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Sparsity based super-resolution in optical measurements

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Abstract—**We propose and experimentally demonstrate a method of exploiting prior knowledge of a signal's sparsity to perform super-resolution in various optical measurements, including: single-shot sub-wavelength Coherent Diffractive Imaging (CDI), i.e. algorithmic object reconstruction from Fourier amplitude measurements, and ultra-fast pulse measurement, i.e. exceeding the temporal resolution imposed by the rise time of the photodiode. The prior knowledge of the signal's sparsity compensates for the loss of phase information and the loss of high spatial frequencies in the case of CDI, and for the loss of temporal frequencies accompanying the photodiode measurement process.**

I. Introduction

Many problems in Optics are ill posed inverse problems. These include for example - lensless imaging (also known as CDI [1], [2]), in which the goal is to recover an object from the measurement of its Fourier magnitude, imaging in sub-wavelength resolution, and measuring an ultrafast pulse using a relatively slow measurement device (photodiode). In these cases, some kind of prior knowledge can be used to help regularize the problem. In this work we demonstrate that such prior knowledge can be that the sought object is sparse in a representation that is either known or taken from a known set.

In Section II we demonstrate the application of the prior knowledge of sparsity to help solve the sub-wavelength CDI problem, in which it is used to overcome the lack of both high-spatial frequencies and phase from the measurements. Mathematically, CDI comes down to recovering a signal from the magnitude of its Fourier transform, a problem known as phase-retrieval. The problem is in general ill-posed, and a common approach to overcome this ill-posedeness is to exploit prior information on the signal. A variety of methods have been developed that use such prior information, which may be the signal's support, non-negativity, or real-space magnitude [3], [4]. The problem becomes more difficult when attempting to recover features that are sub-wavelength (i.e. to achieve super-resolution) - as this corresponds to measuring only a truncated part of the Fourier spectrum - where high spatial frequencies (above the cutoff frequency $\nu_c = \frac{1}{\lambda}$) are lost. We suggest to use the signal's sparsity as prior knowledge to help regularize the combined problem of phase-retrieval and super-resolution. Existing approaches aimed at recovering sparse signals from their Fourier magnitude belong to two main categories: SDP-based techniques [5], [6], [7], [8] and algorithms that use alternate projections (Fienup-type methods) [9]. To solve the combined problem of phase-retrieval and super-resolution, we formulate the problem as a quadratic optimization problem, and solve it using a recently developed efficient sparse quadratic solver named GESPAR [10].

Section III describes the use of sparsity to determine the shape of a temporal signal, namely an optical pulse, from measurements taken by a slow photodiode. This problem is equivalent to linear deconvolution of a low-pass filtered signal, and to regularize it we use the fact that the signal is sparse in a Gauss-Hermite function basis, where the two parameters of the exact representation are found as part of the algorithm.

II. Coherent Diffractive Imaging (CDI)

Coherent Diffractive Imaging (CDI) is an imaging technique where intricate features are algorithmically reconstructed from measurements of the freely-diffracting intensity pattern ([1], [2]). In CDI, an object is illuminated by a coherent plane wave (LASER light), and the far-field diffraction intensity is measured, corresponding to the squared absolute value of the Fourier transform of the object. Recent advances in making lasers in the x-ray regime and in the extreme ultraviolet have made this technique very important for a variety of applications, among them structural biology: mapping out the structure of proteins that cannot be crystallized. However, the physics underlying the propagation of electromagnetic waves acts as a low-pass filter, effectively truncating high Fourier components, setting a fundamental constraint on imaging systems: the finest feature that can be recovered in imaging microscopes is larger than one half of the optical wavelength (the so-called diffraction limit). This condition naturally also limits CDI: the resolution in all current work on CDI is limited by the diffraction limit [11].

Over the past few decades, several sub-wavelength imaging techniques were developed, but none of them works at a 'single shot': they all involve scanning or integration over many acquired images generated by sub-wavelength light sources. These methods include Scanning Near-Field Microscope ([12], [13]), scanning a sub-wavelength "hot spot" ([14], [15]), or using multiple exposures with fluorescent particles ([16], [17], [18]). Due to the nature of these techniques - they cannot be used for real-time imaging of dynamic processes (say, a chemical reaction that evolves with time). On the other hand, CDI, being a ‘single shot’ imaging technique, is suitable for ultra-fast imaging, but it lacks sub-wavelength resolution. Here, we present and demonstrate experimentally a method to enhance CDI resolution beyond the diffraction limit, based on prior knowledge that the object is sparse in a known basis.

