Coupled Tensor Decompositions for Applications in Array Signal Processing

Mikael Sørensen and Lieven De Lathauwer
KU Leuven - E.E. Dept. (ESAT) - STADIUS Center for Dynamical Systems, Signal Processing and Data Analytics, and iMinds Future Health Department, Kasteelpark Arenberg 10, B-3001 Leuven-Heverlee, Belgium.
Group Science, Engineering and Technology, KU Leuven Kulak, E. Sabbelaan 53, 8500 Kortrijk, Belgium.
Email: {Mikael.Sorensen, Lieven.DeLathauwer}@kuleuven-kulak.be.

Abstract—**For the case of a single colocated receive antenna array and additional linear diversity (e.g. oversampling or polarization), tensor decomposition based signal separation is now well-established. For increasing the spatial diversity of communication systems, the use of several widely separated colocated antenna arrays has been suggested. However, for such problems no algebraic framework has been proposed. We explain that recently developed coupled tensor decompositions provide such a framework. In particular, we explain that the use of several widely separated colocated antenna arrays may lead to improved identifiability results.**

I. Introduction

In the late nineties it was realized that several signal separation or source localization problems in telecommunication are inherently multilinear when the full diversity is exploited. Besides temporal and spatial diversity, diversity yielding multilinear structure could for instance be due to MI-ESPRIT-type subarrays [12], oversampling [13] or polarization [6]. Exploitation of the multilinearity suddenly made it possible to solve array processing problems in an entirely deterministic manner through the computation of tensor decomposition, such as the Canonical Polyadic Decomposition (CPD).

Original work considered the case of a single colocated receive antenna. In order to increase the spatial diversity of a communication system more elaborate antenna array configurations have been proposed in the meantime. We mention multistatic MIMO radar systems [20], [11] where both the receive and transmit antenna arrays are composed of several widely separated colocated antenna arrays. However, no firm algebraic tensorial framework for array processing based on widely separated colocated antenna arrays has yet been presented. Consequently, no dedicated uniqueness conditions and algorithms are available. The goal of the paper is to explain that some of the coupled tensor decompositions recently proposed in [14], [15] are good candidate models for some problems involving widely separated colocated antenna arrays. In particular, the coupled tensor decomposition framework is able to explain that the use of several widely separated colocated antenna arrays leads to improved identifiability results.

The paper is organized as follows. The rest of the introduction will present the notation followed by a quick review of the CPD in section II. Section III briefly reviews two coupled tensor decompositions studied that are used in this paper. In section IV we demonstrate the usefulness of coupled tensor decompositions in the context of array signal processing problems involving widely separated antenna arrays with at least triple diversity. We end the paper with a conclusion in section V.

A. Notation

Vectors, matrices and tensors are denoted by lower case boldface, upper case boldface and upper case calligraphic letters, respectively. The number of non-zero entries of a vector $\mathbf{x}$ is denoted by $\omega(\mathbf{x})$. The transpose, rank, k-rank, and rth column vector of a matrix $\mathbf{A}$ are denoted by $\mathbf{A}^T$, $r(\mathbf{A})$, $k(\mathbf{A})$ and $\mathbf{a}_r$, respectively. The symbols $\otimes$ and $\odot$ denote the Kronecker and Khatri-Rao products, defined as

$$\mathbf{A}\otimes\mathbf{B} \triangleq \begin{bmatrix} a_{11}\mathbf{B} & a_{12}\mathbf{B} & \dots \\ a_{21}\mathbf{B} & a_{22}\mathbf{B} & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}, \ \mathbf{A}\odot\mathbf{B} \triangleq [\mathbf{a}_1 \otimes \mathbf{b}_1 \ \ \mathbf{a}_2 \otimes \mathbf{b}_2 \ \ \dots]$$

in which $(\mathbf{A})_{mn} = a_{mn}$. The outer product of three vectors $\mathbf{a} \in \mathbb{C}^I$, $\mathbf{b} \in \mathbb{C}^J$ and $\mathbf{c} \in \mathbb{C}^K$ is denoted by $\mathbf{a} \circ \mathbf{b} \circ \mathbf{c} \in \mathbb{C}^{I\times J\times K}$, such that $(\mathbf{a} \circ \mathbf{b} \circ \mathbf{c})_{ijk} = a_i b_j c_k$. $D_k(\mathbf{A}) \in \mathbb{C}^{J\times J}$ denotes the diagonal matrix holding row k of $\mathbf{A} \in \mathbb{C}^{I\times J}$ on its diagonal.

