Invariant Target Detection of MIMO Radar with Unknown Parameters

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Abstract—**In this paper, three target detectors Uniformly Most Powerful Invariant (UMPI), Generalized Likelihood Ratio Test (GLRT) and a Separating Function Estimation Test (SFET) based on the scale group of transformations are proposed and applied to Widely Separated Antennas Multiple-Input Multiple-Output (WSA MIMO) radars. It is shown that for this problem the UMPI test depends on the scatter to noise ratio, hence the UMPI test provides the upper performance bound for all invariant tests. To derive the asymptotically optimal SFET the Maximum Likelihood Estimation (MLE) of unknown parameters are replaced into the induced maximal invariant, which is equal to the scatter to noise ratio. The MLE of the scatter to noise ratio does not have a closed form, hence we propose an iterative estimator to calculate the SFET statistic. Similarly an iterative GLRT is also proposed for this problem. The simulation results show that the performance of SFET tends to the optimal invariant bound by increasing the scatter to noise ratio.**

I. Introduction

Multiple-Input Multiple-Output (MIMO) radar is a type of multi-static radar which transmits different waveforms employing multiple antennas for target detection and identification [1]–[3]. It is shown that the detection performance of MIMO radars is better than that of the other multi-static radars such as the distributed or phase array radars [1], [4], [5]. The MIMO radars are classified into the Widely Separated Antennas (WSA) and Co-Located Antennas (CLA) [1], [6]. In [4], it is shown that for a WSA MIMO radar with orthogonal waveforms , the received signals are independent and a model for the received signal is proposed. Furthermore, the optimal Neyman-Pearson target detector in white Gaussian noise is obtained in [4] assuming that all parameters are known. Moving target detection by MIMO radar is investigated in [7] where the clutter is assumed to be zero-mean, complex Gaussian with known covariance matrix. An adaptive algorithm is proposed in [8] for target detection using a MIMO radar system when the clutter has the K-distribution. In [9], a combination of WSA and CLA approaches is considered by assuming that the transmitter and receiver have multiple well-separated subarrays and each subarray contains closely spaced antennas. They have introduced a Generalized Likelihood Ratio Test (GLRT) and a conditional GLRT considering the channel parameters as unknown deterministic values. An adaptive GLRT is developed based on an auto-regressive model for clutter with known order but unknown parameters [10]. In [11], the GLRT is derived for the case that covariance matrix is assumed to be known. For this test a closed form for detection and false alarm probability is achieved.

In this paper, we consider a target detection problem for a WSA MIMO radar. It is shown that the Uniformly Most Powerful (UMPI) test practically is not realizable and the achieved test when the scatter to noise ratio is known, provides an upper bound for the invariant tests. We also propose a Separating Function Estimation Test (SFET) and GLRT for this problem. The simulation results show that the performance of the SFET and GLRT tends to the optimal invariant bound when the scatter to noise ratio increases. Finally two iterative methods for the SFET and GLRT are proposed to implement the SFET and GLRT.

Assume that the transmit signals $\{s_k(t)\}_{k=1}^{K}$ are narrowband [4], the baseband representation of the received signal from the l^{th} receiver, sampled at time $t = nT_s$ is given by,

$$r_{l,n} = \sqrt{\frac{E_s}{K}} \sum_{k=1}^{K} h_{lk} s_k(nT_s - \tau) + w_{l,n}, \forall l = 1, \cdots, L, \quad (1)$$

