Wave Equation Receiver Deghosting

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***Abstract*— Current solutions to receiver deghosting generally involve making complementary measurements of the wavefield or, alternatively, involve estimation of data not recorded due to ghost interference. Both solutions offer challenges in practice in that marine multimeasurement streamers are commercially available only on a limited basis and existing single-measurement deghosting methods must estimate unrecorded frequencies near the ghost notches. Here, we develop a new wave equation-based approach for single measurement deghosting that does not rely on such estimation procedures.**

I. INTRODUCTION

The deleterious nature of the so-called receiver ghost effect, which is defined as the interference of up and downgoing portions of a seismic wavefield at the receiver, are well-known. The ghost, which generally refers to the downgoing wavefield that is generated when the upgoing wavefield reflects or scatters off the air-water interface, is problematic in two respects. It introduces amplitude and phase distortions that, for example, attenuate frequencies where destructive interference is significant. Certain frequencies can be totally annihilated at the receiver and are known as notch frequencies. Perhaps more importantly, many seismic applications are founded on the assumption that the recorded data are an upward travelling wavefield. As a result, accuracy, bandwidth and resolution of the data may be limited. Many specific and detailed approaches have been proposed for solving this problem but, in general, these efforts can be classified in three groups. The first approach involves making two or more complementary measurements of the seismic wavefield and then combining the measurements, exploiting the complementary nature of their ghosts, to eliminate the ghost effect. This approach was first introduced for sources ([1]) and then for receivers ([2], [3]). While this concept gained early commercial acceptance for ocean-bottom cable acquisition using collocated hydrophones and geophones ([4]), commercial implementations of multimeasurement marine streamers are relatively new ([5], [6]).

Our second category of solutions to the ghost problem is broad and diverse; however, we see a common theme in methods that utilize only one measurement and attempt to rectify the ghost effects by estimating the desired upcoming wavefield through statistical or other means. Deconvolution, recursive filters and other approaches based on a spectral model of the ghost fall into this category. Other single-measurement approaches attempt to aid the estimation process by introducing diversity in the notches by deploying receivers at varying depths, but not collocated ([7]-[9]). The third approach involves first estimating an unrecorded complementary quantity such as particle velocity or pressure gradient from the data and then deghosting the data as in the first category ([10], [11]).

Each of these approaches offers advantages and disadvantages. In theory, making complementary measurements in the field is the most desirable; however, in practice, this requires special equipment and perhaps extra field effort and precision of measurements. On the other hand, existing processing-only solutions involving single measurements can be limited in their ability to estimate unrecorded data. Significantly, nearly all of the existing marine seismic data are single measurement (pressure only) and were acquired with nearly flat streamers and thus could benefit from an accurate, single-measurement deghosting procedure. While a tutorial would be beneficial for those not familiar with the problem, this is beyond the scope of this paper. A very nice exposition of common commercial deghosting approaches is given in [12].

In this work, we introduce a new approach to receiver deghosting based on separation of the up and downgoing wavefields using the wave equation. While applicable to other data types, we focus on the marine streamer case and illustrate that a long-held but unproven belief – that more than one measurement is required to rigorously do an up/down wavefield separation – is not justified. Our approach is similar to migration in that we use the wave equation to simulate propagation of the up and downgoing wavefields between the receivers and the water surface to effect a separation. The key to our approach involves our observation that the desired upcoming wavefield is causal with respect to the downgoing wavefield, a fact we find absent – at least explicitly – in other approaches.

II. MOTIVATION AND KIRCHHOFF FORMULATION

We begin with a simple example that illustrates the ghost problem in a manner different from traditional approaches. By definition, the ghost results when the downgoing wavefield interacts with the upgoing wavefield causing constructive and destructive interference. Figure 1 illustrates this interference from the perspective of the downgoing wavefield at the streamer. For the downgoing wavefield (associated with the seismic source), $D_i(t)$, to exist at time t and location i on the streamer, there must have been earlier upcoming events that generate it. Figure 1 shows two components of the downgoing wavefield that arise due to upcoming events that arrive and are

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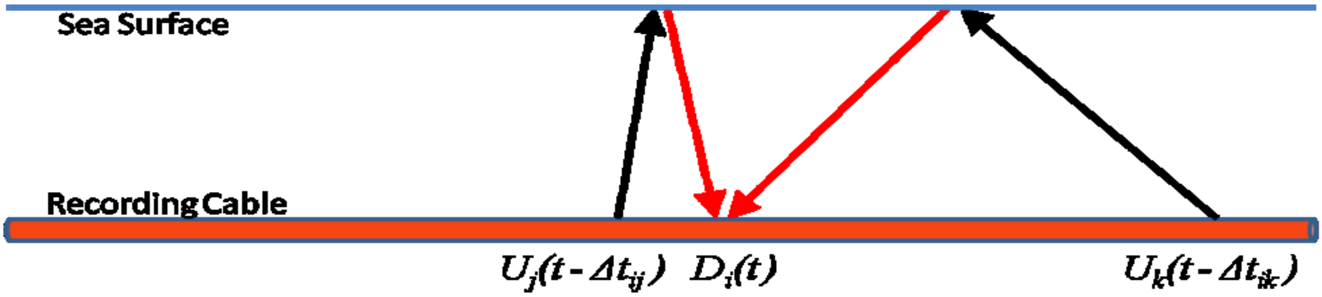