II. Canonical Polyadic Decomposition

Consider the third-order tensor $\mathcal{X} \in \mathbb{C}^{I\times J\times K}$. We say that $\mathcal{X}$ is a rank-1 tensor if it is equal to the outer product of non-zero vectors $\mathbf{a} \in \mathbb{C}^I$, $\mathbf{b} \in \mathbb{C}^J$ and $\mathbf{c} \in \mathbb{C}^K$ such that $x_{ijk} = a_i b_j c_k$. The Polyadic Decomposition (PD) is a decomposition of $\mathcal{X}$ into rank-1 terms

$$\mathbb{C}^{I\times J\times K} \ni \mathcal{X} = \sum_{r=1}^{R} \mathbf{a}_r \circ \mathbf{b}_r \circ \mathbf{c}_r \,. \tag{1}$$

The rank of a tensor $\mathcal{X}$ is equal to the minimal number of rank-1 tensors that yield $\mathcal{X}$ in a linear combination. Assume that the rank of $\mathcal{X}$ is R, then (1) is called the

978-1-4673-3146-3/13/$31.00 ©2013IEEE

Canonical PD (CPD) of $\mathcal{X}$. Let us stack the vectors $\{\mathbf{a}_r\}$, $\{\mathbf{b}_r\}$ and $\{\mathbf{c}_r\}$ into the matrices

$$\mathbf{A} = [\mathbf{a}_1, \ldots, \mathbf{a}_R], \quad \mathbf{B} = [\mathbf{b}_1, \ldots, \mathbf{b}_R], \quad \mathbf{C} = [\mathbf{c}_1, \ldots, \mathbf{c}_R].$$

The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ will be referred to as the factor matrices of the CPD of the tensor $\mathcal{X}$ in (1).

A CPD of $\mathcal{X} \in \mathbb{C}^{I \times J \times K}$ is said to be unique if all the triplets $\left(\widehat{\mathbf{A}}, \widehat{\mathbf{B}}, \widehat{\mathbf{C}}\right)$ satisfying (1) are related via

$$\widehat{\mathbf{A}} = \mathbf{A}\mathbf{P}\Delta_{\mathbf{A}}, \quad \widehat{\mathbf{B}} = \mathbf{B}\mathbf{P}\Delta_{\mathbf{B}}, \quad \widehat{\mathbf{C}} = \mathbf{C}\mathbf{P}\Delta_{\mathbf{C}},$$

where $\mathbf{P}$ is a permutation matrix and $\{\Delta_{\mathbf{A}}, \Delta_{\mathbf{B}}, \Delta_{\mathbf{C}}\}$ are diagonal matrices satisfying $\Delta_{\mathbf{A}}\Delta_{\mathbf{B}}\Delta_{\mathbf{C}} = \mathbf{I}_R$. Necessary conditions for CPD uniqueness are that $k(\mathbf{A}) \geq 2$, $k(\mathbf{B}) \geq 2$ and $k(\mathbf{C}) \geq 2$ and that the matrices $\mathbf{A} \odot \mathbf{B}$, $\mathbf{A} \odot \mathbf{C}$ and $\mathbf{B} \odot \mathbf{C}$ have full column rank (e.g. [18]). The development of sufficient uniqueness conditions for the CPD has been the subject of intense investigation, see [9], [8], [3], [18], [17], [4], [5], [16] and references therein. For the case where one factor matrix has full column rank, say $\mathbf{C}$, the following necessary and sufficient condition has been obtained.