where K and L are the number of transmitters and receivers, $n = 1, 2, \cdots, N$, and N is the number of signal samples for a given cell. E_s is the total energy of transmit signals, and τ is a delay representing the distance between the target location and the antennas [4]. The complex amplitude response, h_{lk}, between the k^{th} transmitter and the l^{th} receiver is proportional to the radar cross section of target. We denote $\mathbf{r}_l \triangleq [r_{l,1}, r_{l,2}, \cdots, r_{l,N}]^T$ as the received vector by the l^{th} antenna. We assume that the clutter $\mathbf{w}_l \triangleq [w_{l,1}, w_{l,2}, \cdots, w_{l,N}]^T$ is a zero-mean Gaussian process with covariance matrix $\mathbf{R}_{w,l}$. The clutter can assumed to be spatially stationary provided that the distance between the sources of clutter and the receiver is significantly larger than the largest distance between the receive antennas. Furthermore, the channel coefficients h_{lk} are assumed to be zero-mean jointly Gaussian, and uncorrelated [1], [4], [7]. In this paper, we assume that h_{lk} is spatial-temporally stationary, i.e., $E[h_{l_1k_1}h^*_{l_2k_2}] = \sigma^2\delta_{l_1-l_2}\delta_{k_1-k_2}$, where σ^2 is unknown. Consider the target detection problem in the Cell Under Test (CUT) as a binary hypothesis test, testing $\mathcal{H}_0$ against $\mathcal{H}_1$ where $\mathcal{H}_0$ and $\mathcal{H}_1$ denote no target and a target in CUT hypotheses, respectively. Since the transmit signals $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_K(t)]^T$ is known and deterministic under both $\mathcal{H}_0$ and $\mathcal{H}_1$, the received vectors are jointly zero-mean and Gaussian , and their cross covariance under $\mathcal{H}_1$ is given by

$$[\mathbf{R}_{l_1,l_2}]_{n_1,n_2} = E[r_{l_1,n_1}r^*_{l_2,n_2}] = \frac{E_s}{K}\delta_{l_1-l_2} \times \sum_{k_1=1}^{K}\sum_{k_2=1}^{K} E[h_{l_1k_1}h^*_{l_2k_2}]s_{k_1}(n_1T_s-\tau)s^*_{k_2}(n_2T_s-\tau) + \delta_{l_1-l_2}[\mathbf{R}_{w,l_1}]_{n_1,n_2}, \tag{2}$$

for all $n_1, n_2 \in \{1, \cdots, N\}$ and $l_1, l_2 \in \{1, \cdots, L\}$. Denoting $[\mathbf{R}_s]_{n_1,n_2} \triangleq \frac{E_s}{K}\sum\limits_{k=1}^{K} s_k(n_1T_s-\tau)s^*_k(n_2T_s-\tau)$, we obtain $\mathbf{R}_{l_1,l_2} = \delta_{l_1-l_2}(\sigma^2\mathbf{R}_s + \mathbf{R}_{w,l_1})$. Therefore, the distributions of the received signal vector $\mathbf{r} \triangleq [\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_L]$ under hypotheses $\mathcal{H}_1$ and $\mathcal{H}_0$ are zero-mean complex circular symmetrical Gaussian as follows

$$f_{\mathbf{r}|\mathcal{H}_0}(\mathbf{r};\mathbf{R}_{w,l}) = \frac{\exp\left(-\sum\limits_{l=1}^{L}\mathbf{r}_l^H\mathbf{R}_{w,l}^{-1}\mathbf{r}_l\right)}{\pi^{LN}\prod\limits_{l=1}^{L}|\mathbf{R}_{w,l}|},$$
$$f_{\mathbf{r}|\mathcal{H}_1}(\mathbf{r};\sigma^2,\mathbf{R}_{w,l}) = \frac{\exp\left(-\sum\limits_{l=1}^{L}\mathbf{r}_l^H(\sigma^2\mathbf{R}_s+\mathbf{R}_{w,l})^{-1}\mathbf{r}_l\right)}{\pi^{LN}\prod\limits_{l=1}^{L}|\sigma^2\mathbf{R}_s+\mathbf{R}_{w,l}|}, \tag{3}$$

where $|\cdot|$ represents the absolute value of the determinant of a square matrix, $\mathbf{R}_{w,l} = \sigma^2_{w,l}\mathbf{I}$ and σ^2 and $\sigma^2_{w,l}$ are unknown.

