

# Speckle Reducing Diffusion for Ultrasound Image Enhancement using the Structural Similarity Image Measure

Scott T. Acton

*C.L. Brown Dept. of Electrical & Computer Engineering, Dept. of Biomedical Engineering,  
University of Virginia  
Charlottesville, Virginia USA  
acton@virginia.edu*

**Abstract**—An adaptive approach to enhancing images obtained from an array of ultrasonic transducer elements is proposed and evaluated. The basic algorithm is driven by a system of partial differential equations that 1) reduce speckle by way of the instantaneous (local) coefficient of variation and 2) force congruence with an anatomical model using a well-known perceptual quality metric. A differential form of the quality metric, the structural similarity image measure (SSIM), is derived and applied. This update mechanism registers the image data to the model, thus solving segmentation simultaneously with enhancement. The algorithm, called SSIM diffusion, is tested on a needle placement application in phlebotomy in which delineation of a vessel boundary is required. A group of images obtained from a portable C-scan ultrasonic sensor is used to evaluate the enhancement and segmentation algorithm. Comparisons to a standard speckle reducing diffusion algorithm show that the model-based SSIM diffusion superior enhancement with a 67% increase in measurable image quality over the original.

## I. INTRODUCTION

Ultrasound imaging uses beamforming to form an image from an array of transducer elements [10]. This type of imaging has salient value to the medical imaging community given that it is non-radiating, inexpensive and provides high temporal resolution. One application of ultrasonic imaging, explored in this paper, is phlebotomy [2]. Phlebotomy is the practice of making an incision in a vein. Ultrasound can be used to guide such a needle placement. In fact, portable ultrasound units have emerged recently, extending the use of such imaging outside the hospital.

One drawback of ultrasound is the difficulty of segmentation due to the presence of speckle. We apply an adaptive technique based on computing an instantaneous coefficient of variation (ICOV) (as opposed to standard gradient operators) for the reduction of speckle [6][11][12]. This speckle reducing partial differential operator is combined in a novel way with an operator that attempts to fit an anatomical model to the image data.

In a venipuncture process, the basic anatomy of vasculature is known *a priori*, which suggests model-based image analysis is appropriate. With a model-based approach to enhancement, segmentation or registration, we need a numerical measure of

congruence with the model. In the last decade, the signal processing community has emerged from the fog of mean-squared error and has seen the attractive properties of perceptual image quality measures [3][4][5][7][9]. Despite their numerical convenience, traditional measures such as mean squared error or similar manifestations of signal-to-noise ratio do not agree with human perception.

Here, we explore the evaluation of agreement with a model by way of a perceptual image quality metric. Perhaps no other measure has received as much attention as Wang, Bovik, Sheikh and Simoncelli's structural similarity image measure (SSIM) [8]. In SSIM, three main terms are combined to measure agreement with a reference image: a luminance term, a contrast term and a structure term. This paper uses differential versions of these three terms to maximize image quality with respect to a model.

The overall strategy presented here is to combine speckle reducing anisotropic diffusion (a method tailored to ultrasound imaging) with a model-driven term based on SSIM. Section II reviews the necessary fundamentals in speckle reduction, while Section III introduces the SSIM diffusion that promotes agreement with an anatomical model. Results are given in Section IV with a conclusion in the following section.

## II. SPECKLE REDUCTION

In our ultrasound image acquisition process, we acquire the radio frequency (RF) signal directly. Then, the RF data are modulated to baseband (a lowpass signal that is viewable). The intensity signal,  $\mathbf{I}$ , is the square of the baseband signal.

To implement speckle reducing anisotropic diffusion (SRAD) on ultrasound image intensity, the following partial differential equation (PDE) is applied:

$$\frac{\partial I(i,j)}{\partial t} = \text{div}[c(q(i,j))\nabla I(i,j)], \quad (1)$$

where  $t$  represents diffusion time, and  $q(i,j)$  is the ICOV at position  $(i,j)$  and is given by (see [11], [12] for derivation):

$$q(i,j) = \sqrt{\frac{\frac{1}{2}|\nabla I(i,j)|^2 - \frac{1}{16}[\nabla^2 I(i,j)]^2}{[I(i,j) + \frac{1}{4}[\nabla^2 I(i,j)]]^2}}. \quad (2)$$

The function  $c(q(i,j))$  in (1) is a diffusion coefficient that approaches unity in homogeneous regions and approaches zero at boundaries. This diffusion coefficient can be computed

by way of

$$c(q(i, j)) = \frac{1}{1 + \frac{q^2(i, j) - q_0^2}{q_0^2(1 + q_0^2)}} \quad (3)$$

where  $q_0$  is the estimate of the coefficient of variation in a homogeneous region. Of course, as discussed in [1] and [12], (1)-(3) can be implemented on a discrete grid by way of typical discretization techniques.