Theorem II.1. *Consider the PD of $\mathcal{X} \in \mathbb{C}^{I \times J \times K}$ in* (1). *Define $\boldsymbol{E}(\boldsymbol{w}) = \sum_{r=1}^{R} w_r \boldsymbol{a}_r \boldsymbol{b}_r^T$. Assume that $\boldsymbol{C}$ has full column rank. The rank of $\mathcal{X}$ is R and the CPD of $\mathcal{X}$ is unique if and only if [19], [8], [17], [2]:*

$$r(\boldsymbol{E}(\boldsymbol{w})) \geq 2, \quad \forall \boldsymbol{w} \in \left\{\boldsymbol{x} \in \mathbb{C}^R \,\middle|\, \omega(\boldsymbol{x}) \geq 2\right\}. \tag{2}$$

III. Coupled tensor decompositions

The idea to couple several tensors seems to be first suggested in [7], albeit in a very informal way, by means of the coupled CPD briefly discussed in subsection III-A. Coupled tensor decompositions are currently gaining interest in several engineering disciplines, such as chemometrics, data mining, biomedical engineering and bioinformatics. However, only recently algebraic studies of coupled tensor decompositions have been reported in [14], [15]. Furthermore, the authors have developed several new coupled tensor decompositions in [14], [15] suited for array signal processing. In particular, we briefly explain that by taking the coupling between several tensor decompositions into account better identifiability results are obtained. In subsection III-A we briefly review the coupled CPD suggested in [7] and formally studied in [14], [15]. Subsection III-B briefly introduces the mixed coupled Block Term Decomposition (BTD) proposed in [14], [15].

A. Coupled CPD

To the authors' knowledge the first algebraic study and definition of the coupled CPD was presented in [14], [15]. We say that a collection of tensors $\mathcal{X}^{(n)} \in \mathbb{C}^{I_n \times J_n \times K}$, $n \in \{1, \ldots, N\}$ with $N \geq 2$, admits an R-term coupled PD if each tensor $\mathcal{X}^{(n)}$ can be written as

$$\mathcal{X}^{(n)} = \sum_{r=1}^{R} \mathbf{a}_r^{(n)} \circ \mathbf{b}_r^{(n)} \circ \mathbf{c}_r, \quad n \in \{1, \ldots, N\}, \tag{3}$$

with factor matrices $\mathbf{A}^{(n)} = \left[\mathbf{a}_1^{(n)}, \ldots, \mathbf{a}_R^{(n)}\right] \in \mathbb{C}^{I_n \times R}$, $\mathbf{B}^{(n)} = \left[\mathbf{b}_1^{(n)}, \ldots, \mathbf{b}_R^{(n)}\right] \in \mathbb{C}^{J_n \times R}$ and $\mathbf{C} = [\mathbf{c}_1, \ldots, \mathbf{c}_R] \in \mathbb{C}^{K \times R}$. We define the coupled rank of the tensors $\{\mathcal{X}^{(n)}\}$ as the minimal number of rank-1 tensors $\mathbf{a}_r^{(n)} \circ \mathbf{b}_r^{(n)} \circ \mathbf{c}_r$ that yield $\{\mathcal{X}^{(n)}\}$ in a linear combination. Assume that the coupled rank of $\{\mathcal{X}^{(n)}\}$ is R, then (3) will be called the coupled CPD of $\{\mathcal{X}^{(n)}\}$. In section IV we illustrate how the coupled CPD can be used in the context of array processing.

We call the coupled CPD of $\{\mathcal{X}^{(n)}\}$ unique if any alternative set of factors $\{\{\widehat{\mathbf{A}}^{(n)}\}, \{\widehat{\mathbf{B}}^{(n)}\}, \widehat{\mathbf{C}}\}$ satisfies

$$\widehat{\mathbf{A}}^{(n)} = \mathbf{A}^{(n)}\mathbf{P}\Delta_{\mathbf{A}^{(n)}}, \quad \widehat{\mathbf{B}}^{(n)} = \mathbf{B}^{(n)}\mathbf{P}\Delta_{\mathbf{B}^{(n)}}, \quad \widehat{\mathbf{C}} = \mathbf{C}\mathbf{P}\Delta_{\mathbf{C}},$$

where $\mathbf{P}$ is a permutation matrix and $\Delta_{\mathbf{A}^{(n)}}$, $\Delta_{\mathbf{B}^{(n)}}$ and $\Delta_{\mathbf{C}}$ are diagonal matrices satisfying $\Delta_{\mathbf{A}^{(n)}}\Delta_{\mathbf{B}^{(n)}}\Delta_{\mathbf{C}} = \mathbf{I}_R$, $\forall n \in \{1, \ldots, N\}$. Sufficient uniqueness conditions for the coupled CPD have been developed in [14]. For the case where the common factor matrix $\mathbf{C}$ has full column rank, the following version of Theorem II.1 was obtained.

