Distributed Throughput Maximization for Multi-Channel ALOHA Networks

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***Abstract*— We consider the problem of distributed throughput maximization for multi-channel ALOHA networks. We focus on networks containing a large number of users that transmit over a low number of channels. First, we consider the problem of constrained distributed rate maximization, where user rates are subject to total transmission probability constraints. We propose a distributed best-response algorithm to solve the rate maximization problem, where each user updates its strategy using its local channel state information (CSI) and by monitoring the channel utilization. We then consider the case where users are not restricted by transmission probability constraints. Distributed optimization of the network throughput under uncertainty is mandatory since the transmission probabilities of other users are unknown. We propose a distributed scheme to solve the throughput optimization problem under uncertainty, where users adjust their transmission probability to maximize their rates, but maintain the desired load on the channels. We propose sequential and parallel algorithms for this purpose.**

***Index Terms*—** Collision channels, multi-channel ALOHA, distributed optimization, best-response dynamics.

I. Introduction

The opportunistic spectrum access problem in cognitive networks has been widely investigated recently due to the increasing demand for radio spectrum. The technology enabling different intelligent devices and networks to co-exist in the same frequency band is called cognitive radio. In this paper we examine Medium Access Control (MAC) schemes used to enable a large number of users to co-exist in a typically low number of shared channels. We investigate multi-channel ALOHA networks, where users access a channel according to a specific transmission probability. Transmission is successful if only a single user transmits over a shared channel in a given time-slot. However, if two or more users transmit simultaneously over the same channel, a collision occurs. ALOHA-based protocols are widely used in wireless communication primarily because of their ease of implementation and their random nature. Simple transmitters can randomly access a channel without a carrier sensing operation. A related work on single and multi-channel ALOHA networks can be found in [1]–[4].

In wireless communication networks, distributed algorithms are generally preferred over centralized solutions. In this paper we focus on distributed algorithms in multi-channel ALOHA networks. The problem of multi-radio multi-channel allocation was investigated in [3], [5]–[7]. In [6], a distributed learning algorithm was proposed that converges in some special cases. In the multi-radio multi-channel allocation model, the utility of each channel decreases with the number of radios transmitting over it. This is generally done by a TDMA protocol, for instance, among users who transmit over the same channel. As a result, users are encouraged to spread resources over channels. This is not the case in our setup. In [8], the multi-channel ALOHA protocol in cognitive radio networks was analyzed, where secondary users choose randomly one of the idle channels for transmission. In [9], [10], the opportunistic multi-channel ALOHA scheme was analyzed for i.i.d Rayleigh fading channels. In this scheme, a user transmits over channels with gains greater than some threshold. Good surveys of networking games can be found in [11], [12]. In [13]–[16], distributed optimization algorithms of a single-channel ALOHA networks using game theoretic tools are studied, where the utility of each user increases with the transmission probability. Here we consider a generalization of that model to the multichannel case.

First, we consider the case where heterogenous users exploit the CSI and the channel utilization to increase their utility under fixed transmission probability constraint. In our previous work [17], [18] we mainly focused on networks containing homogeneous users. However, here we extend the result to general heterogenous networks, where each user in the network may have a different transmission probability constraint. We propose a best-response dynamics, where users make autonomous decisions using their local CSI and by monitoring the load on the channels. The proposed best-response dynamics converges in finite time starting from any point. Next, we consider the case where users are not restricted by a transmission probability constraint. Users are required to implement distributed optimization of the network throughput under uncertainty due to unknown transmission probabilities of other users. We propose distributed algorithms to solve the throughput optimization problem under uncertainty, where users adjust their transmission probability to maximize their rates, but maintain the desired load on the channels. We propose sequential and parallel algorithms for this purpose.