In the following, the UMPI decision statistic is derived in Section II which depends on some of unknown parameters. The GLRT and SFET are proposed in Section III and Section IV, respectively. In Section V, simulation results are presented and the performances of the SFET and GLRT are compared with the upper invariant bound. Section VI is the conclusion of paper.

II. UMPI Test

Consider the scale group defined as bellow

$$G_s = \{g_c(\mathbf{r}) = c\mathbf{r}, c \in \mathbb{C}, c \neq 0\}, \tag{4}$$

where $\mathbb{C}$ is set of complex numbers. The hypothesis test problem is invariant under G_s, because for all $c \in \mathbb{C}, c \neq 0$, the probability density function (pdf) of $c\mathbf{r}$ is zero-mean complex Gaussian with covariance matrix $|c|^2\sigma^2_w\mathbf{I}$, and the pdf of $c\mathbf{r}$ under $\mathcal{H}_1$ is zero-mean complex Gaussian vector with covariance matrix $|c|^2\sigma^2\mathbf{R}_s + |c|^2\sigma^2_w\mathbf{I}$. According to pdf of $c\mathbf{r}$ under $\mathcal{H}_0$ and $\mathcal{H}_1$, the induced transformations on the parameters under each hypothesis is explained as below

$$\overline{G}_s|\mathcal{H}_0 = \left\{\overline{g}_s; \overline{g}_s\begin{pmatrix} 0 \\ \sigma^2_w \end{pmatrix} = \begin{pmatrix} 0 \\ |c|^2\sigma^2_w \end{pmatrix}\right\} \tag{5}$$

$$\overline{G}_s|\mathcal{H}_1 = \left\{\overline{g}_s; \overline{g}_s\begin{pmatrix} \sigma^2 \\ \sigma^2_w \end{pmatrix} = \begin{pmatrix} |c|^2\sigma^2 \\ |c|^2\sigma^2_w \end{pmatrix}\right\}. \tag{6}$$

σ^2_w in each hypothesis and σ^2 are positive parameters, on the other hand, $|c|^2 > 0$, therefore, $|c|^2\sigma^2_w$ in each hypothesis and $|c|^2\sigma^2$ are positive parameters. It is shown that the origin parameter space is invariant under G_s. The maximal invariant vector with respect to G_s is given by $\mathbf{m}_s = \left[\frac{r_0}{r_{NL-1}}, \cdots, \frac{r_{NL-2}}{r_{NL-1}}, 1\right]^T$ [13]. It can be shown that the pdf of $\mathbf{m}$ under $\mathcal{H}_i$ for $i = 0, 1$ is given as

$$f_{\mathbf{m}_s|\mathcal{H}_i}(\mathbf{m}_s) = \frac{(NL-1)!}{\pi^{NL-1}\beta_i^{NL}|i\rho\mathbf{R}_{tot}+\mathbf{I}|}, \tag{7}$$

where ρ is defined by $\rho \triangleq \frac{\sigma^2}{\sigma^2_w}$ and β_i is given by

$$\beta_i = \mathbf{m}_s^H(i\rho\mathbf{R}_{tot}+\mathbf{I})^{-1}\mathbf{m}_s. \tag{8}$$

Also $\mathbf{R}_{tot}$ is a $NL \times NL$ block diagonal matrix that its diagonal elements are $\mathbf{R}_s$ and other elements are zero. The UMPI statistic is given by constructing the likelihood ratio of $\mathbf{m}_s$ using the pdf of the maximal invariant statistic in (7). Hence the likelihood ratio is

$$\frac{f_{\mathbf{m}_s|\mathcal{H}_1}(\mathbf{m}_s)}{f_{\mathbf{m}_s|\mathcal{H}_0}(\mathbf{m}_s)} = \frac{1}{|\rho\mathbf{R}_{tot}+\mathbf{I}|}\left(\frac{\beta_0}{\beta_1}\right)^{NL}, \tag{9}$$