Fig. 1. Interaction of the up and downgoing wavefields between the streamer and water surface (blue). Components of the downgoing wave (red) are generated by earlier elements of the upcoming wavefield (black). In general, the downgoing wavefield is comprised of the sum of all such components.

recorded at earlier times and other locations on the receiver cable.

Assuming noise and first arrivals have been removed, a common assumption in other deghosting methods, the total downgoing wavefield $D_i(t)$ is the sum of all such possible elements and can be written as

$$D_i(t) = \sum_j A_{ij} U(t-\Delta t_{ij}) \tag{1}$$

where Δt_{ij} is the traveltime for the upgoing wave recorded at location j to reach location i and A_{ij} is an amplitude term to account for reflection and geometrical spreading. Details such as derivations of A_{ij}, Δt_{ij}, and phase (rho filter) are not developed here as actual implementation will be done using wave equation propagators.

The total wavefield W at time t is the sum of the up and downgoing parts so $W(t) = U(t) + D(t)$ and substituting for D from equation (1) we find that $W_i(t) = U_i(t) + \sum_j A_{ij} U_j(t-\Delta t_{ij})$. Solving for U gives the relation

$$U_i(t) = W_i(t) - \sum_j A_{ij} U_j(t-\Delta t_{ij}). \tag{2}$$

As U is causal, initial values of U are available (at least for $t < 2d/V$ where d is the receiver depth and V is water velocity) so equation (2) suggests an iterative scheme for computing the upcoming wavefield. It is easy to see that going from a discrete sum to an integral formulation of equation (2) yields something reminiscent of migration. Indeed the Green's function looks like an inverted and shifted Kirchhoff migration operator applied to the upgoing wavefield. This integral is calculated at earlier times and then subtracted from the full wavefield to give the upgoing wavefield at the new time. This formulation is simple and intuitive, but handling such near-field Kirchhoff operators is difficult in practice. Moreover, allowing for variable sea surface, water velocity variations and other complexities is more natural in a wave equation setting, which we develop in the next section.

III. Wave Equation Deghosting

By definition, deghosting, or "solving the ghost problem" amounts to extracting the upcoming wavefield U from the total wavefield W. Motivated by the previous section, let $W(x,y,z,t) = U(x,y,z,t) + D(x,y,z,t)$ where W is the recorded wavefield for a single shot (direct arrival removed) and U and D are the up and downgoing wavefields, respectively. To find the up and downgoing wavefields, we wish to calculate $U(x_r,y_r,z_r,t)$ and $D(x_r,y_r,z_r,t)$ for all $t > 0$ where (x_r, y_r, z_r) denotes the receiver positions. Appealing to causality, note that, for t sufficiently small $(t < 2d/V)$, $U(x_r,y_r,z_r,t) = W(x_r,y_r,z_r,t)$. For simplicity, assume a sea surface with reflection coefficient of -1, so the downgoing wavefield is $D(x_s,y_s,z_s,t) = -U(x_s,y_s,z_s,t)$ where (x_s,y_s,z_s) is the sea surface at time t. This boundary condition can be replaced by more complex reflection or scattering mechanisms if desired and, in particular, it can accommodate time-varying sea surfaces. Having defined U and D at early times using causality, we calculate the desired up and downgoing wavefields at the receivers iteratively in time. Given $U(x,y,z,t)$ and $D(x,y,z,t)$ first extrapolate the existing U and D forward in time with time step Δt:

$$U(x,y,z,t + \Delta t) = P^+(\Delta t)\, U(x,y,z,t),\ z_s \le z < z_r \text{ and } D(x,y,z,t + \Delta t) = P^-(\Delta t)\, D(x,y,z,t),\ z_s < z \le z_r \tag{3}$$

where $P^+(\Delta t)$ and $P^-(\Delta t)$ are upward and, respectively, downward wave propagation operators that propagate wavefields forward in time by the time step Δt. After a time step is taken using equation (3), the iteration is completed by computing the boundary values for the next iteration:

$$U(x_r,y_r,z_r,t + \Delta t) = W(x_r,y_r,z_r,t + \Delta t) - D(x_r,y_r,z_r,t + \Delta t) \text{ and } D(x_s,y_s,z_s,t + \Delta t) = -U(x_s,y_s,z_s,t + \Delta t). \tag{4}$$

Equations (3) and (4) are iterated to generate $U(x_r,y_r,z_r,t)$ and $D(x_r,y_r,z_r,t)$ for all $t > 0$.

