

# Distributed Demand Response for Plug-in Electrical Vehicles in the Smart Grid

*Zhao Tan, Peng Yang, and Arye Nehorai*

Preston M. Green Department of Electrical and Systems Engineering  
Washington University in St. Louis, St. Louis, MO, USA

**Abstract**—In this paper, we propose a new model of demand response management for the future smart grid that integrates plug-in electric vehicles. A price scheme considering fluctuation cost is developed. We consider a market where users have the flexibility to sell back the energy generated from their distributed generators or the energy stored in their plug-in electric vehicles. A distributed optimization algorithm based on the alternating direction method of multipliers is developed to solve the optimization problem, in which consumers need to report their aggregated load only to the utility company, thus ensuring their privacy. Consumers can update their load scheduling simultaneously and locally to speed up the optimization computing. Using numerical examples, we show the demand curve is flattened after the optimization, thus reducing the cost paid by the utility company. The distributed algorithm is also shown to reduce the users' daily bills.

## I. INTRODUCTION

In the electricity market, demand response [1] is a mechanism to manage users' consumption behavior under specific supply conditions. The goal of demand response is to benefit both consumers and utilities via a more intelligent resources scheduling method. Whereas the classical rule for operating the power system is to supply all the demand whenever it occurs, the new philosophy focuses on the concept that the system will be more efficient when the fluctuations in demand are kept as small as possible [2]. The goal of demand response is to flatten the demand curve by shifting the peak hour load to off-peak hours. Traditionally this is achieved by setting a time of use (TOU) price scheme [3],[4], which normally assigns high prices to the peak hours and low prices to the off-peak hours.

With the incorporation of the plug-in electric vehicles (PEV) into the power grid [5],[6], users have more flexibility to schedule their load and tend to charge them when the electrical price is low. Therefore the effect of the TOU price scheme is simply to move the peak demand from previous peak hours into previous off-peak hours. Even so, the cost arising from load variation still remains high in this situation. With the introduction of advanced metering infrastructures (AMI) [7] and energy-management controllers (EMC) [8], [9], more effective and distributed smart demand response algorithms can be proposed to solve the peak shifting and electric vehicle charging problems [9],[10]. In [9], the authors

proposed a distributed game-theoretical approach for users. But their distributed algorithm can be applied only sequentially among the users, and the communication time and cost will reduce the effectiveness of the method.

In this paper we consider a smart grid with a certain penetration level of PEVs and also with some on-site renewable distributed generators [11]. The price model in this paper considers the base price, and also takes the fluctuation cost into account. We consider the case where users can sell back the energy they generate to the grid. The PEVs can also be used as batteries to store electricity, which can be either consumed or sold back to the grid, whichever is more advantageous. We use the alternating direction method of multipliers (ADMM) to solve the optimization [12]. Unlike in [9], our algorithm is computed in parallel and thus save the computing time of the algorithm.

In Section II, we build a model for different kinds of loads and set the pricing policy for the utility company. In Section III, using the alternating direction method of multipliers, we reformulate the optimization problem into a distributed optimization problem. In Section IV, we show numerical examples to demonstrate the performance of the proposed methods. In Section V, we conclude the paper.

## II. SYSTEM MODEL

We consider a smart grid model with certain number of residences provided with electricity from the same utility company. Each consumer has an energy-management controller (EMC) that controls and communicates with different appliances within the household and also has an advanced metering infrastructure (AMI) to perform two-way communication with the utility company. We also assume that there are a certain number of user-owned distributed generators and plug-in electric vehicles in the grid. Users can sell back the energy generated from their own distributed generators or stored in the batteries of their PEVs.

### A. Electricity Usage Model for Users

We divide a day into  $T$  time periods. We assume there are four kinds of loads in our model: the base load, schedulable load, plug-in electric vehicle load, and the distributed generation.

Let  $l_k^B(t)$  denote the base load of the  $k$ -th user at time  $t$ . It supplies users' basic needs, such as lighting, air conditioning and refrigerators, which can not be scheduled.

