A Bilateral-Market based Mechanism for Spectrum Allocation in Cognitive Radio Networks

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***Abstract*—We address the problem of spectrum allocation by a moderator employed in a cognitive radio (CR) network in the presence of multiple primary users (PUs). It is assumed that the moderator is uncertain about the state of the PU channel (busy or vacant) and therefore, does not have perfect knowledge about spectrum availability. In order to improve the performance, secondary users (SUs) send their sensing decisions to the moderator. The moderator fuses the data from these SUs and makes an inference about spectrum availability. We assume that the spectrum allocation is made by the moderator only when it decides that spectrum is available. In this paper, we model the interaction between the PUs, SUs and the moderator within a bilateral trading framework, and present an iterative allocation mechanism that guarantees improvement in the revenue of the moderator in every iteration.**

I. INTRODUCTION

For the past few years, spectrum scarcity has motivated many researchers to actively pursue the problem of dynamic spectrum access (DSA) in cognitive radio (CR) networks [1]. In this paper, we propose a *bilateral trading framework* for trading spectrum between multiple PUs and SUs, where availability of the traded commodity (spectrum) is dynamic, time-varying and unpredictable in nature, thereby requiring spectrum sensing to be performed by participating SUs.

Several other efforts have been made to address the problem of spectrum allocation in CR networks using trading mechanisms. Wang et al. in [2] assume that the PU behaves as an auctioneer (for its own profit) and allocates a part of the available bandwidth to the CRs without compromising its own performance. Within a bilateral trading framework, where multiple SUs and PUs participate for their individual gains, only a few efforts have been made. Kasbekar *et al*. in [3], considered the auction-based framework for two different categories of networks, namely primary (high-priority access) and secondary (low-priority access) networks. The access allocation problem is solved based on the bids placed by the different users (from the two different types of networks), so as to maximize the auctioneer's revenue. Niyato *et al*. in [4], on the other hand, addressed the problem of spectrum pricing in the case of multiple PUs which compete to offer spectrum opportunities to maximize their individual profits. However, in a DSA framework, PU's spectrum usage is dynamic, and therefore, the availability of the PU channel at the SU is uncertain. Earlier works have failed to capture the dynamics of the PU activity in their models.

Moreover, in practice, PU networks may have other priorities and therefore, can find the task of spectrum allocation to be burdensome. This can be due to one of the following reasons:

1) Minimal/No-change, as suggested by FCC [1].
2) PUs may possess wide spectrum-bandwidths over large geographic areas, resulting in a cumbersome management of spectrum usage everywhere.

In order to minimize the effort, the seller may outsource the task of selling the spectrum to a moderator, from whom revenue is collected. In such a case, the moderator (auctioneer) does not have perfect knowledge about the availability of the PUs' channels. Therefore, we addressed the issue of spectrum uncertainty within the design of our auction in [5], where we designed the optimal spectrum allocation mechanism in the presence of spectrum uncertainty at the auctioneer when there is only one PU.

Our contributions in this paper are two-fold. First, we present a novel model for the problem of spectrum allocation as a bilateral-trading framework in the presence of spectrum uncertainty at the moderator, when multiple PUs (along with multiple SUs) participate in the trade. In other words, PUs behave as sellers, SUs behave as buyers and the moderator is a trade-broker. Note that the bilateral-trading framework has been addressed in the past by Myerson in [6] and McAfee in [7], in scenarios where the item-to-be-traded can be allocated by the moderator with certainty. Our second contribution in this paper is the design a novel spectrum allocation algorithm in a bilateral-trading framework, where the moderator experiences uncertainty in PU activity. The moderator makes inferences (based on the sensing results of SUs) about spectrum availability of PUs and allocates PUs' spectrum to SUs so that the moderator's revenue is maximized.

The remainder of the paper is organized as follows. Section II presents a detailed description of our system model and defines the utilites of the participating nodes in the proposed bilateral-trading framework. In addition, a formal problem-statement is also included in Section II. Section III presents some of the necessary matrix transformations needed in designing the proposed algorithm. In Section IV, we present an spectrum allocation algorithm. Finally, concluding remarks are made in Section V.