A. Problem Formulation

In a CDI setting, an object is illuminated by a coherent plane wave, and the far field diffraction pattern intensity is measured. This measurement, in the Fraunhofer approximation, is proportional to the magnitude of the object’s Fourier transform, up to the cut-off frequency $1/\lambda$, where λ is the wavelength of the light [11]. Therefore, mathematically, the sub-wavelength CDI problem becomes the problem of recovering a 2D signal from only the magnitude of its truncated Fourier transform. This relation can be written as:

$$I = |LFb|^2, \tag{1}$$

where I is the measured far-field intensity, F is the 2D Fourier transform operator, L is a low-pass filter with cutoff frequency $1/\lambda$, and b is the sought 2D object. The operator $|\cdot|$ denotes element-wise absolute value.

Inverting (1), i.e. finding b from I, L, F is an ill-posed problem, both because the high frequency information is lost, and due to the loss of Fourier phase information. The problem, therefore, is phase-retrieval of a 2D object, combined with bandwidth-extrapolation. In order to invert this ill-posed problem, some additional information is needed, e.g. prior knowledge on the sought signal.

In this work, we focus on objects that can be represented compactly in a known basis, i.e. $b = Ax$ where A is a known basis and x is a sparse vector, namely, containing a small number of nonzero elements. In this case, (1) can be rewritten as:

$$I = |LFAx|^2, \tag{2}$$

where x is sparse. The sparsity prior has been used for sub-wavelength imaging [19], but only when the Fourier phase was also known, yielding a linear problem. However, since the measurements in our setting are not linear in the unknown (but quadratic), standard linear sparse inversion algorithms cannot be used, and a method to find a sparse solution to a set of quadratic equations is required.

B. Solution Method and Experimental Results

The problem of sub-wavelength CDI can be viewed as consisting of two sub-problems: Phase retrieval, and bandwidth extrapolation. The problem of phase retrieval, i.e. recovering a signal from the magnitude of its Fourier transform arises in applications such as holography and crystallography, and there has been a vast amount of work dealing with it ([4], [3]). Usually, some prior knowledge about the object is used (e.g. known support or known real-space magnitude), and the different constraints are imposed iteratively. These techniques

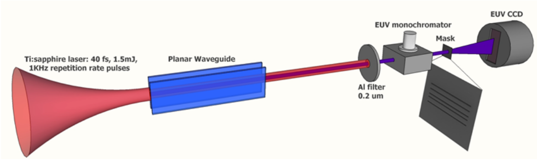

Fig. 1. EUV Experimental setup.

have been used in the context of CDI [2], but their application has always been limited to the information contained within numerical aperture of the system.

Here, we use a recently developed efficient phase-retrieval method called GESPAR [10] that is shown here to also be able to deal with the loss of high-frequencies, by using the prior knowledge that the sought object is sparse in a known basis. GESPAR is described in detail in [10], including extensive numerical performance evaluation. Basically, it seeks a solution to the following problem:

$$\begin{aligned} min_x \;\; f(x) \;\; &\equiv \;\; \sum_{i=1}^{N}(x^T A_i x - c_i)^2 \\ s.t. \;\; & \;\; ||x||_0 \leq s, \end{aligned} \tag{3}$$

where A_i is a set of measurement matrices - in the Fourier phase retrieval problem they are simply defined by $A_i = F_i F_i^T$ with F_i being the ith row of the DFT matrix (or the truncated DFT matrix, in the low-pass scenario), s is an upper limit to the signal’s cardinality, and c_i are the measurements (Fourier magnitude squared). GESPAR iteratively performs a local search to update the signal’s support, and given a current support - uses the damped Gauss Newton method for finding a local minimum of the objective function.

We demonstrate sparsity based super-resolution CDI experimentally, using the setup shown in Fig. 1 [20]. A mask made of titanium foil (200nm thick) containing a pattern of 900nm wide slits, is coherently illuminated by a coherent beam of wavelength λ= 35nm. The far field diffraction pattern is then measured on an EUV CCD. Although the wavelength is shorter than the size of the features (slit width), there is still loss of high spatial frequencies, due to the finite size of the EUV CCD, that imposes a practical cutoff spatial frequency on the measured Fourier magnitude.

