}[|\boldsymbol{u}_{k,i}\widetilde{\boldsymbol{w}}_{k,i-1}|^2 | \boldsymbol{u}_{k,i} = u_{k,i}] \tag{7}$$

which we rewrite as:

$$\text{EMSE}_{k,i} = \mathbb{E}|u_{k,i}\widetilde{\boldsymbol{w}}_{k,i-1}|^2 \ge 0 \tag{8}$$

Smaller values for $\text{EMSE}_{k,i}$ correspond to enhanced estimation accuracy. The analysis that follows explains how smaller values for $\text{EMSE}_{k,i}$ enhance the detection accuracy as well.

We reconsider the detection problem (2) by examining the statistics of the random variable $u_{k,i}\boldsymbol{w}_{k,i-1}$, which can be interpreted as an estimate for the received signal power. For small step-sizes and after sufficient iterations, the iterated $\boldsymbol{w}_{k,i-1}$ approaches w^o with a small mean-square error. We therefore approximate the mean of $u_{k,i}\boldsymbol{w}_{k,i-1}$ by

$$\mathbb{E}u_{k,i}\boldsymbol{w}_{k,i-1} \approx u_{k,i} w^o \tag{9}$$

Likewise, the variance of $u_{k,i}\boldsymbol{w}_{k,i-1}$ is approximated by:

$$\begin{aligned}\text{Var}(u_{k,i}\boldsymbol{w}_{k,i-1}) &\triangleq \mathbb{E}|u_{k,i}\boldsymbol{w}_{k,i-1} - \mathbb{E}(u_{k,i}\boldsymbol{w}_{k,i-1})|^2 \\ &\approx \mathbb{E}|u_{k,i}\widetilde{\boldsymbol{w}}_{k,i-1}|^2 = \text{EMSE}_{k,i}\end{aligned} \tag{10}$$

Thus, after a sufficient number iterations, we can replace the detection problem in (2) by

$$\begin{cases} \mathcal{H}_0 : \mathbb{E}(u_{k,i}\boldsymbol{w}_{k,i-1}) \approx 0 \\ \mathcal{H}_1 : \mathbb{E}(u_{k,i}\boldsymbol{w}_{k,i-1}) \approx u_{k,i} w^s \end{cases} \tag{11}$$

Now each SU k will decide on $\mathcal{H}_0$ or $\mathcal{H}_1$ by comparing the statistics $u_{k,i}\boldsymbol{w}_{k,i-1}$ with a threshold:

$$u_{k,i}\boldsymbol{w}_{k,i-1} \underset{\mathcal{H}_1}{\overset{\mathcal{H}_0}{\lessgtr}} \eta_{k,i} \tag{12}$$

We consider the Neyman-Pearson test in which the threshold $\eta_{k,i}$ is chosen to maximize the detection probability under a constraint on the false-alarm probability, namely,

$$\begin{aligned} \max_{\eta_{k,i}} \quad & P_{k,i}^{\text{D}} \triangleq \Pr\{u_{k,i}\boldsymbol{w}_{k,i-1} \ge \eta_{k,i}; \mathcal{H}_1\} \\ \text{subject to} \quad & P_{k,i}^{\text{FA}} \triangleq \Pr\{u_{k,i}\boldsymbol{w}_{k,i-1} \ge \eta_{k,i}; \mathcal{H}_0\} = \kappa \end{aligned} \tag{13}$$

We note that other than the mean and variance, the statistics of $u_{k,i}\boldsymbol{w}_{k,i-1}$ are generally unknown. Therefore, the optimization problem (13) cannot be solved explicitly. To continue, we assume that the probability distribution of $u_{k,i}\boldsymbol{w}_{k,i-1}$ is symmetric around the mean under both $\mathcal{H}_0$ and $\mathcal{H}_1$. With this assumption, we can utilize Chebyshev's inequality to ensure that an upper bound on $P_{k,i}^{\text{FA}}$ is smaller than κ. Thus, note that

$$\begin{aligned} P_{k,i}^{\text{FA}} &= \Pr\{u_{k,i}\boldsymbol{w}_{k,i-1} \ge \eta_{k,i}; \mathcal{H}_0\} \\ &= \frac{1}{2}\Pr\{|u_{k,i}\boldsymbol{w}_{k,i-1}| \ge \eta_{k,i}; \mathcal{H}_0\} \\ &\le \frac{\text{EMSE}_{k,i}}{2\eta_{k,i}^2} \end{aligned} \tag{14}$$