II. Network Model

Consider a wireless network containing N users who transmit over K orthogonal channels, where $N > K$. The users transmit over the shared channels using the slotted ALOHA protocol. In each time slot each user is allowed to access a single channel according to a specific transmission probability. Transmission is successful if only a single user transmits over a shared channel in a given time-slot. However, if two or more users transmit simultaneously over the same channel, a collision occurs. The collision-free achievable rate of user n at channel k is denoted by $u_n(k) \geq 0$. Let $u_n(0) = 0 \ , \ \forall n$ be a virtual zero-rate channel. Transmitting over a channel where $k = 0$ refers to no-transmission. The achievable rate $u_n(k)$ is given by $u_n(k) = W \log\left(1 + \frac{1}{\lambda}\text{SNR}|h_n(k)|^2\right)$, where W is the channel bandwidth, λ is the SNR gap to capacity and $h_n(k)$ is the fading channel experienced by user n on channel k. In our model $u_n(k)$ represents the instantaneous rate, or the long-term rate (i.e., mean rate). Note that in practical systems, $u_n(k)$ is generally estimated by user n from a pilot signal.

Let $p_n(k)$ be the probability that user n transmits over channel k, where $\sum_{k=0}^{K} p_n(k) = 1 \ \forall n$. When user n perfectly monitors the k^{th} channel utilization, it observes:

$$v_n(k) \triangleq \prod_{i \neq n} (1 - p_i(k)) = 1 - q_n(k) \ , \tag{1}$$

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which is the success probability of user n on channel k.
We further define

$$b_n(k) \triangleq \prod_{i=1}^{N} (1 - p_i(k)) \ , \tag{2}$$

which is the probability that a channel k is available.
The expected rate of user n in the k^{th} channel is given by:

$$r_n(k) \triangleq u_n(k) v_n(k) \ . \tag{3}$$

Hence, the expected rate of user n is given by:

$$R_n \triangleq \sum_{k=1}^{K} p_n(k) r_n(k) \ . \tag{4}$$

III. The Distributed Rate Maximization Problem

In this section we extend the result reported in [17], [18] for the special case of a homogenous network to the general case of a heterogenous network, where every user may have different probability constraint. We are interested in solving the distributed rate maximization problem, where each user tries to maximize its own expected rate subject to a total transmission probability constraint:

$$\max_{p_n(1),\dots,p_n(K)} \quad R_n \qquad \text{s.t.} \qquad \sum_{k=1}^{K} p_n(k) \le P_n \ . \tag{5}$$

To avoid collisions, typically $P_n < 1 \ \forall n$.
The goal of every user n is to set a probability vector $(p_n(1), ..., p_n(K))$ defining the transmission probability over every channel, based on its utility vector and the strategies of all other users. We focus on stationary strategies, where every user n maps the current observed state $\left(\{u_n(k)\}_{k=1}^{K}, \{\{p_i(k)\}_{i \neq n}\}_{k=1}^{K}\right)$ to $(p_n(1), ..., p_n(K))$.
Note that in practical systems, $u_n(k)$ is generally estimated from a pilot signal. On the other hand, complete information on the strategies of all other users is not required. Monitoring the channel to obtain $v_n(k)$ is sufficient to make a decision.

Next, we propose a best-response dynamics to solve the distributed rate maximization problem. First, we show the following result:

Theorem 1: Let $k^* = \arg\max_k \{r_n(k)\}$, where $r_n(k)$ is defined in (3). Then, each user n that solves (5) plays the strategy:

$$p_n(k) = \begin{cases} 1 - P_n \ , & \text{if } k = 0 \\ P_n \ , & \text{if } k = k^* \\ 0 \ , & \text{otherwise} \end{cases} \ , \tag{6}$$

with probability 1.

Theorem 1 extends the result reported in [17]. It states that users transmit over a single channel in each iteration, and may switch strategies according to the load on the channels.
As a result, we obtain a distributed best-response dynamics to solve the rate maximization problem (5). We initialize the algorithm by a simple solution where every user picks the channel with the highest collision-free utility $u_n(k)$. In the learning process step, each user occasionally monitors the channel utilization to obtain $v_n(k)$ for all k. Then the user updates its strategy by selecting the channel with the maximal achievable rate $r_n(k) = u_n(k)v_n(k)$ based on the estimated load. When users cannot increase their rates by unilaterally changing their strategy, an equilibrium is obtained. Similar to [17], [18], it can be shown that the proposed best response dynamics converges in finite time, starting from any point.