The reconstruction of the slit pattern is shown in Fig. 2. Since the slits are much longer than their width, the problem is effectively treated as 1D. Figure 2a shows the measured part of the Fourier magnitude. Figure 2b shows the recovery that would have been obtained from the measured part of the spectrum if the Fourier phase had been measured. The blurring effect due to the low-pass filtering of the system is clearly visible, compared to Fig. 2c which contains the true object in blue. Also in Fig. 2c, the recovered object, using GESPAR, is shown in dotted red. The sparsity basis used here was the basis of 900nm wide rectangles (overlapping). Figure 2d shows the super-resolution explicitly by plotting the original and recovered Fourier spectrum, containing spatial frequencies up to 3 times higher than the measured data (Fig. 2a). Similar

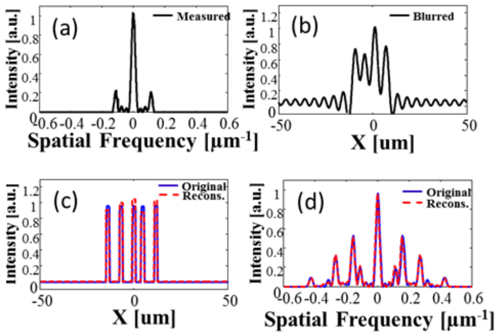

Fig. 2. EUV recovery results. a) Measured Fourier magnitude. b) Blurred object, obtained using (Unmeasured) Fourier phase. c) True (blue) and recovered (dotted red) object. d) True (blue) and recovered (red) Fourier magnitude, exhibiting the method's super resolution (bandwidth extrapolation) capability.

sparsity based super-resolution from Fourier magnitude has also been demonstrated for a truly sub-wavelength and 2D case [21], where an object consisting of an arrangement of holes 100nm in diameter was illuminated by a 532nm laser, and successfully recovered from measurements of the magnitude of its truncated Fourier spectrum.

III. Temporal Super Resolution - Ultrashort Pulse Measurement

The resolution limits of instruments for diagnostics of short laser pulses are defined by their physical properties. For example, the spectral response functions of photodiodes exhibit a low-pass-filter form, with a characteristic cutoff-frequency f_c, where $f_c \sim 1/\tau_c$, with τ_c being the response time of the photodiode. These physical resolution limits, however, do not take into account additional information about the structure of the measured pulse, e.g. that the pulse is sparse in some representation. Here, we propose and demonstrate experimentally the employment of sparsity-based concepts for enhancing the resolution of time-resolved instruments, significantly beyond their inherent physical limits. The algorithm used for this problem uses the measured data to find a proper basis for compact mathematical representation of the input signal, and then utilizes it for extrapolating the resolution significantly beyond the inherent physical limit of the measurement device.

A. Problem Formulation

The sought pulse $x(t)$ is assumed to be sparse in a Gauss Hermite (GH) basis, comprising of the following set of functions: $\Psi_n = H_n(t) \cdot e^{-\frac{(t-t_0)^2}{\Delta t^2}}, n \in \mathbb{N}$, where $H_n(t)$ is the nth Hermite polynomial, and the values of the two parameters Δt and t_0 are unknown in advance. The detected signal is obtained by measuring the output of a photodiode with a temporal PSF of $u(t)$, so that the measured signal is given by the convolution: $y(t) = x(t) * u(t)$. Since the temporal response of the photodiode, which determines $u(t)$ is slower than the features in $x(t)$, the system acts as a low-pass filter.

The problem is therefore to recover $x(t)$ given $y(t)$ and $u(t)$, where it is assumed that $x(t)$ is sparse under the correct GH representation, i.e. using the proper values for the unknown parameters Δt and t_0.

B. Solution Method and Experimental Results

The solution algorithm first finds the parameter t_0, which is determined by the center of the mass of the blurred measured signal $y(t)$. Then, scanning over possible values of the remaining parameter Δt, for each value a basis pursuit denoising problem is solved - i.e. minimizing the l_1 norm while conforming to the measurements. The value that is selected for Δt is the one that yields the sparsest representation. The sought input signal $x(t)$ is given directly by selecting the solution to the l_1 minimization problem corresponding to the chosen parameters Δt and t_0.