Therefore, in order for (14) to be bounded by κ, the threshold should be selected to satisfy:

$$\eta_{k,i} \ge \sqrt{\frac{\text{EMSE}_{k,i}}{2\kappa}} \tag{15}$$

Likewise, we maximize a lower bound on $P_{k,i}^{\text{D}}$. Thus, note again that using the assumed symmetry of the distribution of $u_{k,i}\boldsymbol{w}_{k,i-1}$, we obtain

$$\begin{aligned} &\Pr\{u_{k,i}\boldsymbol{w}_{k,i-1} \le \eta_{k,i}; \mathcal{H}_1\} \\ &\quad = \frac{1}{2}\Pr\{|u_{k,i}\boldsymbol{w}_{k,i-1} - u_{k,i}w^s| \ge u_{k,i}w^s - \eta_{k,i}; \mathcal{H}_1\} \\ &\quad \le \frac{\text{EMSE}_{k,i}}{2(u_{k,i}w^s - \eta_{k,i})^2} \end{aligned} \tag{16}$$

where we assume $u_{k,i}w^s > \eta_{k,i}$ in the second equality. This assumption is reasonable in most environments when the signal power $u_{k,i}w^s$ is sufficiently large, which means sufficiently high signal-to-noise ratio (SNR). Then,

$$\begin{aligned} P_{k,i}^{\text{D}} &= \Pr\{u_{k,i}\boldsymbol{w}_{k,i} \ge \eta_{k,i}; \mathcal{H}_1\} \\ &= 1 - \Pr\{u_{k,i}\boldsymbol{w}_{k,i-1} \le \eta_{k,i}; \mathcal{H}_1\} \\ &\ge 1 - \frac{\text{EMSE}_{k,i}}{2(u_{k,i}w^s - \eta_{k,i})^2} \end{aligned} \tag{17}$$

Therefore, the optimization problem (13) is approximated and replaced by

$$\max_{\eta_{k,i}} \quad \bar{P}^{\mathrm{D}}_{k,i} \triangleq 1 - \frac{\mathrm{EMSE}_{k,i}}{2(u_{k,i}w^s - \eta_{k,i})^2} \quad \text{subject to} \quad \eta_{k,i} \geq \sqrt{\frac{\mathrm{EMSE}_{k,i}}{2\kappa}} \tag{18}$$

Under the assumption $u_k w^s > \eta_{k,i}$, the objective function $\bar{P}^{\mathrm{D}}_{k,i}$ is monotonically decreasing with respect to $\eta_{k,i}$. Thus, the solution to (18) occurs at

$$\eta^o_{k,i} = \sqrt{\frac{\mathrm{EMSE}_{k,i}}{2\kappa}} \tag{19}$$

and the resulting $\bar{P}^{\mathrm{D}}_{k,i}$ is

$$\bar{P}^o_{k,i} = 1 - \frac{\mathrm{EMSE}_{k,i}}{2\left(u_{k,i}w^s - \sqrt{\frac{\mathrm{EMSE}_{k,i}}{2\kappa}}\right)^2} \tag{20}$$

It can be verified that $\bar{P}^o_{k,i}$ increases when $\mathrm{EMSE}_{k,i}$ decreases.

B. Combined Cost Function

It follows that SUs should be motivated to cooperate in order to enhance the estimation accuracy and the detection probability. However, as shown in [16], the cost of sharing information discourages selfish agents from sharing data unless these agents are enticed to become willing participants. We can achieve this condition by employing an adaptive reputation mechanism whereby selfish SUs are dynamically rated according to their willingness to cooperate with other SUs. By doing so, a dynamic reward/punishment mechanism is incorporated into the cooperative process in order to enhance the performance of spectrum sensing.