Theorem III.1. *Consider the coupled PD of $\mathcal{X}^{(n)} \in \mathbb{C}^{I_n \times J_n \times K}$, $n \in \{1, \ldots, N\}$, in* (3). *Define*

$$\boldsymbol{E}^{(n)}(\boldsymbol{w}) = \sum_{r=1}^{R} w_r \boldsymbol{a}_r^{(n)} \boldsymbol{b}_r^{(n)T} \quad \text{and} \quad \Omega = \left\{\boldsymbol{x} \in \mathbb{C}^R \,\middle|\, \omega(\boldsymbol{x}) \geq 2\right\}.$$

Assume that $\boldsymbol{C}$ has full column rank. The coupled rank of $\{\mathcal{X}^{(n)}\}$ is R and the coupled CPD of $\{\mathcal{X}^{(n)}\}$ is unique if and only if [14]:

$$\forall \boldsymbol{w} \in \Omega, \, \exists n \in \{1, \ldots, N\} : r\left(\boldsymbol{E}^{(n)}(\boldsymbol{w})\right) \geq 2. \tag{4}$$

Note that condition (4) does not prevent that some of the factors are collinear, i.e., we may have coupled CPD uniqueness despite $k\left(\mathbf{A}^{(n)}\right) = 1$ or $k\left(\mathbf{B}^{(n)}\right) = 1$. We may also have coupled CPD uniqueness despite rank deficient matrices $\mathbf{A}^{(n)} \odot \mathbf{B}^{(n)}$. This result tells us that the coupled CPD is unique under more mild conditions than the ordinary CPD.

B. Mixed coupled BTD

More generally, the so-called mixed coupled multilinear rank-$(L_{r,n}, L_{r,n}, 1)$ term decomposition of the tensors $\mathcal{X}^{(n)} \in \mathbb{C}^{I_n \times J_n \times K}$, $n \in \{1, \ldots, N\}$ with $N \geq 2$, was proposed in [14]:

$$\mathcal{X}^{(n)} = \sum_{r=1}^{R} \sum_{l=1}^{L_{r,n}} \mathbf{a}_l^{(r,n)} \circ \mathbf{b}_l^{(r,n)} \circ \mathbf{c}^{(r)} = \sum_{r=1}^{R} \left(\mathbf{A}^{(r,n)}\mathbf{B}^{(r,n)T}\right) \circ \mathbf{c}^{(r)}, \tag{5}$$

and with factor matrices $\mathbf{A}^{(r,n)} = \left[\mathbf{a}_1^{(r,n)}, \ldots, \mathbf{a}_{L_{r,n}}^{(r,n)}\right] \in \mathbb{C}^{I_n \times L_{r,n}}$, $\mathbf{B}^{(r,n)} = \left[\mathbf{b}_1^{(r,n)}, \ldots, \mathbf{b}_{L_{r,n}}^{(r,n)}\right] \in \mathbb{C}^{J_n \times L_{r,n}}$ and $\mathbf{C} = \left[\mathbf{c}^{(1)}, \ldots, \mathbf{c}^{(R)}\right] \in \mathbb{C}^{K \times R}$. Note that in the special case where $N = 1$ and the matrices $\mathbf{A}^{(r,n)}\mathbf{B}^{(r,n)T}$ have rank L_r, (5) corresponds to the multilinear rank-$(L_r, L_r, 1)$ term decomposition introduced in [1].

We define the mixed coupled rank of $\{\mathcal{X}^{(n)}\}$ given by (5) as the minimal number of multilinear rank-$(L_{r,n}, L_{r,n}, 1)$ terms of the form $\left(\mathbf{A}^{(r,n)}\mathbf{B}^{(r,n)T}\right) \circ \mathbf{c}^{(r)}$ that yield $\{\mathcal{X}^{(n)}\}$ in a linear combination. If the mixed coupled rank of $\{\mathcal{X}^{(n)}\}$ is R, then we call (5) the mixed coupled BTD of $\{\mathcal{X}^{(n)}\}$.

In section IV we illustrate how the mixed coupled BTD can be used in the context of array processing.