### III. SSIM DIFFUSION

#### A. SSIM Model

In this problem, we assume the existence of a model  $\mathbf{M}$ , which can be considered an atlas of the anatomy. Such a model matches the actual imaged data in structure and in mean intensity over homogeneous regions. The model  $\mathbf{M}$ , however, is not necessarily registered in space to the sensed image  $\mathbf{I}$ . In fact, the model has a coordinate system in 2-D of  $\mathbf{i}' = (i', j')$  that is related to the image coordinate system  $\mathbf{i} = (i, j)$  by  $\mathbf{i}' = \mathbf{f}(\mathbf{i})$ . Without loss of generality, we can assume an affine model of

$$\mathbf{i}' = \mathbf{A}\mathbf{i} + \mathbf{B}. \quad (4)$$

Our hypothesis is that the similarity measure used to compare  $\mathbf{I}$  and  $\mathbf{M}$  should be structurally based, as opposed to mean squared error (MSE). Wang *et al.* [8] show that at constant levels of MSE there are widely varying levels of perceptual quality. Since we want our atlas model to model the image data structurally, we choose SSIM as the similarity measure.

Their similarity measure is expressed [8]:

$$\text{SSIM}(\mathbf{M}, \mathbf{I}) = [l(\mathbf{M}, \mathbf{I})]^\alpha [c(\mathbf{M}, \mathbf{I})]^\beta [s(\mathbf{M}, \mathbf{I})]^\gamma \quad (5)$$

where  $l(\mathbf{M}, \mathbf{I})$  is the luminance term,

$$l(\mathbf{M}, \mathbf{I}) = \frac{2\mu_M\mu_I + C_1}{\mu_M^2 + \mu_I^2 + C_1}$$

and  $c(\mathbf{M}, \mathbf{I})$  is the contrast term,

$$c(\mathbf{M}, \mathbf{I}) = \frac{2\sigma_M\sigma_I + C_2}{\sigma_M^2 + \sigma_I^2 + C_2}$$

and, finally,  $s(\mathbf{M}, \mathbf{I})$  is the structure term,

$$s(\mathbf{M}, \mathbf{I}) = \frac{\sigma_{MI} + C_3}{\sigma_M\sigma_I + C_3}.$$

In the above, the  $\mu$  terms are local means, the  $\sigma$  terms are local standard deviations and  $\sigma_{MI}$  represents the local correlation coefficient (between model and image). The constants  $C_1, C_2, C_3$  are defined in [8], and we set  $\alpha = \beta = \gamma = 1$  as in the Wang *et al.* paper [8].

#### B. Diffusion term for SSIM-based Model Adherence

To implement model adherence via a PDE, we need a differential term that represents the change in SSIM( $\mathbf{M}, \mathbf{I}$ ) based on a change in a single pixel  $I(i, j)$ . Thus, we need  $\frac{\partial \text{SSIM}(\mathbf{M}, \mathbf{I})}{\partial I(i, j)}$ . In such a computation, we treat all other pixel intensities (aside from  $I(i, j)$ ) in  $\mathbf{I}$  and  $\mathbf{M}$  as constants.

So, we can represent SSIM( $\mathbf{M}, \mathbf{I}$ ) as:

$$\frac{4\mu_M\mu_I\sigma_{MI} + 2C_2\mu_M\mu_I + 2C_1\sigma_{MI} + C_1C_2}{\mu_M^2\sigma_I^2 + \mu_I^2\sigma_M^2 + \mu_I^2\sigma_I^2 + C_2\mu_I^2 + C_1\sigma_I^2 + \mu_M^2\sigma_M^2 + C_2\mu_M^2 + C_1\sigma_M^2 + C_1C_2}$$

and then take a partial derivative with respect to  $I(i, j)$ . If the expression for the SSIM( $\mathbf{M}, \mathbf{I}$ ) is represented by the constituent numerator and denominator, i.e.,  $\text{SSIM}(\mathbf{M}, \mathbf{I}) = \text{num}(\text{SSIM}) / \text{denom}(\text{SSIM})$  then we need to compute:

$$\frac{\frac{\partial \text{SSIM}(\mathbf{M}, \mathbf{I})}{\partial I(i, j)}}{\text{denom}(\text{SSIM})\text{num}'(\text{SSIM}) - \text{num}(\text{SSIM})\text{denom}'(\text{SSIM})} = \frac{[\text{denom}(\text{SSIM})]^2}{\text{denom}(\text{SSIM})\text{num}'(\text{SSIM}) - \text{num}(\text{SSIM})\text{denom}'(\text{SSIM})} \quad (6)$$

Hence, that leaves the derivation of  $\text{num}'(\text{SSIM})$  and  $\text{denom}'(\text{SSIM})$ , which involves numerous terms but is tractable.