Equations (3) and (4) essentially replace full two-way wave propagation with one-way propagators linked by boundary conditions at the sea surface and at the receivers which is reminiscent of the so-called Bremmer coupling series [13]. The key that allows this decomposition is that, by causality of the upcoming wavefield, an initial boundary condition is available. In general two-way wave propagation problems, this would not be the case. We have said earlier that the method is reminiscent of seismic migration in that we simulate and reverse the effects of propagation with wave extrapolation operators. However, our approach differs from migration in that we do not attempt to focus or image the

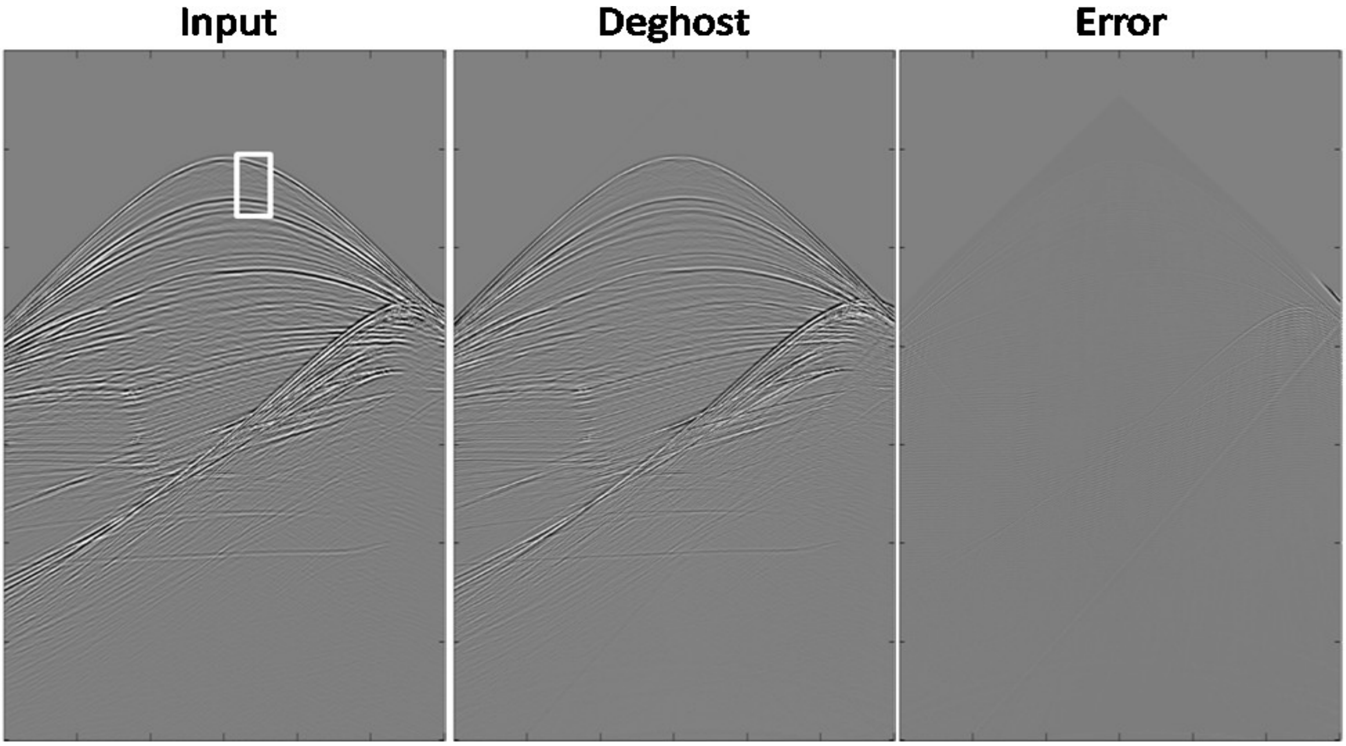

Fig. 2. Wave equation deghosting applied to a shot from the Gulf of Mexico SEAM model. The error is calculated as the difference between the deghosted result and data (not shown) modeled with no free surface boundary condition. Multiples were not modeled. The error shows small differences between the two. The small box on the input data indicates the position of spectra shown in Figure 3.

recorded wavefield, but instead only seek to remove the downgoing wavefield. An excellent discussion of migration and one- and two-way wave propagators can be found in [14].