Let $\{\{\widehat{\mathbf{A}}^{(n)}\},\{\widehat{\mathbf{B}}^{(n)}\},\widehat{\mathbf{C}}\}$ yield an alternative mixed coupled BTD of the tensors $\{\mathcal{X}^{(n)}\}$ in (5). We say that the mixed coupled BTD of $\{\mathcal{X}^{(n)}\}$ is unique if it is unique up to a permutation of the coupled multilinear rank-$(L_{r,n},L_{r,n},1)$ terms $\{\widehat{\mathbf{A}}^{(r,n)T}\widehat{\mathbf{B}}^{(r,n)}\circ\widehat{\mathbf{c}}^{(r)}\}$ and up to the following indeterminacies within each term:
$\widehat{\mathbf{A}}^{(r,n)}=\alpha^{(r,n)}\mathbf{A}^{(r,n)}\mathbf{F}_{r,n}$, $\widehat{\mathbf{B}}^{(r,n)}=\beta^{(r,n)}\mathbf{B}^{(r,n)}\mathbf{F}_{r,n}^{-1}$, $\widehat{\mathbf{c}}^{(r)}=\gamma^{(r)}\mathbf{c}^{(r)}$

where $\mathbf{F}_{r,n}\in\mathbb{C}^{L_{r,n}\times L_{r,n}}$ are nonsingular matrices and $\alpha^{(r,n)},\beta^{(r,n)},\gamma^{(r)}\in\mathbb{C}$ are scalars satisfying $\alpha^{(r,n)}\beta^{(r,n)}\gamma^{(r)}=1$, $\forall n\in\{1,\ldots,N\}$. Uniqueness conditions for the mixed coupled BTD can be found in [14]. For the case where the common factor matrix $\mathbf{C}$ has full column rank, the following extension of Theorems II.1 and III.1 was obtained.

Theorem III.2. *Consider the mixed coupled multilinear rank-$(L_{r,n},L_{r,n},1)$ term decomposition of $\mathcal{X}^{(n)}\in\mathbb{C}^{I_n\times J_n\times K}$, $n\in\{1,\ldots,N\}$ in (5). Define $\boldsymbol{E}^{(n)}(\boldsymbol{w})=\sum_{r=1}^{R}w_r\boldsymbol{A}^{(r,n)}\boldsymbol{B}^{(r,n)T}$ and $\Omega=\left\{\boldsymbol{x}\in\mathbb{C}^R\,\middle|\,\omega(\boldsymbol{x})\geq 2\right\}$. Assume that $\boldsymbol{C}$ has full column rank. The mixed coupled rank of $\{\mathcal{X}^{(n)}\}$ is R and the mixed coupled BTD of $\{\mathcal{X}^{(n)}\}$ is unique if and only if*

$$\forall\boldsymbol{w}\in\Omega,\ \exists\, n\in\{1,\ldots,N\}\ :\ r\left(\boldsymbol{E}^{(n)}(\boldsymbol{w})\right)>\max_{r|w_r\neq 0}L_{r,n}\,. \tag{6}$$

Conditions (2), (4) and (6) may not be easy to verify in practice. On the other hand, they are necessary for uniqueness when $\mathbf{C}$ has full column rank. For conditions that are not necessary but easier to verify and for examples, we refer to [14], [15].

IV. Application in array Processing

Let us explain how coupled tensor decompositions may be used in array processing for a scenario with widely separated colocated antenna arrays and oversampling as third diversity. More precisely, we extend the approach in [13] to the case of incoherent channels with small delay spread and widely separated colocated antenna arrays. We note in passing that the coupled tensor decomposition approach can also be extended to the case of large delay spread. Due to space considerations this is not further discussed.