The likelihood ratio in (9) is increasing by $\frac{\beta_0}{\beta_1}$. Hence, the UMPI detector rejects $\mathcal{H}_0$, if $\frac{\beta_0}{\beta_1} > \eta_{\text{UMPI}}$, where, η_{UMPI} is set to P_{fa} requirement and $\frac{\beta_0}{\beta_1} = \frac{\|\mathbf{m}_s\|^2}{\mathbf{m}_s^H(\rho\mathbf{R}_{tot}+\mathbf{I})^{-1}\mathbf{m}_s}$. This statistic depends on ρ which is unknown, hence the UMPI test does not exist for this problem. However, we can use the achieved UMPI test as a performance bound and refer to it as the Most Powerful Invariant (MPI) test.

III. GLRT

The GLRT is given by substituting the Maximum Likelihood Estimation (MLE) of the unknown parameters into the likelihood ratio. To derive the MLE of unknown parameters, consider the pdf of $\mathbf{r}$ under $\mathcal{H}_1$ in (3) as follows

$$f_{\mathbf{r}|\mathcal{H}_1}(\mathbf{r};\sigma^2,\sigma^2_w) = \frac{\exp(-\mathbf{r}^H(\sigma^2\mathbf{R}_{tot}+\sigma^2_w\mathbf{I})^{-1}\mathbf{r})}{\pi^{NL}|\sigma^2\mathbf{R}_{tot}+\sigma^2_w\mathbf{I}|}. \tag{10}$$

Considering $\rho \triangleq \frac{\sigma^2}{\sigma^2_w}$ and using the eigenvalue decomposition of $\mathbf{R}_{tot} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, we have

$$f_{\mathbf{r}|\mathcal{H}_1}(\mathbf{r};\rho,\sigma^2_w) = \frac{\exp\left(\frac{-1}{\sigma^2_w}\sum\limits_{n=0}^{NL-1}\frac{|x_n|^2}{\rho\lambda_n+1}\right)}{\pi^{NL}\sigma_w^{2NL}\prod\limits_{n=0}^{NL-1}(\rho\lambda_n+1)}, \tag{11}$$

where, x_n's are elements of $\mathbf{x} \triangleq \mathbf{U}^H\mathbf{r}$ and λ_n's are the elements of Λ. The MLEs of σ^2_w under $\mathcal{H}_1$ and $\mathcal{H}_0$ are

$$\widehat{\sigma}^2_{w|\mathcal{H}_i} = \frac{1}{NL}\sum_{n=0}^{NL-1}\frac{|x_n|^2}{i\rho\lambda_n+1}, i = 0, 1. \tag{12}$$

The MLE of ρ is $\widehat{\rho}$ at which the pdf under $\mathcal{H}_1$ is maximum. Substituting $\widehat{\sigma}^2_{w|\mathcal{H}_1}$ and $\widehat{\rho}$ into $f_{\mathbf{r}|\mathcal{H}_1}(\mathbf{r};\rho,\sigma_w^2)$ gives

$$f_{\mathbf{r}|\mathcal{H}_1}(\mathbf{r};\widehat{\rho},\widehat{\sigma}^2_{w|\mathcal{H}_1}) = \max_{\rho\in(0,\infty)}\left\{\frac{\exp(-NL)}{\left(\frac{\pi}{NL}\sum_{n=0}^{NL-1}\frac{|x_n|^2}{\rho\lambda_n+1}\right)^{NL}\prod_{n=0}^{NL-1}(\rho\lambda_n+1)}\right\} = \frac{\exp(-NL)}{\min_{\rho\in(0,\infty)}\left\{\left(\frac{\pi}{NL}\sum_{n=0}^{NL-1}\frac{|x_n|^2}{\rho\lambda_n+1}\right)^{NL}\prod_{n=0}^{NL-1}(\rho\lambda_n+1)\right\}} \quad (13)$$