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II. System Model

Consider a bilateral market with M buyers (SUs), N sellers (PUs) and one moderator. In order to avoid the overhead of managing the spectrum themselves, PUs outsource the task of spectrum sensing and trading to the moderator (note that PUs may own large amounts of spectrum over vast geographic areas). Since the moderator does not have any knowledge about the state of any of the PUs' spectrum, the SUs acquire measurements of the spectrum and provide their local decisions about each of the PU's activity to the moderator, so that the moderator ensures minimal collisions at the PUs.

We assume that the SUs perform wide-band spectrum sensing, similar to [8], and the moderator fuses all the local decisions to make global inferences about the availability of the licensed channels. Let $H_0(j)$ and $H_1(j)$ denote the state of the j^{th} PU's channel being vacant and busy, and whose prior probabilities are denoted as π_{j0} and π_{j1} respectively. Let the probabilities of global false alarm and detection at the moderator (after the fusion of local SU decisions) be denoted as $\mathbf{P_f} = \{P_{f_1}, \cdots, P_{f_N}\}$ and $\mathbf{P_d} = \{P_{d_1}, \cdots, P_{d_N}\}$ respectively. Therefore, the probability with which the moderator makes the decision that j^{th} channel is vacant, is given by $\alpha_j = \pi_{j0}(1 - P_{f_j}) + \beta_j$, where $\beta_j = \pi_{j1}(1 - P_{d_j})$ is the probability of the moderator making a decision that the j^{th} PU is vacant and the true hypothesis is $H_1(j)$.

During the process of acquiring measurements and sharing the local decisions with the moderator, we assume that each SU incurs a cost of c_i. Once the moderator assigns a PU channel to an SU, due to the uncertainty in the PU state, collisions might occur. We assume that the moderator is responsible for any collisions and, therefore, impose a penalty p_j when there is a collision with the j^{th} PU.

In this paper, we denote the probability of allocating j^{th} PU's spectrum to the i^{th} SU as ψ_{ij}. Therefore, the moderator tries to maximize its utility by choosing an appropriate allocation matrix Ψ, defined as follows.

$$\Psi = \begin{bmatrix} \psi_{11} & \psi_{12} & \cdots & \psi_{1N} \\ \psi_{21} & \psi_{22} & \cdots & \psi_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{M1} & \psi_{M2} & \cdots & \psi_{MN} \end{bmatrix} \tag{1}$$

Let us denote the valuations of the i^{th} buyer (SU) and j^{th} seller (PU) posted per unit spectrum as $\tilde{v}_i$ and $\tilde{t}_j$ respectively. We assume a fixed pricing mechanism in our bilateral-trading model, where v_i is the amount paid by the i^{th} SU to the moderator, while t_j is the amount paid by the moderator to the j^{th} PU. Therefore, if the moderator allocates the total spectrum of the j^{th} PU to one of the SUs, then the moderator is expected to pay t_j to the j^{th} PU, while the i^{th} SU is expected to pay v_i to the moderator. Note that the prices v_i and t_j, for all $i = 1, \cdots, M$ and $j = 1, \cdots, N$, are fixed prior to the auction mechanism and therefore, the individual players have to obey the pricing scheme whenever there is a transaction.

In practice, the moderator (auctioneer, or the decision maker, e.g., an IEEE 802.22 BS) maximizes its revenue by appropriately assigning the PUs' channels to the SUs within the network. In order to acquire the spectrum, SUs pay the moderator for its service. In contrast to the benefits (payments from the CRs) the moderator acquires, it also bears the cost of making wrong decisions on the channel availability and, therefore, possible collisions with the PU.

We define the utility of the i^{th} SU as follows.

Definition 1 (Utility of the i^{th} SU)**.** *The utility of the i^{th} SU, denoted as U_i, is given by*

$$U_{SU_i}(\Psi) = (\tilde{v}_i - v_i) \sum_{j=1}^{N} \alpha_j \psi_{ij} - c_i. \tag{2}$$

Similarly, we define the utility of the j^{th} PU as follows.