Consider a system with R users in which the transmitted signal from user r is of the form $x_r(t)=\sum_{k\in\mathbb{N}}g(t-kT)s^{(r)}(t)$, where $g(t)$ is a pulseshaping function with support $[0,L_g]$, T is the normalized pulse period which we set to $T=1$ and $s^{(r)}(t)$ is the transmitted symbol at time instant t. The nth receive antenna array is equipped with I_n antennas and samples the data at a rate J_n times the symbol rate. As in [13] we assume that the channel between user r and the nth receive antenna array can be characterized by $P_{r,n}$ distinct paths each characterized by a delay $\tau_{p,r,n}\in\mathbb{R}$ and a gain factor $\beta_{p,r,n}\in\mathbb{C}$. The outputs of the nth receive antenna array at oversampling periods $1\leq j\leq J_n$ and symbol periods $1\leq k\leq K$ can be stored in a tensor $\mathcal{Y}^{(n)}\in\mathbb{C}^{I_n\times J_n\times K}$ with decomposition (e.g. [13]):

$$\mathcal{Y}^{(n)}=\mathcal{X}^{(n)}+\mathcal{V}^{(n)}=\sum_{r=1}^{R}\sum_{p_r=1}^{P_{r,n}}\mathbf{a}_{p_r}^{(r,n)}\circ\mathbf{h}_{p_r}^{(r,n)}\circ\mathbf{s}^{(r)}+\mathcal{V}^{(n)}, \tag{7}$$

where $\mathbf{a}_{p_r}^{(r,n)}\in\mathbb{C}^{I_n}$ is the array response vector for the p_rth path of user r to the nth receive array, $\mathbf{h}_{p_r}^{(r,n)}=\beta_{p,r,n}\left[g\left(-\tau_{p,r,n}\right),\ldots,g\left(\frac{J_n-1}{J_n}-\tau_{p,r,n}\right)\right]\in\mathbb{C}^{J_n}$ is the channel impulse response for the p_rth path of user r to the nth receive array, where it is assumed that the delay spread is sufficiently small so that $\max_{p,r,n}\left(L_g+\tau_{p,r,n}\right)<T=1$, $\mathbf{s}^{(r)}=\left[s^{(r)}(1),\ldots,s^{(r)}(K)\right]^T\in\mathbb{C}^K$, and $\mathcal{V}^{(n)}\in\mathbb{C}^{I_n\times J_n\times K}$ represents noise. The vectors $\mathbf{h}_{p_r}^{(r,n)}$ add little diversity since they are all similar, making (7) a difficult signal separation problem. In practice, $P_{r,n}$ may not be known in advance. For the case where $\min\left(\sum_{n=1}^{N}I_nJ_n,K\right)\geq R$ we can in some instances first determine R via a singular value decomposition of a matrix representation of $\{\mathcal{Y}^{(n)}\}$, see [14], [15] for details. Choose a safe estimate of $P_{r,n}$ which is denoted by $\widehat{P}_{r,n}$. Next, we compute the mixed coupled $(\widehat{P}_{r,n},\widehat{P}_{r,n},1)$-BTD of $\{\mathcal{Y}^{(n)}\}$. Finally, we may estimate the integers $\{P_{r,n}\}$ from an investigation of the linear (in)dependencies among the columns of the factor matrices of the $(\widehat{P}_{r,n},\widehat{P}_{r,n},1)$-BTDs of $\mathcal{Y}^{(n)}$, $n\in\{1,\ldots,N\}$. In sections III-A and III-B we have explained that coupled tensor decompositions lead to better identifiability conditions. We now demonstrate that coupled tensor decompositions can also lead to more robust computations.

Let us compare a signal separation method which exploits the coupling in (7) with a method that only exploits the individual tensor decomposition structure in (7). The distance between the symbol matrix $\mathbf{S}=\left[\mathbf{s}^{(1)},\ldots,\mathbf{s}^{(R)}\right]\in\mathbb{C}^{K\times R}$ and its estimate $\widehat{\mathbf{S}}$, is measured as $\mathrm{P}(\mathbf{S})=\min_{\Pi\Lambda}\left\|\mathbf{S}-\widehat{\mathbf{S}}\Pi\Lambda\right\|_F/\|\mathbf{S}\|_F$. where Π denotes a permutation matrix and Λ denotes a diagonal matrix. The Signal-to-Noise Ratio (SNR) is measured as SNR [dB] $=10\log\left(\sum_{n,i,j,k}\left|x_{ijk}^{(n)}\right|^2/\sum_{n,i,j,k}\left|v_{ijk}^{(n)}\right|^2\right)$.