According to (3), if ρ is equal to zero, $f_{\mathbf{r}|\mathcal{H}_1}(\mathbf{r};\rho,\sigma_w^2)$ is the same as the pdf of $\mathbf{r}$ under $\mathcal{H}_0$. By substituting $\widehat{\sigma}^2_{w|\mathcal{H}_0} = \frac{1}{NL}\sum_{n=0}^{NL-1}|x_n|^2$ into the pdf of $\mathbf{r}$ under $\mathcal{H}_0$ the maximum of this probability function is given as

$$f_{\mathbf{r}|\mathcal{H}_0}(\mathbf{r};\widehat{\sigma}^2_{w|\mathcal{H}_0}) = \frac{\exp(-NL)}{\left(\frac{\pi}{NL}\sum_{n=0}^{NL-1}|x_n|^2\right)^{NL}}. \quad (14)$$

Using (14) and (13), the GLRT statistic is obtained by

$$L_{\mathrm{GLR}}(\mathbf{x}) = \frac{\left(\sum_{n=0}^{NL-1}|x_n|^2\right)^{NL}}{\min_{\rho\in(0,\infty)}\left\{\left(\sum_{n=0}^{NL-1}\frac{|x_n|^2}{\rho\lambda_n+1}\right)^{NL}\prod_{n=0}^{NL-1}(\rho\lambda_n+1)\right\}}. \quad (15)$$

Thus the GLRT rejects $\mathcal{H}_0$ if $L_{\mathrm{GLR}} > \eta_{\mathrm{GLR}}$, where, η_{GLR} is set to satisfy P_{fa}.

The test in (15) is not a practical test, because we need to minimize the denominator of (15). In order to maximize the pdf under $\mathcal{H}_1$ with respect to ρ, we set $\frac{\partial}{\partial\rho}\ln(f_{\mathbf{r}|\mathcal{H}_1}(\mathbf{r};\rho,\sigma_w^2)) = 0$. Hence, we have

$$\frac{1}{\sigma_w^2}\sum_{n=0}^{NL-1}\frac{\lambda_n|x_n|^2}{(\rho\lambda_n+1)^2} - \sum_{n=0}^{NL-1}\frac{\lambda_n}{\rho\lambda_n+1} = 0 \Rightarrow$$
$$\rho = \frac{1}{\frac{1}{\sigma_w^2}\sum_{n=0}^{NL-1}\frac{\lambda_n|x_n|^2}{(\rho\lambda_n+1)^2} - \sum_{n=1}^{NL-1}\frac{\lambda_n}{\rho\lambda_n+1}} - \frac{1}{\lambda_0} \quad (16)$$

Now, by considering (12) and (16) , an iterative sequence of $\hat{\rho}^{(i)}$ can be defined as follow to reach $\hat{\rho}$

$$\begin{cases} \widehat{\sigma^2}^{(i)}_{w|\mathcal{H}_1} = \frac{1}{NL}\sum_{n=0}^{NL-1}\frac{|x_n|^2}{\hat{\rho}^{(i)}\lambda_n+1} \\ \hat{\rho}^{(i+1)} = \frac{1}{\frac{1}{\widehat{\sigma^2}^{(i)}_{w|\mathcal{H}_1}}\sum_{n=0}^{NL-1}\frac{\lambda_n|x_n|^2}{(\hat{\rho}^{(i)}\lambda_n+1)^2} - \sum_{n=1}^{NL-1}\frac{\lambda_n}{\hat{\rho}^{(i)}\lambda_n+1}} - \frac{1}{\lambda_0} \end{cases} \quad (17)$$

where, $\rho^{(0)} = 0$. Hence, we can define an iterative GLRT by substituting $\hat{\rho}^{(i)}$ into (15), and the iterative GLRT rejects $\mathcal{H}_0$ if

$$\frac{\left(\sum_{n=0}^{NL-1}|x_n|^2\right)^{NL}}{\left(\sum_{n=0}^{NL-1}\frac{|x_n|^2}{\hat{\rho}^{(i)}\lambda_n+1}\right)^{NL}\prod_{n=0}^{NL-1}(\hat{\rho}^{(i)}\lambda_n+1)} > \eta^{(i)} \quad (18)$$

where, $\eta^{(i)}$ is the threshold for the i^{th} iteration of the iterative GLRT. In the simulation results we show that this method has an acceptable convergence rate.