Definition 2 (Utility of the j^{th} PU)**.** *The utility of the j^{th} PU, denoted as U_i, is given by*

$$U_{PU_j}(\Psi) = \left[\alpha_j(t_j - \tilde{t}_j) + \beta_j p_j\right] \sum_{i=1}^{M} \psi_{ij}. \tag{3}$$

Note that the utilities of the PUs and SUs have to be non-negative so that every user has an incentive to participate in the bilateral trade under all circumstances. We ignore the pricing mechanism design, as it is beyond of the scope of this paper.

Similarly, we define the expected utility of the moderator as follows.

Definition 3 (Utility of the Moderator)**.** *The utility of the moderator, denoted as U_M, is given by*

$$U_M(\Psi) = \sum_{i=1}^{M} \sum_{j=1}^{N} u_{ij} \tag{4}$$

where u_{ij} is the utility attained by allocating j^{th} PU's spectrum to the i^{th} SU, which is given as follows.

$$u_{ij} = \alpha_j \psi_{ij}(v_i - t_j) - \beta_j \psi_{ij} p_j. \tag{5}$$

Note that, in Definitions 1, 2 and 3, $\alpha_j \psi_{ij}$ is the probability with which the moderator allocates the j^{th} PU's channel to the i^{th} SU. Similarly, $\beta_j \psi_{ij}$ is the probability with which the i^{th} SU collides with the j^{th} PU, given that the moderator allocated the j^{th} PU's channel to the i^{th} SU. The utility loss of the j^{th} PU due to collision with the i^{th} SU, for any i due to the moderator's erroneous decision, is assumed to be accounted in the penalty p_j that the moderator pays. Therefore, in addition to the performance loss, the moderator is expected to pay an additional penalty for any erroneous decision it makes about spectrum allocation, driving it to behave in a very cautious manner.

In this paper, we assume that each SU is interested in only one channel, for its own communication needs. We also assume that each PU offers one channel for sale, and that the moderator allocates the channels. In such a framework, the moderator would be interested in finding the optimal allocation that maximizes its revenue, given that the PUs and SUs are willing to participate in the bilateral trade with their declared prices they will pay/receive. This can be formally expressed as follows.

Problem P1.

$$\arg\max_{\Psi} \quad U_M(\Psi) \quad \textit{s.t.}$$
$$\textit{1.} \quad \sum_{i=1}^{M} \psi_{ij} \leq 1, \quad \forall\ j = 1, \cdots, N.$$
$$\textit{2.} \quad \sum_{j=1}^{N} \psi_{ij} \leq 1, \quad \forall\ i = 1, \cdots, M.$$

Note that the matrix Ψ has stochastic entries, whose rows and columns sum up to any real value between zero and one. We call such matrices, *weakly-stochastic* matrices.

In this paper, we present an iterative-allocation mechanism in Section IV, that guarantees an improvement in the moderator's utility at every iteration.

III. Necessary Matrix Transformations

Before delving into the details of our solution to the allocation problem posed in Problem P1, in this section, we first define the necessary matrix transformations in our proposed algorithm.

Consider a matrix $A = [a_{ij}]$. We define three matrix-transformations in our paper. These transformations are necessary in finding the optimal spectrum allocation at the moderator.

First, we define a *non-negative* matrix transformation of a given matrix A in Definition 4, where only the non-negative entries are preserved.

Definition 4 (Non-Negative Matrix Transformation)**.** *The non-negative matrix transformation, denoted $\mathscr{P}$, of a matrix A is defined as*

$$\mathscr{P}(A) = [a_{ij}^{+}] \tag{6}$$

where

$$a_{ij}^{+} = \begin{cases} a_{ij} & \textit{if } a_{ij} \geq 0 \\ 0 & \textit{otherwise.} \end{cases}$$

The second is the *row-column-void* transformation of matrix A, where a given row and column entries are voided as stated in Definition 5.