Consider first the case where $P_{r,n}=1$, $\forall r,n$. In that case the decomposition (7) corresponds to a perturbed coupled CPD. We set $R=3$, $N=2$ $P_{r,n}=1$, $I_n=3$, $J_n=5$ and $K=50$. The individual CPDs will be computed by the Simultaneous Matrix Diagonalization (SMD) method described in [3], while the coupled CPD will be computed by an extension of the SMD method to the coupled CPD case, described in [15]. The mean $\mathrm{P}(\mathbf{S})$ value over 100 trials for varying SNR can be seen in figure 1. It is clear that the SMD method for coupled CPD yields a better performance than the SMD method for ordinary CPD based on $\mathcal{Y}^{(1)}$.

Consider now the case where $P_{r,n}\geq 2$ for at least one pair (r,n). In that case (7) corresponds to a perturbed mixed coupled BTD. Let us compare a signal separation method which exploits the mixed coupled BTD structure in (7) with a method that only exploits the individual multilinear rank-$(P_{r,n},P_{r,n},1)$ term decomposition structure in (7). We set $R = 2$, $N = 2$, $I_n = 3$, $J_n = 5$, $K = 50$ and $P_{r,n} = 2$, $\forall r, n$. The individual multilinear rank-$(P_{r,n}, P_{r,n}, 1)$ term decompositions will be computed by means of an extension of the SMD method described in [10] while the mixed coupled BTD will be computed by means of an extension of the SMD described in [15]. The mean $\mathrm{P}(\mathbf{S})$ value over 100 trials for varying SNR can be seen in figure 2. Again it is clear that the SMD method for mixed coupled BTD yields a better performance than the SMD method for the decomposition of $\mathcal{Y}^{(1)}$ into multilinear rank-$(P_{r,1}, P_{r,1}, 1)$ terms.

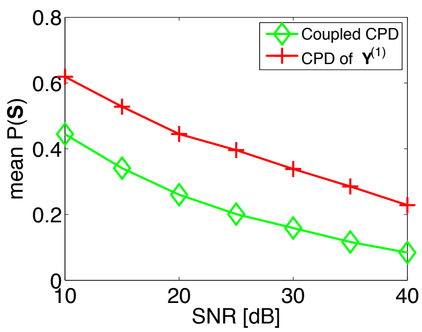

Fig. 1. Mean $\mathrm{P}(\mathbf{S})$ for varying SNR, case 1

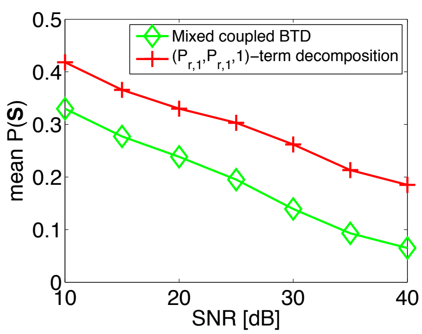

Fig. 2. Mean $\mathrm{P}(\mathbf{S})$ for varying SNR, case 2.

V. Conclusion

Tensor decompositions have already proven to be useful in array signal processing. To increase the spatial diversity of a communication system, the use of several widely separated antenna arrays has been proposed. However, the existing tensor-based methods are mainly limited to array processing problems involving a single colocated antenna array. To accommodate the use of several widely separated antenna arrays, we have studied and developed several new coupled tensor decomposition models, of which two are briefly discussed in this paper. We first briefly explained that coupled tensor decompositions lead to better uniqueness conditions. Thereafter, we demonstrated by means of computer simulations that they also lead to a more robust estimation of the transmitted symbol vectors.

Acknowledgment

Research supported by: (1) Research Council KU Leuven: GOA-MaNet, CoE EF/05/006 Optimization in Engineering (OPTEC), CIF1, STRT1/08/23, (2) F.W.O.: project G.0427.10N, (3) the Belgian Federal Science Policy Office: IUAP P7 (DYSCO II, Dynamical systems, control and optimization, 2012-2017).

References

[1] L. De Lathauwer, "Decomposition of a higher-order tensor in block terms — Part II: Definitions and uniqueness," *SIAM J. Matrix Anal. Appl.*, vol. 30, no. 3, pp. 1033–1066, 2008.

[2] ——, "Blind separation of exponential polynomials and the decomposition of a tensor in rank-$(L_r, L_r, 1)$ terms," *SIAM J. Matrix Anal. Appl.*, vol. 32, no. 4, pp. 1451–1474, 2011.