IV. SFET

In [12], the SFET as a suboptimal test is proposed . This test is given by the estimation of a Separating Function (SF). In order to find an SF, we rewrite the hypothesis testing problem as $\mathcal{H}_0 : \theta \in \Theta_0$ against $\mathcal{H}_1 : \theta \in \Theta_1$, where, $\theta = [\sigma^2, \sigma_w^2]$ and $\Theta_i = \{[i\sigma^2, \sigma_w^2]; \sigma_w^2 > 0, \sigma^2 > 0\}$ for $i = \{0, 1\}$. For this problem the Signal to Noise Ratio (SNR) $\rho = \sigma^2/\sigma_w^2$ is a SF for (3), because $\rho > 0$ if and only if $\theta \in \Theta_1$ and $\rho = 0$ if and only if $\theta \in \Theta_0$. Thus, the SFET rejects $\mathcal{H}_0$ if $\widehat{\rho} > \eta_{\mathrm{SFET}}$. We must note that the MLE of ρ is calculated with respect to $\mathcal{H}_1 \cup \mathcal{H}_0$, i.e., $\rho \in [0,\infty)$. The MLE of ρ is one parameter at which the pdf in (13) is maximum, therefore, we have

$$\hat{\rho} = \arg\max_{\rho\in[0,\infty)} f_{\mathbf{r}|\mathcal{H}_1}(\mathbf{r};\widehat{\rho})$$
$$= \arg\max_{\rho\in[0,\infty)} \frac{\exp(-NL)}{\left(\frac{\pi}{NL}\sum_{n=0}^{NL-1}\frac{|x_n|^2}{\rho\lambda_n+1}\right)^{NL}\prod_{n=0}^{NL-1}(\rho\lambda_n+1)}$$
$$= \arg\min_{\rho\in[0,\infty)} \left(\frac{\pi}{NL}\sum_{n=0}^{NL-1}\frac{|x_n|^2}{\rho\lambda_n+1}\right)^{NL}\prod_{n=0}^{NL-1}(\rho\lambda_n+1).$$

This test statistic is invariant with respect to G_s, because for any $c \in \mathbb{C}$, we have $c\mathbf{r} = \mathbf{U}c\mathbf{x}$ then

$$\arg\min_{\rho\in[0,\infty)} \left(\frac{\pi}{NL}\sum_{n=0}^{NL-1}\frac{|cx_n|^2}{\rho\lambda_n+1}\right)^{NL}\prod_{n=0}^{NL-1}(\rho\lambda_n+1)$$
$$= \arg\min_{\rho\in[0,\infty)} \left(\frac{\pi}{NL}\sum_{n=0}^{NL-1}\frac{|x_n|^2}{\rho\lambda_n+1}\right)^{NL}\prod_{n=0}^{NL-1}(\rho\lambda_n+1).$$

Note that in the latest equation, $|c|^2$ is the amplitude of statistic and we can remove it in the maximization.

According to (17), we can find $\hat{\rho}^{(i)}$ by an iterative method. Hence, using the estimation in (17), the SFET rejects $\mathcal{H}_0$ if $\hat{\rho}^{(i)} > \eta^{(i)}$, where, $\eta^{(i)}$ set to P_{fa} requirements and $\hat{\rho}^{(i)}$ is given by (17).