IV. Distributed Algorithms for Adaptive Throughput Maximization

In this section we consider a different problem in multi-channel ALOHA networks, where users are not restricted by a transmission probability constraint. Due to space limitation, we provide a general discussion under this setting. For more details, the reader is referred to [19]. Here, users are required to implement a distributed optimization of their rate under uncertainty due to unknown transmission probabilities of the other users. Note that unconstrained distributed rate maximization without self-control on users' transmission probabilities will lead users to increase their probability to 1 and cause collisions. Hence, users should maximize their rate, but maintain the desired load on the channels. Based on the observation that for a large number of users the optimal throughput on each channel is e^{-1} [20], the goal of the proposed algorithms in this section is to cause the system to operate with the desired load on each channel in a distributed fashion. Hence, each user is required to solve the following optimization problem for throughput maximization:

$$\max_{k, P_n} \qquad R_n \qquad \text{s.t.} \qquad b_n(k) = e^{-1} \ . \tag{7}$$

Note that solving (7) may lead to undesirable solutions depending on the dynamic updating of the transmission probabilities across users. For instance, if user n detects channel k as a free channel, i.e., $v_n(k) = 1$, maximizing its transmission probability to get $P_n = 1 - e^{-1}$ (which achieves the desired load) causes any other user that accesses this channel to force its transmission probability to zero (to keep the desired load). Hence, we propose two schemes to obtain the target solutions for all users; namely, parallel and sequential mechanisms, presented in Tables I and II, respectively.

In the sequential updating mechanism, presented in Table I, users adjust their transmission probability until they get the desired channel load. Let $\delta_n(k) \triangleq |b_n(k) - e^{-1}|$. The users' goal is to reduce $\delta_n(k)$ sequentially until convergence.
In the initialization step, all users select the channels with the highest collision-free utility and set their transmission probability to $P_n = p_n^{(0)}(k^*) = p_0 << 1$.
Next, in the learning step, each user occasionally monitors the channel utilization $v_n(k)$ of all channels. After the user has estimated $v_n(k)$ it does the following. First, it computes the highest transmission probability $\tilde{p}_n(k)$ allowed on each channel based on the estimated load, given in step 10. This operation will encourage users to move to channels with low loads. Next, the user compares its current rate $R_n = R_n(k^*)$, given in step 12 to the potential achievable rates on all other channels, $\tilde{R}_n(k)$, given in step 11. For convenience denote $\tilde{R}_n(k^*) = R_n(k^*)$. If there is a channel with a higher potential rate than its current rate, the user switches to this channel; i.e., it updates k^* as presented in step 13. Next, the user reduces $\delta_n(k^*)$ to obtain the desired load, as presented in steps $16, 17$.
Note that as $b_n(k)$ approaches e^{-1} for all k, the potential transmission probability $\tilde{p}_n(k)$ that user n computes for all other channels $k \neq k^*$ approaches zero to maintain the desired load. Hence, users are encouraged to switch to channels with low loads in the beginning of the process, and to remain in their channels as the load approaches the desired load.
To stabilize the algorithm, we allow user n to switch to channel k_2 from k_1 only if it gains at least $\delta_R(n)$ percents of its current rate. For $\delta_R(n) = 0$ users play their best response, while for $\delta_R(n) \to \infty$ users select the channel with the highest collision-free utility.

The parallel algorithm, presented in Table II, is based on the observation that for a large number of users, and when the rates are independent across users and identically distributed across channels,

TABLE I. SEQUENTIAL UPDATING ALGORITHM

1. Initialize:
2. for $n = 1, ..., N$ users do:
3. estimate $u_n(k)$ for all $k = 1, ..., K$
4. $k^* \leftarrow \arg \max_k \{u_n(k)\}$
5. $P_n \leftarrow p_0$ for all k
6. end for