To explain the procedure, we denote the action of SU k at time i by $\boldsymbol{a}_k(i)$, where $\boldsymbol{a}_k(i) = 1$ means "to share" information with neighbor ℓ and $\boldsymbol{a}_k(i) = 0$ means "not to share" information. With each SU k, we associate an instantaneous combined cost, $J_{k,i}$, that takes into account the cost of communication and the predicted estimation benefit for user k, which is a function of the actions by both SUs. Since the action $\boldsymbol{a}_\ell(i)$ affects the resulting $\boldsymbol{w}_{k,i}$ in (5), and thus $\widetilde{\boldsymbol{w}}_{k,i}$, we use $\mathbb{E}[\mathrm{EMSE}_{k,i+1}]$ to represent the predicted estimation accuracy. Thus, given the current state estimate $\boldsymbol{w}_{k,i-1}$, the expression for $J_{k,i}$ is defined as follows:

$$J_{k,i}(\boldsymbol{a}_k(i), \boldsymbol{a}_\ell(i)|\widetilde{\boldsymbol{w}}_{k,i-1}) \triangleq \mathbb{E}[\mathrm{EMSE}_{k,i+1}(\boldsymbol{a}_\ell(i)|\widetilde{\boldsymbol{w}}_{k,i-1})] + \boldsymbol{a}_k(i)\cdot c \tag{21}$$

Expressions for $\mathbb{E}[\mathrm{EMSE}_{k,i+1}]$ for both cases of $\boldsymbol{a}_\ell(i) = 0$ or $\boldsymbol{a}_\ell(i) = 1$ can be derived in a manner similar to [16]; this step is unnecessary for the discussion in the remainder of this article and will be skipped. It was argued in [16] in the broader context of parameter estimation, where it was shown that, if left attended, the dominant strategies for all SUs is not to share information, i.e., to set $\boldsymbol{a}_k(i) = 0$ for all k. This situation arises because SUs do not have a mechanism in place to predict what the actions of their neighbors will be.

One way to encourage SUs to cooperate is to associate a *dynamic* reputation score with each SU. Users that cooperate are rewarded with higher scores and users that do not cooperate are penalized with lower scores. Under these conditions, it becomes important for users to be able to assess the long-term benefit of their decisions to cooperate or not. For this purpose, SUs need to be foresighted and minimize instead a discounted long-term cost of the form:

$$\min_{a_k(i)} \quad J^{\infty}_{k,i}[a_k(i)] \triangleq \sum_{t=i}^{\infty} \delta^{t-i}\mathbb{E}[J_{k,t}(\boldsymbol{a}_k(t), \boldsymbol{a}_\ell(t)|\widetilde{\boldsymbol{w}}_{k,t-1})|\widetilde{\boldsymbol{w}}_{k,i-1}, \boldsymbol{a}_k(i) = a_k(i)] \tag{22}$$

where the discount factor $\delta \in (0, 1)$ models the probability that SUs leave the network in the future, and the expectation is taken over the random processes $\widetilde{\boldsymbol{w}}_{k,t-1}$, and $\boldsymbol{a}_\ell(t)$, conditioned on $\widetilde{\boldsymbol{w}}_{k,i-1}$ and $\boldsymbol{a}_k(i)$.

IV. Adaptive Reputation Design

A reputation design mechanism proposed in [16], [17] can be used to encourage cooperation and to approximate the solution to (22). Readers may refer to [16] for more details. We summarize the construction as follows.