V. Simulation Results

In this section, we provide numerical examples to illustrate the performances of the UMPI bound, the iterative GLRT, and the iterative SFET. As we discussed in Section II, since ρ is unknown, the UMPI test cannot be implemented. Nevertheless, in computer simulations that the values of ρ is known, we can use the UMPI test as an upper performance bound. In these simulations, the MIMO radar is considered as a narrow-band system with K transmit and L receive antennas. Let $s_k(n) = e^{j\omega_k n}$ for $n = 0, \cdots, N-1$ and $k = 1, \cdots, K$, be a complex exponential transmit signal at the k^{th} transmitter, where $\omega_k = (k-1)\Delta\omega$ and $\Delta\omega$ is the two by two increment between transmitter carriers, which assumed $\Delta\omega = 2\pi/K$. In all simulations, we consider $N = 10$. In the first simulation, we have selected $K = L = 5$. Figure 1 provides the results of this simulations as the probability of detection P_d versus ρ for $P_{\mathrm{fa}} = 0.001$. It is seen that the performance of the iterative SFET is close to MPI bound and the SFET outperforms the GLRT. In [12], it is shown that the SFET is asymptotically optimal. However in this case the performance of SFET tends to the MPI bound when the ρ increases. Figure 2 shows the probability of detection versus the false alarm probability

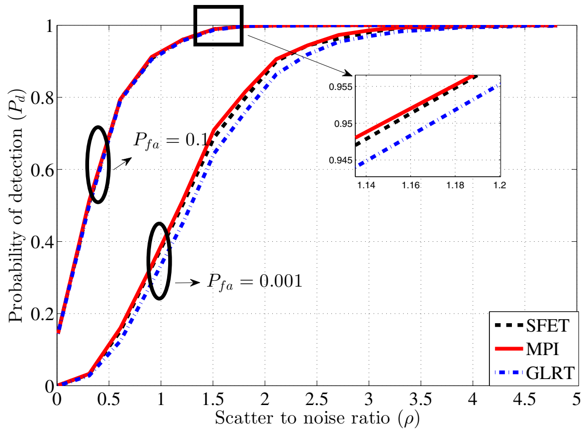

Fig. 1. Probability of detection versus the scatter to noise ratio (ρ), for $P_{\rm fa} = 0.001$, $K = L = 5$, $N = 10$.

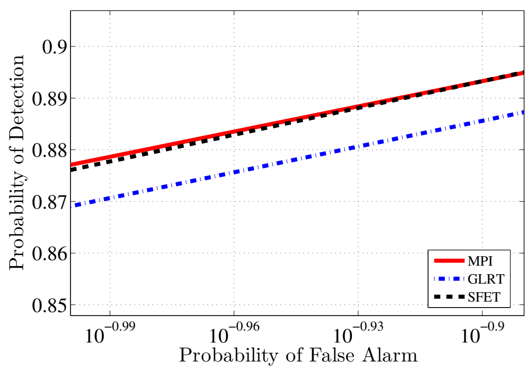

Fig. 2. Probability of detection versus Probability of false alarm, for $\rho = 0.12$, $K = L = 5$, $N = 10$.

for the MPI bound, iterative SFET and itrative GLRT for $N = 10, K = L = 5$, and $\rho = 2$. This simulation shows that the SFET outperforms the GLRT.

VI. Conclusion

In this paper, the target detection problem for a WSA MIMO radar was addressed. The UMPI test was derived for this problem and it was shown that this test depends on the scatter to noise ratio. Hence the UMPI for this problem does not exist and the proposed test in simulations provides the upper bound for the invariant tests when the scatter to noise ratio is assumed to be known. We proposed an SFET, which is asymptotically optimal. The simulation results showed that the performance of this test tends to the optimal invariant bound by increasing the scatter to noise ratio. A GLRT was also proposed for this problem. Since in this problem the SFET and GLRT did not have a closed form, we proposed two iterative methods to implement the SFET and GLRT.

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