In contrast to the approach suggested by (6), taking the partial derivative of the SSIM expression as a whole, we take an alternate approach. By differentiating the  $l(\mathbf{M}, \mathbf{I})$ ,  $c(\mathbf{M}, \mathbf{I})$ , and  $s(\mathbf{M}, \mathbf{I})$  terms separately, we can control the rate of diffusion for luminance, contrast and structure individually. We can then combine the terms as a derivative of products or simply use a sum of weighted partial derivative terms.

For the three individual terms, we have:

$$\frac{\partial l(\mathbf{M}, \mathbf{I})}{\partial I(i, j)} = \frac{\frac{2}{5}(\mu_M^2 + \mu_I^2 + C_1)\mu_M - \frac{2}{5}\mu_I(2\mu_M\mu_I + C_1)}{(\mu_M^2 + \mu_I^2 + C_1)^2}$$

$$\frac{\partial c(\mathbf{M}, \mathbf{I})}{\partial I(i, j)} = \frac{\frac{2\sigma_M}{\sigma_I}(\sigma_M^2 + \sigma_I^2 + C_2)(\frac{2}{5}I(i, j) - \frac{1}{10}\bar{I}) - (2\sigma_M\sigma_I + C_2)(\frac{2}{5}I(i, j) - \frac{1}{10}\bar{I})}{(\sigma_M^2 + \sigma_I^2 + C_2)^2}$$

$$\frac{\partial s(\mathbf{M}, \mathbf{I})}{\partial I(i, j)} = \frac{\sigma'_{MI}(\sigma_M\sigma_I + C_3) - \frac{\sigma_M}{\sigma_I}(\sigma_{MI} + C_3)(\frac{2}{5}I(i, j) - \frac{1}{10}\bar{I})}{(\sigma_M\sigma_I + C_3)^2}$$

where  $\bar{I} = I(i-1, j) + I(i, j-1) + I(i+1, j) + I(i, j+1)$ , and  $\sigma'_{MI} = \frac{\partial \sigma_{MI}}{\partial I(i, j)}$ .

Combining the SSIM-related terms with the speckle reducing diffusion in (1), we have

$$\frac{\partial I(i, j)}{\partial t} = \text{div}[c(q(i, j))\nabla I(i, j)] + \lambda \left[ \frac{\partial l(\mathbf{M}, \mathbf{I})}{\partial I(i, j)} + \frac{\partial c(\mathbf{M}, \mathbf{I})}{\partial I(i, j)} + \frac{\partial s(\mathbf{M}, \mathbf{I})}{\partial I(i, j)} \right] \quad (7)$$

where  $\lambda$  is a weight that equalizes the maximum rate of change due to diffusion and due to the SSIM-based model adherence. Because it is a combination of SSIM and diffusion, we call the new approach *SSIM diffusion*.

#### C. Registration of model to image data

Given that the image data and the model are related by an affine transformation in domain, we can simultaneously adjust the registration parameters while enhancing the image. Although the main purpose of this work is to explore the use

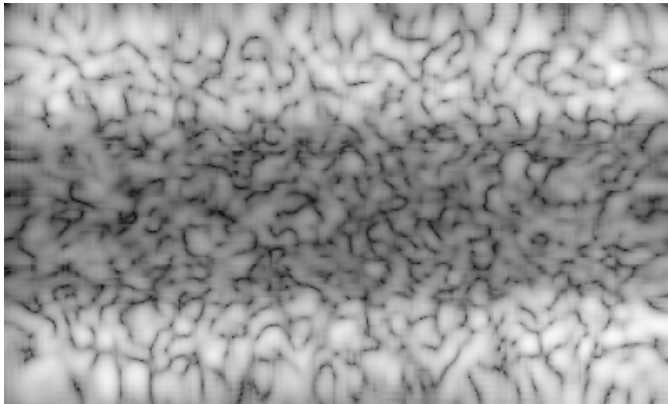
of a structural quality measure in enhancing ultrasound images as combined with speckle reduction, this framework opens the door for the exploration of an automated registration technique. Here, we assume that 1) the model and the image are related by translation, 2) the initialization is close enough that the effects of local optima can be ignored. We perform gradient ascent on the SSIM surface by making changes to the translation  $\mathbf{B}$  in (4).