This work was supported in part by the International Center for Advanced Renewable Energy and Sustainability (I-CARES) in Washington University in St. Louis.

We use  $l_k^S(t)$  to represent the schedulable load of user  $k$  at time  $t$ . It can be scheduled during a day, but it must satisfy the following sum constraint to meet the satisfaction of consumers. It should also meet the maximum physical usage rate constraints. The constraints for schedule load are stated as follows:

$$\sum_{t=1}^T l_k^S(t) = s_k^S, \text{ and } 0 \leq l_k^S(t) \leq l_k^{\max}(t), \forall k. \quad (1)$$

Define  $l_k^P(t)$  to be the electric vehicle load of user  $k$  at time  $t$ . It can be decomposed into two parts, namely the charging energy and discharging energy. The equation can be written as

$$l_k^P(t) = \frac{\tilde{l}_k^{P+}(t)}{\mu_c} + \mu_d \tilde{l}_k^{P-}(t), \quad (2)$$

where  $\mu_c$  and  $\mu_d$  denote the charging efficiency and discharging efficiency of the electric vehicle in this paper.  $\tilde{l}_k^P(t)$  represents the energy change in the battery of the electric vehicles. We have

$$\tilde{l}_k^{P+}(t) = \begin{cases} \tilde{l}_k^P(t) & \text{if } \tilde{l}_k^P(t) \geq 0, \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

$$\tilde{l}_k^{P-}(t) = \begin{cases} \tilde{l}_k^P(t) & \text{if } \tilde{l}_k^P(t) \leq 0, \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

The term  $\tilde{l}_k^P(t)$  must also satisfy the sum constraint. For simplicity, we assume that consumers always want to fully charge their PEV batteries before they leave for work. In addition, the user can use the electric vehicle as a battery to store energy when the electricity price is low and sell it back when the price is high. The charging and discharging rate also have upper bounds to meet physical constraints. The energy remaining in a battery at every time slot should also be larger than zero and less than the battery size. Let  $E_k$  denote the battery size of user  $k$ , and  $r_k^{\max}$  and  $r_k^{\min}$  denote the maximum charging rate and discharging rate. Let  $s_k^P$  denote the energy left in the battery when the user  $k$ 's PEV arrives home. Then these constraints can be stated as follows:

$$\sum_{t=t_{\text{ar}}}^{t_{\text{dep}}-1} \tilde{l}_k^P(t) = E_k - s_k^P, \text{ and } r_k^{\min} \leq \tilde{l}_k^P(t) \leq r_k^{\max}, \quad (5)$$

$$0 \leq s_k^P + \sum_{t=t_{\text{ar}}}^{t_0} \tilde{l}_k^P(t) \leq E_k, t_0 = t_{\text{ar}}, t_{\text{ar}} + 1, \dots, t_{\text{dep}} - 1, \quad (6)$$

where  $t_{\text{ar}}$  indicates the time of arrival and  $t_{\text{dep}}$  indicates the time of departure.

Let  $l_k^D(t)$  denote the distributed generation of user  $k$ . It has a negative value since it is obtained from an external clean energy source and can be sold back to the grid when the user has a surplus. Then the load of user  $k$  at time point  $t$  can be expressed as the sum of these four kinds of loads:

$$l_k(t) = l_k^B(t) + l_k^S(t) + l_k^P(t) + l_k^D(t). \quad (7)$$

These equations are satisfied for all  $t$  and  $k$ .

## B. Electricity Pricing Policy

Let  $c_t$  denote the marginal generation cost for electricity from thermal plants at time  $t$ . In an electrical power system, demand fluctuation can result in ancillary cost to the utility company since larger fluctuation will also lead to inefficient usage of the plants and the need for secondary thermal plants during the peak hours. We model this fluctuation cost as a function of the variance of the electricity load [13]. The total generation cost model for the utility company is described as

$$\text{Cost} = \sum_{t=1}^T c_t \sum_k l_k(t) + \mu \sum_{t=1}^T (\sum_k l_k(t) - m)^2, \quad (8)$$

where  $m$  is the mean usage during a day defined by  $\frac{1}{T} \sum_{t=1}^T \sum_k l_k(t)$ .