[3] ——, "A link between the canonical decomposition in multilinear algebra and simultaneous matrix diagonalization," *SIAM J. Matrix Anal. Appl.*, vol. 28, no. 3, pp. 642–666, 2006.

[4] I. Domanov and L. De Lathauwer, "On the uniqueness of the canonical polyadic decomposition — Part I: Basic results and uniqueness of one factor matrix," *SIAM J. Matrix Anal. Appl.*, vol. 34, no. 3, pp. 855–875, 2013.

[5] ——, "On the uniqueness of the canonical polyadic decomposition — Part II: Overall uniqueness," *SIAM J. Matrix Anal. Appl.*, vol. 34, no. 3, pp. 876–903, 2013.

[6] X. Guo, S. Miron, D. Brie, S. Zhu, and X. Liao, "A CANDECOMP/PARAFAC perspective on uniqueness of DOA estimation using a vector sensor array," *IEEE Trans. Signal Process.*, vol. 59, no. 7, pp. 3475–3481, Jul. 2011.

[7] R. A. Harshman and M. E. Lundy, "Data preprocessing and the extended parafac model," in *Research Methods for Multimode Data Analysis*, H. G. Law, C. W. Snyder, J. R. Hattie, and R. P. McDonald, Eds. Praeger, 1984, pp. 216–284.

[8] T. Jiang and N. D. Sidiropoulos, " Kruskal's permutation lemma and the identification of CANDECOMP/PARAFAC and bilinear model with constant modulus constraints," *IEEE Trans. Signal Process.*, vol. 52, no. 9, pp. 2625–2636, Sep. 2004.

[9] J. B. Kruskal, "Three-way arrays: Rank and uniqueness of trilinear decompositions, with applications to arithmetic complexity and statistics," *Linear Algebra Appl.*, vol. 18, pp. 95–138, 1977.

[10] D. Nion and L. De Lathauwer, "A tensor-based blind DS-CDMA receiver using simultaneous matrix diagonalization," in *Proc. IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, June 17-20, Helsinki, Finland, 2007.

[11] D. Nion and N. D. Sidiropoulos, "Tensor algebra and multidimensional harmonic retrieval in signal processing for MIMO Radar," *IEEE Trans. Signal Processing*, vol. 58, no. 11, pp. 5693–5705, Nov. 2010.

[12] N. D. Sidiropoulos, R. Bro, and G. B. Giannakis, "Parallel factor analysis in sensor array processing," *IEEE Trans. Signal Processing*, vol. 48, no. 8, pp. 2377–2388, Aug. 2000.

[13] N. D. Sidiropoulos and X. Liu, "Identifiability results for blind beamforming in incoherent multipath with small delay spread," *IEEE Trans. Signal Processing*, vol. 49, no. 1, pp. 228–236, Jan. 2001.

[14] M. Sørensen and L. De Lathauwer, "Coupled canonical polyadic decompositions — Part I: Uniqueness," ESAT-SISTA, KU Leuven, Belgium, Tech. Rep. 13-143, 2013.

[15] ——, "Coupled canonical polyadic decompositions — Part II: Algorithms," ESAT-SISTA, KU Leuven, Belgium, Tech. Rep. 13-144, 2013.

[16] ——, "New uniqueness conditions for the canonical polyadic decomposition of third-order tensors," ESAT-SISTA, KU Leuven, Belgium, Tech. Rep. 13-05, 2013.

[17] A. Stegeman, "On uniqueness of the N-th order tensor decomposition into rank-1 terms with linear independence in one mode," *SIAM J. Matrix Anal. Appl.*, vol. 31, pp. 2498–2516, 2010.

[18] A. Stegeman and N. D. Sidiropoulos, "On Kruskal's uniqueness condition for the Candecomp/Parafac decomposition," *Linear Algebra Appl.*, vol. 420, pp. 540–552, 2007.

[19] V. Strassen, "Rank and optimal computation of generic tensors," *Linear Algebra Appl.*, vol. 52, pp. 645–685, 1983.

[20] L. Xu and J. Li, "Iterative generalized-likelihood ratio test for MIMO radar," *IEEE Trans. Signal Process.*, vol. 55, no. 6, pp. 2375–2385, Jun. 2007.