#### IV. RESULTS

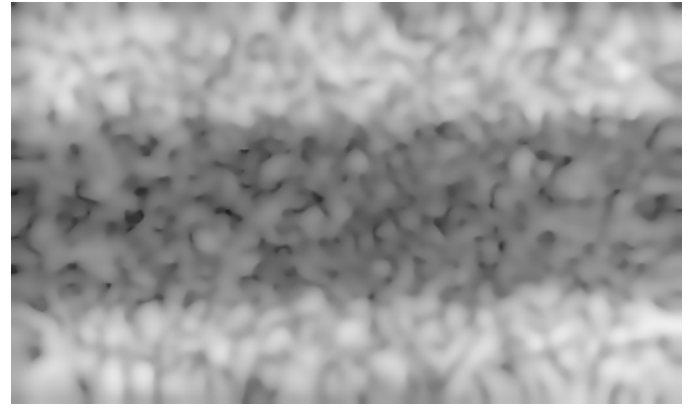
To examine the efficacy of SSIM diffusion, we used a set of 20 C-scan images (from a portable, battery powered scanner) of a cylindrical phantom that mimics a vessel. The accompanying models in the SSIM approach were simply cylinders of the same diameter as imaged. Two sample images are shown in Fig. 1(a) and 2(a). First, results using just SRAD [12] were generated and then compared to results using SSIM diffusion. Given a similar PDE structure (between (1) and (7)), it was straightforward to use matching parameters in terms of diffusion iterations, rate of diffusion and identical numerical schemes. A sample result from SRAD is shown in Fig. 1(b), with a sample result from SSIM diffusion in Fig. 1(c). We realize that many other relevant comparisons are possible. For more extensive comparisons to SRAD, see [11] and [12].

Fig. 2 provides a graph of the results on the 20 images in terms of SSIM (taken with respect to the cylindrical model). The blue bars represent the SSIM values of the original image as acquired, the green bars represent enhancement by SRAD and the red bars enhancement by SSIM diffusion (the method introduced here). The average SSIM for the original images is 0.55. The average SSIM for SRAD-enhanced results is 0.77, giving an improvement of 0.22 over the original images. For SSIM diffusion, the average SSIM produced is 0.92, which is 0.37 over the original images.

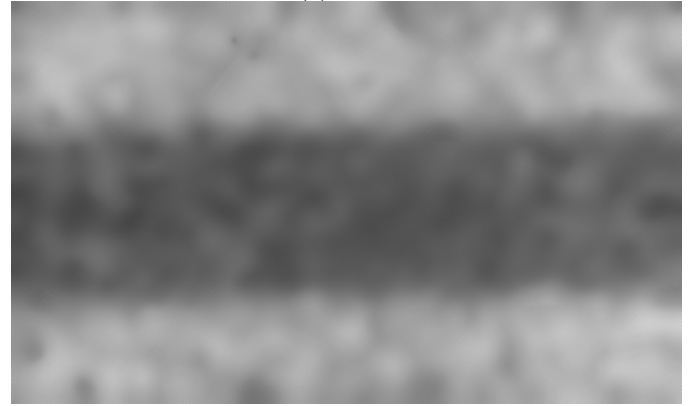
The performance of SSIM diffusion as given in (7) depends upon two main parameters. First, the selection of the number of updates (the diffusion sweeps) affects the result. As shown in Fig. 3, the algorithm needs about 30 diffusion sweeps to achieve sufficient smoothing and model adherence. After 40 iterations, the result levels off in quality but suffers from some over-smoothing.



(a)



(b)



(c)

Fig. 1. Example results from the 20 experiments. (a) The original image. (b) Enhancement by SRAD. (c) Enhancement by SSIM diffusion.