7. repeat:
8. for $n = 1, ..., N$ users do:
9. estimate $v_n(k)$ for all $k = 1, ..., K$
10. compute $\tilde{p}_n(k) = \max\left\{1 - \frac{e^{-1}}{v_n(k)} \ , \ 0\right\}$ for all $k \neq k^*$
11. compute potential rates: $\tilde{R}_n(k) = \tilde{p}_n(k) u_n(k) v_n(k)$ for all $k \neq k^*$
12. compute current rate: $\tilde{R}_n(k^*) = P_n u_n(k^*) v_n(k^*)$
13. if $\max_k \left\{\tilde{R}_n(k)\right\} > \tilde{R}_n(k^*)\,(1 + \delta_R(n))$ do: $k^* \leftarrow \arg \max_k \left\{\tilde{R}_n(k)\right\}$
14. end if
15. compute $b_n(k^*) = (1 - P_n) \cdot v_n(k^*)$
16. if $b_n(k^*) > e^{-1}$ do: $P_n \leftarrow P_n + \epsilon$
17. else, do: $P_n \leftarrow P_n - \epsilon$
18. end if
19. end for
20. until $|b_n(k) - e^{-1}| \leq \delta$ for all $k = 1, ..., K$

TABLE II. PARALLEL UPDATING ALGORITHM

1. Initialize:
2. for $n = 1, ..., N$ users do:
3. estimate $u_n(k)$ for all $k = 1, ..., K$
4. $k^* \leftarrow \arg \max_k \{u_n(k)\}$
5. $p_n(k^*) \leftarrow p_0$
6. end for

7. for $n = 1, ..., N$ users do:
8. estimate $v_n(k)$ for all $k = 1, ..., K$
9. compute $b_n(k) = (1 - p_0) \cdot v_n(k)$ for all $k = 1, ..., K$
10. compute $\hat{N} = \sum_{k=1}^{K} \frac{\log(b_n(k))}{\log(1-p_0)}$
11. end for

12. for $n = 1, ..., N$ users do:
13. $P_n \leftarrow K/\hat{N}$
14. end for

15. perform the best-response dynamics with given P_n until convergence

the maximal network throughput in multi-channel ALOHA networks approaches Ke^{-1}, where users transmit with probability K/N [9], [18]. In the initialization step, all users set their transmission probability to $P_n^{(0)} = p_0$. In the learning step, all users monitor the channel utilization $v_n(k)$ for all $k = 1, ..., K$ and compute $b_n(k) = (1 - p_0)^{\hat{N}(k)}$. The estimated number of users is computed in step 10. Then all users set their transmission probability according to step 13 and implement the best-response dynamics, discussed in section III, with a given transmission probability P_n.

The advantages of the sequential mechanism are twofold. First, even if users start the dynamics with different transmission probabilities, they update their transmission probabilities to approach the desired load $b_n(k) = e^{-1}$. Second, in the case of a non-i.i.d utilities, the users adjust their transmission probability according to the channel load. On the other hand, when users are synchronized and parallel updating can be applied, the parallel mechanism determines the required transmission probability in a single iteration. Then, convergence of the best-response dynamics with a given transmission probability is much faster. Hence, when the rates are independent across users and identically distributed across channels, this is a good solution, since the throughput approaches e^{-1} as N increases [18].

V. SIMULATION RESULTS

In this section we provide numerical examples to illustrate the performance of the proposed algorithms. We focus on the case where users are not restricted by a transmission probability constraint, as discussed in section IV. Users maximize their rate, but still keep the desired load on the channels. We simulated Rayleigh fading channels, where each channel bandwidth was set to 10MHz. In Fig. 1 we present the convergence of the sequential updating algorithm, as shown in Table I, on a single channel (i.e., $K = 1$) to the desired throughput (or normalized rate) e^{-1}. We also present the performance of the parallel scheme, given in Table II in this case. In cases where parallel updating by all users can be implemented, this scheme is preferred on a single channel, since it only requires a single iteration. Next, we