We present below an experimental example of super-resolution in a photodiode [22]: We constructed a laser pulse containing three peaks by splitting and later combining an uncompressed pulse from a Ti:Sapphire laser amplifier system into three routes with different lengths. The experimental results are displayed in Fig. 3. The laser pulse is detected by a "slow" photodiode that is characterized by 1000ps rise-time and also by a "fast" photodiode (175ps rise time) – whose measured signal we use as a comparison (Fig. 3a). We first measure the temporal and spectral responses of the photodiodes by detecting the output for a 30fs pulse (Figs. 3b and 3c). Figure 3d shows the measurement taken by the slow and fast photodiodes, while their Fourier spectra are shown in Fig. 3e. We implement our reconstruction scheme on the detected signals from both the slow and fast detectors. For comparison, we also implement Wiener deconvolution on the two detected signals. The reconstructed intensity and spectral profiles are shown in Figs. 3f and 3g, respectively.

As shown, the Wiener deconvolution reconstruction using the slow photodetector is very different form the Wiener deconvolution and the sparsity-based reconstructions using the fast photodiode. This large deviation shows that Wiener deconvolution reconstruction using the slow photodiode signal completely fails to reconstruct the correct profile. On the other hand, our sparsity-based reconstruction using the slow photodiode signal matches very well the reconstructions using the fast photodiode (compare the solid black curve with the solid blue and dash red curves in Figs. 3f and 3g). This correspondence shows that our reconstruction exhibits super-resolution, significantly better than Wiener deconvolution. Comparing the deviations in Fig. 3g, we conclude that our sparsity-based reconstruction has increased the resolution by a factor of ~ 5 over the Wiener deconvolution.

IV. Conclusion

In this work, we have presented a technique facilitating the reconstruction of sub-wavelength features, along with phase retrieval, at an unprecedented resolution for single-shot experiments. Then, we demonstrated the use of similar sparsity based ideas to obtain temporal super-resolution in ultrashort

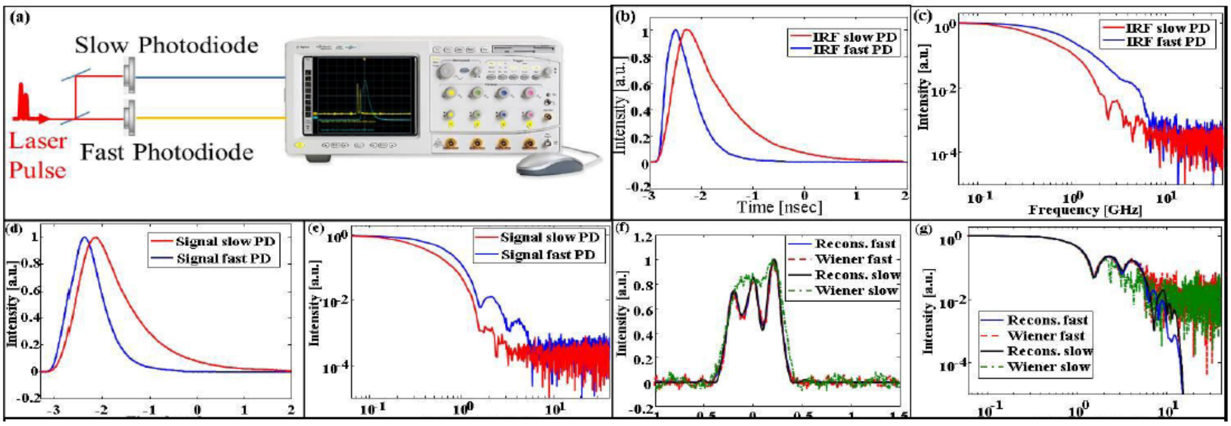

Fig. 3. Experimental super-resolution using a photodiode. a) Experimental setup: a laser pulse probed by a slow (1000ps rise time) and fast (175ps rise-time) photodiodes. b) Measured impulse response of the slow photodiode (red) and the fast photodiode (blue). c) Fourier spectrum of the impulse response functions shown in b). Signals (d) and spectra (e) measured by the slow (red) and fast (blue) photodiodes. Reconstructed pulse-shapes (f) and their spectra (g) obtained using our algorithm, given the measured signals from the slow (solid black) and fast (solid blue) and reconstruction using Wiener deconvolution using the measured signals from the slow (dashed green) and fast (dashed red) photodiode. The reconstructed pulse shape using the slow photodiode signal matches very well the measured fast-photodiode pulse.

pulse measurement. Fundamentally, sparsity-based concepts can be implemented in all imaging systems and achieve sub-wavelength resolution without additional hardware, given only that the image is sparse in a known basis. For example, sparsity-based methods could considerably improve the CDI resolution with x-ray free electron laser [23], without hardware modification.

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