According to the cost function, the price function contains two parts, namely the base price and the fee arising from the demand fluctuation.

The base price  $p_B(t)$  is determined by the sum of the base loads of all individual users at time  $t$ . We assume the base price  $p_B(t)$  is proportional to the sum of base loads at time  $t$ :

$$p_B(t) = C_1 \left( \sum_k l_k^B(t) \right), \quad (9)$$

where  $C_1$  is chosen so that  $c_t \leq p_B(t)$  for all time points  $t$ . If only the base price is used, consumers have limited motivation to reschedule their demand response to lower the fluctuation in the demand curve. In order to align the incentives of consumers to lower the fluctuation cost in (8), an extra fee based on how much each consumer contributes to this demand curve fluctuation needs to be introduced. In our price model, the total fluctuation contribution fee  $f_0$  collected from all the users is:

$$f_0 = C_2 \sum_{t=1}^T (\sum_k l_k(t) - m)^2. \quad (10)$$

$C_2$  is chosen to be larger than  $\mu$  in the cost function (8). This total fluctuation fee will be distributed to the users in time periods when the total load of all users is larger than the mean usage  $m$ . Let  $\mathcal{T}_0$  denote the set containing these time points. Then the fee for user  $k$  can be written as

$$f_k(t) = \begin{cases} f_0 \frac{\sum_{k'} l_{k'}(t) - m}{\sum_{t' \in \mathcal{T}_0} (\sum_{k'} l_{k'}(t') - m)} & \frac{l_k(t)}{\sum_{k'} l_{k'}(t)} \\ 0, & \text{otherwise.} \end{cases} \quad (11)$$

With the introduction of the fluctuation contribution fee, consumers will cooperate with each other to reduce the variance of the overall load curve, and therefore lower the generation cost to the utility company and their own bills.

## III. DISTRIBUTED OPTIMIZATION ALGORITHM

### A. Centralized Optimization of Loads

The goal of the users is to minimize their bills. The utility company or retailer also has the incentive to minimize this total

bill since minimizing the total bills of all the users will lead to a more flattened load curve and thus lower the fluctuation cost for the utility company. Therefore the centralized optimization problem can be formulated as

$$\min_{\{\mathbf{l}_k\}} \sum_k \mathbf{p}_B^T \mathbf{l}_k + f_0(\sum_k \mathbf{l}_k), \quad (12)$$

$$\text{subject to} \quad \mathbf{l}_k \in \mathcal{F}_k, \forall k, \quad (13)$$

where  $\mathbf{p}_B = [p_B(1), p_B(2), \dots, p_B(T)]^T$  and  $\mathbf{l}_k = [l_k(1), l_k(2), \dots, l_k(T)]^T$ .  $\mathcal{F}_k$  denotes the feasible set for load  $\mathbf{l}_k$ , i.e.,  $\mathcal{F}_k = \{\mathbf{l}_k : \mathbf{l}_k \text{ satisfies conditions (1)-(7)}\}$ .

### B. Distributed Optimization of Loads

Solving the optimization problem (12) in a centralized way is inefficient due to the huge dimensionality and thousands of constraints. In addition consumers need to report their specific load usage to the utility company, which will lead to privacy issues. The objective function in problem (12) can be decentralized into parallel programming using the alternating direction method of multipliers [12]. In order to formulate the problem into ADMM framework, the optimization problem (12) is reformulated by introducing auxiliary variables  $\{z_k\}$  as follows:

$$\min_{\{\mathbf{l}_k\}, \{z_k\}} \sum_k \mathbf{p}_B^T \mathbf{l}_k + f_0(\sum_k z_k) \quad (14)$$

$$\text{subject to} \quad \mathbf{l}_k = z_k, \mathbf{l}_k \in \mathcal{F}_k, \forall k. \quad (15)$$