Definition 5 (Row-Column-Void Transformation)**.** *The row-column-void transformation, denoted $\mathscr{V}(A, i, j)$, voids out the i^{th} row and the j^{th} column in the matrix A, such that*

$$\mathscr{V}(A, i, j) = [\tilde{a}_{mn}] \tag{7}$$

where

$$\tilde{a}_{mn} = \begin{cases} 0; \ \textit{if } m = i, \ \textit{or } n = j. \\ a_{mn}; \ \textit{otherwise.} \end{cases} \tag{8}$$

for all $m = 1, \cdots, M$ and $n = 1, \cdots, N$.

Finally, we define the *row-column-replace* transformation of matrix A, where a given position is replaced with one while the remaining entries in the corresponding row and column are voided, as stated in Definition 6.

Definition 6 (Row-Column-Replace Transformation)**.** *The row-column-replace transformation, denoted $\mathscr{R}(A, i, j)$, replaces the $(i, j)^th$ entry with one and voids out the remaining entries in the i^{th} row and the j^{th} column, thus transforming the matrix A as follows.*

$$\mathscr{R}(A, i, j) = [\bar{a}_{mn}] \tag{9}$$

where

$$\bar{a}_{mn} = \begin{cases} 1; \ \textit{if } m = i, n = j \\ 0; \ \textit{if } (m = i, n \neq j) \textit{ or } (m \neq i, n = j) \\ a_{mn}; \ \textit{otherwise.} \end{cases} \tag{10}$$

Using the transformations stated in Definitions 4, 5, and 6, we present an iterative algorithm in Figure 1 that guarantees an improvement in the moderator's utility U_M in every iteration by appropriately finding the allocation matrix Ψ.

IV. Allocation Algorithm

Note that several efforts have been made in the past to address bilateral-trading frameworks, out of which, the most notable ones are the mechanisms proposed by Myerson in [6] and McAfee in [7]. All of these works are geared towards addressing an allocation problem when the item-to-be-traded is known to be available with certainty at the moderator. In this paper, we consider a more general bilateral-trading framework where the item-to-be-traded (more specifically, PUs' spectrum, in our case) is randomly available at the moderator. Given that the moderator does not have perfect knowledge about the PUs' spectrum activity, our goal is to find a mechanism that maximizes the moderator's utility.

Here, we propose an iterative allocation algorithm, as shown in Figure 1, which is a suboptimal solution to Problem P1. By construction, this algorithm guarantees a non-negative utility at the moderator. Also, our algorithm ensures an improvement in the moderator's utility at every iteration.

In order to understand how this algorithm works, let us first investigate the structure of the moderator's utility. Since Ψ is the parameter of interest, we rewrite the moderator's utility, given in Equation (11), as follows.

$$\begin{aligned} U_M(\Psi) &= \sum_{i=1}^{M}\sum_{j=1}^{N} \alpha_j \psi_{ij}(v_i - t_j) - \beta_j \psi_{ij} p_j \\ &= \sum_{i=1}^{M}\sum_{j=1}^{N} \psi_{ij} w_{ij} \end{aligned} \tag{11}$$

where $w_{ij} = \alpha_j(v_i - t_j) - \beta_j p_j$ is the weighting coefficient of the (SU-i, PU-j) association, which is used in our proposed algorithm to decide the appropriate spectrum allocation. For the sake of notational simplicity, we express these coefficients in a matrix form, denoted W, as follows.

procedure SPECALLOC(W, c)
  $W_0 \leftarrow \mathscr{P}(W)$
  $\Psi_0 \leftarrow 0$
  $k = 0$
  while $W_k \neq 0$ **do**
    $(i^*_{k+1}, j^*_{k+1}) \leftarrow \arg\max_{i,j} W_k$
    $W_{k+1} \leftarrow \mathscr{V}(W_k, i^*_{k+1}, j^*_{k+1})$
    $\Psi_{k+1} \leftarrow \mathscr{R}(\Psi_k, i^*_{k+1}, j^*_{k+1})$
    $k \leftarrow k + 1$
  end while
  $\Psi \leftarrow \Psi_k$
  return Ψ
end procedure