Each SU k maintains a reputation score, $\boldsymbol{\theta}^\ell_{k,i}$, for any potential neighbor ℓ. This scalar score assumes values in the range $(0, 1)$ and its value reflects a summary of the history of SU ℓ's actions as viewed by SU k at time i. The reputation update rule takes the following form:

$$\boldsymbol{\theta}^\ell_{k,i+1} = r\boldsymbol{\theta}^{\ell'}_{k,i} + (1-r)\boldsymbol{a}_\ell(i) \tag{23}$$

where $0 < r < 1$ controls the updating rate of the reputation scores which are constrained to be higher than a threshold ϵ to avoid losing the adaptability so that

$$\boldsymbol{\theta}^{\ell'}_{k,i} = \max\{\boldsymbol{\theta}^\ell_{k,i}, \epsilon\} \tag{24}$$

for a small positive coefficient $\epsilon < 1$. The arguments in [16] show that such reputation scores can be used by SU k to predict SU ℓ's future behavior in the following manner:

$$\mathbb{P}(\boldsymbol{a}_\ell(t) = 1) \approx \boldsymbol{\theta}^\ell_{k,t} \cdot \boldsymbol{\theta}^k_{\ell,t}, \quad t \geq i \tag{25}$$

Therefore, SUs would try to maintain their reputation scores high in order to be rewarded with cooperation from other SUs when cooperation is beneficial to them.

It is clear that determining the optimal action-choosing policy of (22) requires prediction of the future. However, the future estimation errors are unavailable and evolve dynamically. The following approximations can be used to approximate the future status for $t > i$:

$$\widetilde{\boldsymbol{w}}_{k,t-1} \approx \widetilde{\boldsymbol{w}}_{k,i-1}, \quad \boldsymbol{\theta}^\ell_{k,t} \approx \boldsymbol{\theta}^\ell_{k,i}, \quad \boldsymbol{u}_{k,t} \approx \boldsymbol{u}_{k,i} \tag{26}$$

As a result, it is reasonable to predict the future actions as

$$\boldsymbol{a}_k(t) \approx \boldsymbol{a}_k(i) \tag{27}$$

The optimal action-choosing policy can be solved under assumptions (26) and (27) to obtain the threshold rule [16]:

$$\boldsymbol{a}_k(i) = \begin{cases} 1, & \text{if } \boldsymbol{\gamma}_{k,i} \triangleq \frac{\boldsymbol{b}_{k,i}}{c} > \frac{1-r\delta}{\delta(1-r)\boldsymbol{\theta}^\ell_{k,i}} \\ 0, & \text{otherwise} \end{cases} \tag{28}$$

where

$$\boldsymbol{b}_{k,i} \triangleq (1-\alpha_k^2)\boldsymbol{s}_{kk}(i) - (1-\alpha_k)^2 s_\ell \tag{29}$$

$$\boldsymbol{s}_{kk}(i) \triangleq \widetilde{\boldsymbol{w}}^*_{k,i-1}u^T_{k,i}u_{k,i}(I - 2\mu u^T_{k,i}u_{k,i})\widetilde{\boldsymbol{w}}_{k,i-1} \tag{30}$$

$$s_\ell \triangleq \frac{\mu\sigma^2_{v,\ell}\|u_{\ell,i}\|^2}{2} \tag{31}$$

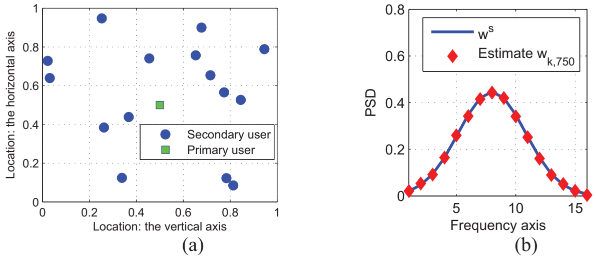

Fig. 1: (a) Spatial distribution of the PU and SUs. (b) The spectrum pattern w^s and the average of estimates $\boldsymbol{w}_{k,750}$ over all SUs at $i = 750$.