Due to the constraint above, it is the same optimization as (12). By introducing auxiliary  $l_2$  norm penalty terms in the objective function, we will have an equivalent optimization problem as

$$\min_{\{\mathbf{l}_k\}, \{z_k\}} \sum_k \mathbf{p}_B^T \mathbf{l}_k + \frac{\rho}{2} \|\mathbf{l}_k - z_k\|_2^2 \quad (16)$$

$$+ f_0(\sum_k z_k) \quad (17)$$

$$\text{subject to} \quad \mathbf{l}_k = z_k, \mathbf{l}_k \in \mathcal{F}_k, \forall k. \quad (18)$$

Introducing the Lagrange multipliers  $\mathbf{v}_k \in \mathbb{R}^T$  for each  $\mathbf{l}_k = z_k$  constraint in the above optimization, we can obtain the augmented Lagrangian function as

$$\begin{aligned} L(\{\mathbf{l}_k\}, \{z_k\}, \{\mathbf{v}_k\}) &= \sum_k (\mathbf{p}_B^T \mathbf{l}_k + \\ &\mathbf{v}_k^T (\mathbf{l}_k - z_k) + \frac{\rho}{2} \|\mathbf{l}_k - z_k\|_2^2) + f_0(\sum_k z_k), \end{aligned} \quad (19)$$

where  $\rho$  is a pre-defined constant. The original optimal problem can be solved using a Gauss-Seidel algorithm on the augmented Lagrangian function  $L(\{\mathbf{l}_k\}, \{z_k\}, \{\mathbf{v}_k\})$  [12]. Basically, the ADMM cycles through the following steps until some kind of convergence is reached:

$$\begin{aligned} \mathbf{l}_k^{i+1} &= \underset{\mathbf{l}_k}{\operatorname{argmin}} \quad \mathbf{p}_B^T \mathbf{l}_k + \mathbf{v}_k^{iT} (\mathbf{l}_k - z_k^i) \\ &+ \frac{\rho}{2} \|\mathbf{l}_k - z_k^i\|_2^2, \mathbf{l}_k \in \mathcal{F}_k, \forall k, \end{aligned} \quad (20)$$

$$\begin{aligned} \{z_k^{i+1}\} &= \underset{z_k}{\operatorname{argmin}} \sum_k (\mathbf{v}_k^{iT} (\mathbf{l}_k^{i+1} - z_k) \\ &+ \frac{\rho}{2} \|\mathbf{l}_k^{i+1} - z_k\|_2^2) + f_0(\sum_k z_k), \end{aligned} \quad (21)$$

$$\mathbf{v}_k^{i+1} = \mathbf{v}_k^i + \rho(\mathbf{l}_k^{i+1} - z_k^{i+1}), \forall k. \quad (22)$$

The optimization problems (20) and (22) can be solved locally and also in parallel. Each user needs to report his/her total usage during each time slot only to the utility company, which then solves the optimization problem (21).

Since in real-world application, the charging and discharging efficiencies are less than one, we can set  $\mu_c$  and  $\mu_d$  to be one in the ADMM iterations to ensure convexity of the problem. After convergence of the ADMM algorithm, we post-process the load of the electric vehicles using equation (2), (3) and (4) to get a suboptimal solution of the distributed optimization algorithm.

## IV. NUMERICAL EXAMPLES

In the examples we consider the case where there is only one electricity supplier and 120 households in the smart grid. There are 30 households with both wind distributed generators and plug-in electric vehicles, 20 households with only distributed generators, 30 households with only plug-in electric vehicles, and 40 households with none of these. The sum of the base load demand and schedulable demand from each household is generated randomly according to the MISO daily report by the U.S Federal Regulatory Commission (FERC) [14]. The distributed wind generation values are taken from the Ontario Power Authority [15]. We set the battery size of the plug-in electric vehicle as either  $10kW$  or  $20kW$ , to reflect different kinds of vehicles. The maximum charging rate is assumed to be  $3.3kW/h$ , and the maximum discharging rate is  $1.5kW/h$ . We also assume the charging and discharging rates can change continuously between the maximum discharging rate and maximum charging rate. The statistical mean of arrival time and departure are 18:00 and 8:00, respectively. The specific time slot is generated according to Gaussian distributions. Both charging efficiency and discharging efficiency are set to be 0.8.