Fig. 1. Pseudo-code for the proposed algorithm to find the optimal spectrum allocation at the moderator

$$W = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1N} \\ w_{21} & w_{22} & \cdots & w_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ w_{M1} & w_{M2} & \cdots & w_{MN} \end{bmatrix} \tag{12}$$

Note that, if $w_{ij} \geq 0$, then there is an incentive for the moderator to allocate the j^{th} PU's spectrum to the i^{th} SU. Otherwise, associating i^{th} SU to the j^{th} PU contributes negatively to the utility of the moderator. In other words, we henceforth work on $W_0 = \mathscr{P}(W)$. Similarly, we assume that the initial value of the output allocation matrix be $\Psi_0 = [0]$.

Therefore, as the first stage of finding the solution Ψ^*, we discard all those possibilities that contribute negatively to the utility of the moderator. Within the available choices (associations between the PUs and the SUs), due to the competition between the players, there are several conflicting allocations that the moderator can make. In order to improve its utility, the moderator would, therefore, prioritize the associations between the PUs and the SUs, and allocate the resources based on the established priority.

Let us assume that the position-index of the maximum entry in W_0 is (i^*_1, j^*_1). The proposed algorithm in Figure 1, immediately allocates PU-j^*_1's channel to SU-i^*_1 by transforming Ψ_0 into $\Psi_1 = \mathscr{R}(\Psi_0, i^*_1, j^*_1)$, and voids out the possibility of allocation of multiple spectra to or from the same node by transforming W_0 into $W_1 = \mathscr{V}(W_0, i^*_1, j^*_1)$. Similarly, in the k^{th} iteration, if (i^*_k, j^*_k) is the position-index of the maximum entry of W_{k-1}, then the updated allocation and the weight matrices are given as $\Psi_k = \mathscr{R}(\Psi_{k-1}, i^*_k, j^*_k)$ and $W_k = \mathscr{V}(W_{k-1}, i^*_k, j^*_k)$. The algorithm terminates at the k^{th} iteration if $W_k = [0]$.

Thus, our proposed algorithm iteratively finds the set of associations (between the PUs and the SUs) that improves the moderator's utility. It also guarantees a non-negative utility at the moderator by construction, thereby providing incentive for the moderator to participate in our bilateral-trading framework.

Our proposed algorithm ensures that the moderator's allocation Ψ is a *weakly-stochastic Boolean* matrix. Intuitively, one can see that the optimal solution of Problem P1 is also a weakly-stochastic Boolean matrix, since the set of weak-stochastic Boolean matrices form the extreme points of the set of weak-stochastic matrices. Note that the algorithm may be suboptimal in the most general sense, since the proposed algorithm may result in a different weak-stochastic Boolean matrix from the optimal solution. But, the proposed algorithm can guarantee optimality if the matrix W has a specific structure. The structure of W for which this algorithm gives the optimal allocation, shall be investigated in our future work.

Example: The symmetric case

In this example, let us assume that all the PUs and SUs are statistically identical. Therefore, we ignore the indices i and j corresponding to the SUs and the PUs respectively, in our notation. Therefore, $w_{ij} = w$, for all $i = 1, \cdots, M$ and $j = 1, \cdots, N$. If w is non-negative, our proposed algorithm ensures that the moderator's allocation Ψ is a *weakly-stochastic Boolean* matrix with a block of a $k^* \times k^*$ identity matrix where $k^* = \min\{m, n\}$. On the other hand, if w is negative, the algorithm drives the moderator into a passive mode so that it does not incur a negative utility by allocating any of the PU's channels to any of the SUs.

V. CONCLUSION

We modelled the problem of spectrum allocation as a bilateral trade between the PUs and the SUs, where the moderator iteratively allocates the spectrum to the SUs. By construction, our algorithm guarantees a non-negative utility at the moderator, and that the moderator's utility improves in every iteration. In our future work, we will attempt to design the optimal allocation mechanism along with the pricing mechanism, that maximizes the moderator's utility, in the proposed bilateral-trading framework.

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