IV. Example from the SEAM Model

We tested the deghosting approach described in equations (3) and (4) (modified for 2D) on a shot from the Gulf of Mexico SEG advanced model (SEAM) which is a geologically complex model that includes significant salt bodies. Modelling and deghosting were all done in 2D and multiples were not modelled. Figure 2 shows a comparison of the input data, the deghosted shot and the error, which is the subtraction of the deghosted shot from a shot modelled with no free surface boundary condition. Other than edge effects related to boundary treatment, very small errors are observed.

Figure 3 shows a comparison of spectra computed in the small box annotated on the input section of Figure 2. The first notch around 15 Hz on the input (red curve) is shown to be nicely recovered and the deghosted spectrum shows excellent agreement with the no-ghost data generated without the free surface boundary conditions.

V. Discussion and Conclusions

We have shown using noise free synthetic data that pressure-only data can be deghosted using the wave equation to propagate the wavefields between the receivers and the sea surface as happens in the actual seismic experiment. This is an extremely interesting theoretical result since it contradicts the traditional wisdom that rigorous deghosting can only be accomplished with multi-measurement acquisition. Naturally, it brings into question the relationship between this approach and other methods that are based on the conventional ghost model typically formulated in the frequency-wavenumber domain as a relationship between the recorded wavefield W and the desired upcoming wavefield U as $W = GU$ where

$$G = G(z, f, k_x, k_y) = 1 - e^{-i4\pi k_z z}$$

with $k_z = \sqrt{\left(\frac{f}{c}\right)^2 - k_x^2 - k_y^2}$, c the water velocity, and z the streamer depth. To solve for U in this setting, one must

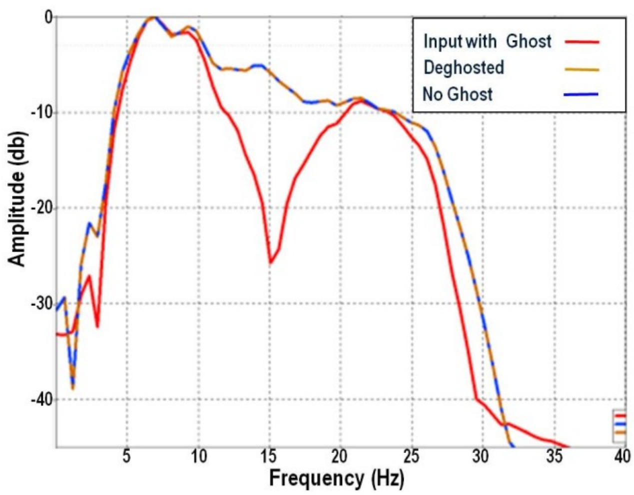

Fig. 3. Average spectra comparing input, deghosted and no-ghost data. There is excellent agreement between the deghosted data and the no-ghost data. Compare brown and blue curves.

invert G which is clearly a problem at the notch frequencies where $G = 0$. While it is beyond the scope of this paper to analyse the relationship of wave equation deghosting to other approaches based on the ghost model, it is clear that it is not simply a reformulation of the traditional ghost model. For instance, as shown in Figure 3, it is possible to directly recover the notch frequency. Again, it is not shown here, but this apparent contradiction is resolved by noting that causality implies a more complicated spectrum than that assumed by the traditional ghost model and in particular, the so-called notch frequencies are not strictly zero. So, inversion is theoretically possible. While wave equation deghosting is different in some respects from most modern approaches to deghosting, it is closely related to Lindsay's recursive filter approach [15]. If one reduces wave equation deghosting to 1D, wave propagation becomes a simple time shift which yields Lindsey's method. Indeed, Lindsey's method points to potential issues with wave equation deghosting since it becomes unstable in the presence of noise. We will conclude with a discussion of such challenges associated with making wave equation deghosting a robust and commercial method.

As might be expected, practical issues arise in application of wave equation deghosting to real data. Water velocity and receiver depth must be known or estimated. Noise that contains a ghost is handled properly but other types of noise such as direct arrivals must be addressed. Fortunately, although noise that is not part of the wavefield can be magnified, unlike the 1D case which becomes unstable in the presence of noise, higher dimensional implementations appear to be stable. Further, the data must be sampled densely for accurate wave propagation, all of which are issues common to other deghosting methods. However, our approach has potential advantage in that it is accurate for all dips, handles wavefield complexity, variable velocity, variable sea state and acquisition geometry, makes no explicit assumptions about the spectrum, and does not estimate unrecorded data; rather we use the wave equation to propagate the recorded data to remove the ghost and also offer the possibility to complement multimeasurement data. Finally, the recursive nature of the algorithm implies that it will be sensitive to errors in estimates of quantities such as water velocity and receiver depth. While this presents a challenge in using wave equation deghosting to deghost the data, there is an upside to this problem; it is likely that this sensitivity could be exploited to invert for quantities such as water velocity or receiver depth. In the future, we expect to pursue this and the general question of how wave equation deghosting might be made more robust in the presence of errors in physical parameters and noise.

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