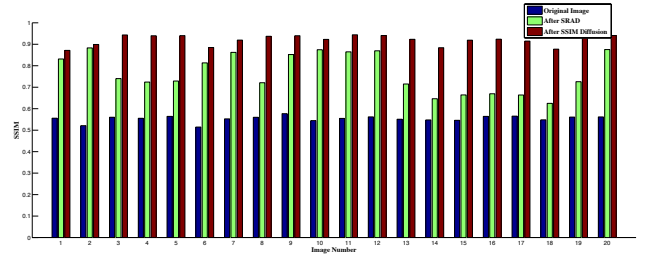


Fig. 2. SSIM values for the original images (blue), the SRAD-processed images (green) and the SSIM diffusion-processed images (red). SSIM diffusion is the SSIM-based anisotropic diffusion introduced in this paper.

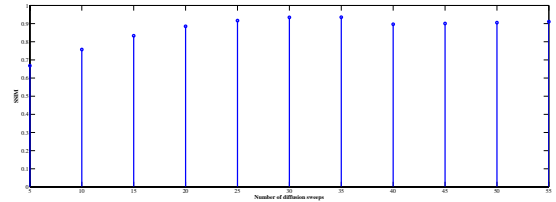


Fig. 3. Sensitivity of SSIM quality to number of diffusion sweeps (iteration) for SSIM diffusion.

The second important parameter selection is the weight  $\lambda$  in (7).  $\lambda$  represents the strength of model adherence in relation to the speckle reducing (smoothing) term. For our experiments, performance is fairly robust with respect to choice of  $\lambda$ . However, small  $\lambda$  values ( $\ll 100$ ) result in no model adherence, and large  $\lambda$  values ( $\gg 220$ ) result in reduced smoothing. See Fig. 4.

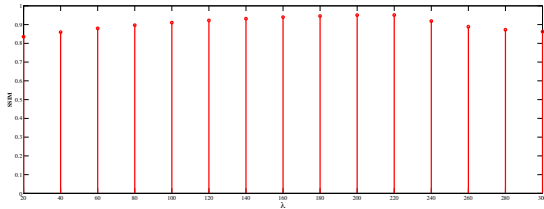


Fig. 4. Sensitivity of SSIM quality to weight  $\lambda$  (see (7)) for SSIM diffusion.

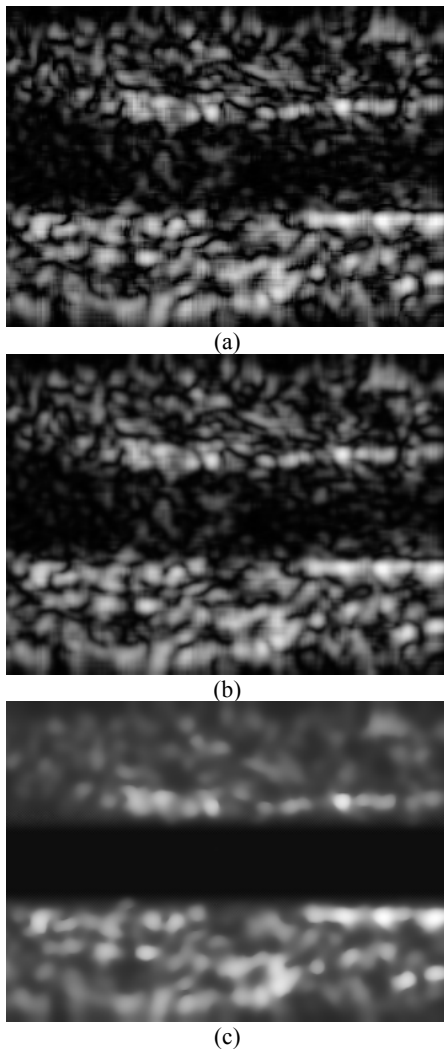


Fig. 5. (a) Human vein as imaged. Enhanced images using: (b) SRAD and (c) SSIM diffusion.

To show efficacy for ultrasound data on an actual vessel, we performed both SRAD and SSIM diffusion on a C-scan image of a human medial cubital vein shown in Fig. 4(a). The SRAD and SSIM diffusion results are shown in Fig. 4(b) and 4(c), respectively. One may observe that segmentation appears straightforward using the SSIM diffusion result.

## V. CONCLUSION

This paper reports the combination of speckle reduction and model adherence for scenarios in which anatomy is roughly known and an atlas can be employed. Both speckle reduction and model adherence in this approach are implemented via PDEs. In the application of the model, the perceptually significant SSIM measure is used, which emphasizes structure. The results demonstrate that the addition of the SSIM image quality terms dramatically improves enhancement. Specifically, a 67% increase in SSIM value is produced by SSIM diffusion as compared to the original SSIM values. More work is needed to fully explore the registration process that complements enhancement in this framework.

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