### A. Valley filling properties of the ADMM scheduling method

In the first part of the results, we show three different types of distributed scheduling algorithms. The first one uses no optimization, in which users randomly select time slots to put their schedulable loads and charge their PEVs as soon as their vehicles are in the garages. The second algorithm is a greedy algorithm, in which everyone tries to lower their total electricity bill according to the pre-determined base price. We compare them with the ADMM scheduling method proposed earlier. we present the experiment when 20% of the load (except PEV and load supplied by distributed generators) of an individual user is schedulable. Thus the ratio of the base load and schedulable load is 4 : 1.  $\rho$  in the ADMM iteration is chosen to be 0.006. We show the valley filling results of the ADMM scheduling algorithm in Figure 1. We can see that the load without optimization creates a peak when

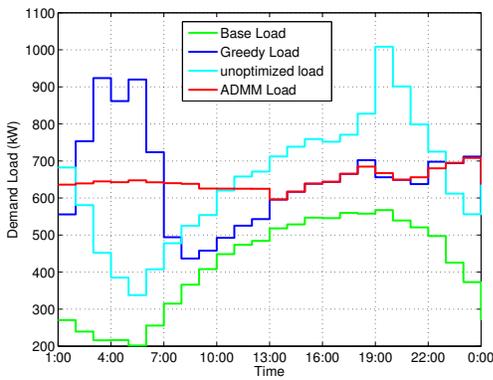


Fig. 1: Total load of four scheduling methods.

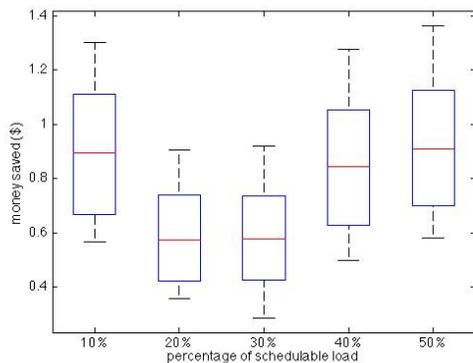


Fig. 2: Money saved with changing proportion of schedulable load.

most PEVs arrive home. The greedy algorithm simply moves the peak to another time period, which has the lowest base electricity price. The ADMM-based distributed optimization model proposed in this paper can fill the valley of the original base load and thus lead to a lower fluctuation of the load curve. The unoptimized method has a total bill as high as \$689, and the greedy method has a bill amount as \$592. While the ADMM scheduling method leads to a total bill as low as \$521.

### B. Results with different proportions of schedulable energy

In Figure 2, we show the daily bill reduction achieved by using the proposed distributed ADMM optimization algorithm, compared with the greedy algorithm. The percentage of schedulable load of each individual user changes from 10% to 50%.  $\rho$  in ADMM equals 0.006 in the algorithm. Every user will gain under the proposed distributed algorithm, and thus everyone has the incentive to be cooperative. In Figure 2 we observe that when percentage of schedulable load is 20% and 30%, the difference between ADMM scheduling method and greedy method is less than other cases. This is due to the fact that with this percentage of schedulable energy, the valley in the base load can be filled without creating a new peak at

the same time therefore greedy method in both cases is more efficient than greedy algorithm in other cases.

## V. CONCLUSION

In this paper, we first built an electricity usage model for four kinds of loads. A price scheme considering both base price and demand fluctuation in the demand response was proposed. By applying the alternating direction method of multipliers, we decomposed the centralized optimization problem into distributed and parallel optimization problems. Using numerical examples, we demonstrated that by using the ADMM based distributed scheduling method the demand response was flattened and the electrical bill was reduced for each individual user. In our future work, we will employ a more detailed cost function for the utility company. We will also develop more advanced machine learning models to predict the users' future electricity usage behavior.

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