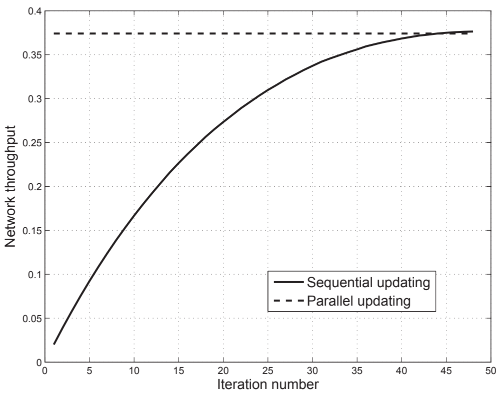

Fig. 1. Network throughput achieved by the sequential and parallel updating algorithms, given in Tables I and II, for $N = 30$.

illustrate the performance of the sequential updating algorithm for a multi-channel system. We simulated a common scenario where users transmit over channels $K = 1, 2$ with SNR=$20dB$, and over channels $K = 3, 4$ with SNR=$0dB$, due to significant interference in channels $K = 3, 4$. We compare the algorithm performance for $\delta_R \rightarrow \infty$ (i.e., users transmit over the channel with the highest collision-free utility, as studied in [9], [18] for the case of fixed transmission probability) and $\delta_R = 0.2$ (i.e., users change channels only if their rates are improved by at least 20%). We set δ_R to be equal for all users. In Fig. 2 we present the convergence of the algorithm for $N = 20$

as a function of the normalized number of iterations (i.e., scaled to 100 iterations). In Fig. 3 we present the average number of users that transmit over the inferior channels ($k = 3, 4$). For $\delta_R \to \infty$, the average number of users that transmit over the inferior channels approaches zero. The number of iteration for $\delta_R = 0.2$ was 400. It can be seen that implementing the sequential dynamics mechanism using $\delta_R = 0.2$ (i.e., approaching the best-response dynamics) significantly outperforms the sequential dynamics using $\delta_R \to \infty$. As discussed in section IV, low δ_R leads the users to use inferior channels when the load on good channels increases significantly. On the other hand, increasing δ_R leads to a high load on good channels and inefficient exploitation of the inferior channels.

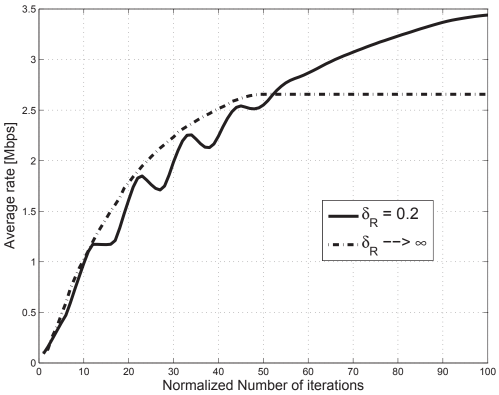

Fig. 2. Convergence of the sequential updating for $N = 20$ as a function of the normalized number of iterations.

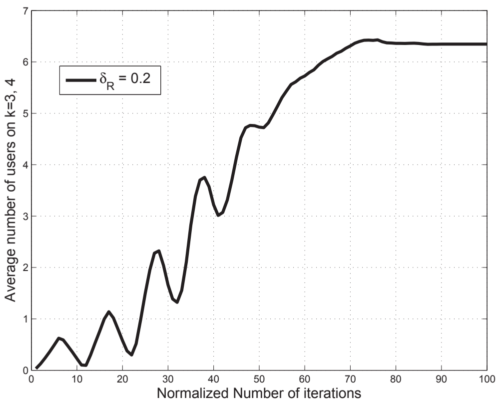

Fig. 3. Number of users that select the inferior channels by the sequential updating for $N = 20$ as a function of the normalized number of iterations.

VI. Conclusion

In this paper we examined the problem of distributed throughput maximization in multi-channel ALOHA networks. We focused on networks containing a large number of users that transmit over a typically low number of channels. First, we proposed a distributed best-response dynamics for rate maximization. In this scheme, users exploit both CSI and the channel utilization to increase their rates. Then, we considered the case where users can adjust their transmission probability to achieve the desired throughput. We examine the problem of distributed optimization of network throughput under uncertainty due to unknown transmission probabilities of other users. For this purpose, we proposed parallel and sequential learning mechanisms to adjust the transmission probabilities to increase the network throughput. Simulation results are provided to demonstrate the performance of the algorithms.

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