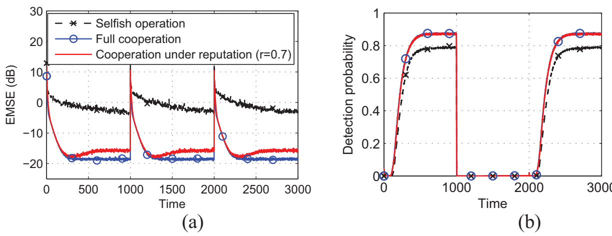

Fig. 2: The PU is active during $i \in [0, 1000)$ and $i \geq 2000$. (a) EMSE learning curve under different cooperative scenarios. (b) Evolving probability of detection under different cooperative scenarios.

We use the following instantaneous approximation for the quantity $\widetilde{\boldsymbol{w}}_{k,i-1}$ that appears in the above expressions:

$$\widetilde{\boldsymbol{w}}_{k,i-1} \approx \hat{\boldsymbol{w}}^o_{k,i} - \boldsymbol{w}_{k,i-1} \tag{32}$$

where

$$\hat{\boldsymbol{w}}^o_{k,i} = (1 - \nu_k)\hat{\boldsymbol{w}}^o_{k,i-1} + \nu_k \boldsymbol{\psi}_{k,i} \tag{33}$$

and $0 < \nu_k < 1$ is a positive forgetting factor. At each time instant i, each SU k can use this estimated $\widetilde{\boldsymbol{w}}_{k,i-1}$ to approximate $\text{EMSE}_{k,i} \approx |u_{k,i}\widetilde{\boldsymbol{w}}_{k,i-1}|^2$ which is then used to determine the threshold $\eta^o_{k,i}$ in (19) and to detect the spectrum status $\mathcal{H}_0$ and $\mathcal{H}_1$ in (12).

V. Simulation Results

In the simulations, we assume there are $N = 15$ SUs. The locations of the PU and SUs are shown in Fig. 1(a). The PU is initially active at time $i = 0$, becomes inactive at $i = 1000$, and becomes active again at $i = 2000$. We assume that the SUs are randomly paired at each time instant. The spectrum pattern of w^s with $\|w^s\| = 1$ is represented by $M = 16$ samples and is illustrated in Fig. 1(b) along with the estimated $\boldsymbol{w}_{k,i}$ averaged across all SUs after sufficient iterations. The channel power gain $\boldsymbol{u}_{k,i}$ between the PU and each SU is assumed to be a constant path loss gain with a random disturbance:

$$\boldsymbol{u}_{k,i} = g_{\text{p},k}\mathbb{1} + \boldsymbol{g}_{k,i} \tag{34}$$

where the notation $\mathbb{1}_M$ denotes a vector with all its entries equal to one, $g_{\text{p},k} = K_L \cdot (r_k/r_0)^{-2}$, $K_L = 0.1$ is a path-loss parameter, $r_0 = 1$ is a reference distance, and r_k is the distance between the PU and the k-th SU. The disturbance $\boldsymbol{g}_{k,i}$ is a zero-mean Gaussian random vector with covariance matrix $1.5I$. The measurement noise $\boldsymbol{v}'_k(i)$ is temporally white and spatially independent Gaussian distributed with mean $\bar{v}_k = 0.1$ and uniform variance $\sigma^2_{v,k} = \sigma^2_v = -10$ (dB). We set the step-size to $\mu = 0.005$, the transmission cost to $c = 10^{-6}$, the discounted parameter to $\delta = 0.99$, the minimum reputation $\epsilon = 0.1$, and the combination coefficients $\alpha_k = 1/2$ for all SUs when the shared estimates are available. All reputation scores are set to 1 at time $i = 0$ and discounted by $r = 0.7$.

In Fig. 2(a), the average EMSE over all SUs is simulated. Without the reputation scheme, the selfish SUs have no incentive to cooperate and their learning curve attains the worst EMSE performance. On the other hand, the reputation scheme encourages cooperation by selfish SUs and leads to better estimation performance. In Fig. 2(b), we simulate the detection performance in terms of the average $P^{\text{D}}_{k,i}$ over all SUs. The threshold is determined by (19) and (32). The upper bound probability κ is 0.1.

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