Root-MUSIC Based Source Localization Using Transmit Array Interpolation in MIMO Radar With Arbitrary Planar Arrays

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***Abstract*—We consider the problem of target localization in MIMO radar with arbitrary planar arrays. A method for mapping a two-dimensional (2D) arbitrary transmit array into a uniform rectangular array or an L-shaped array with uniform element displacement is developed. The uniform structure of the mapped array enables applying the computationally efficient 2D root-MUSIC algorithm for estimating the elevation and azimuth parameters of the targets. The transmit array interpolation is achieved by properly designing the mapping matrix via solving a minmax convex optimization problem. We show that there is a tradeoff between the mapping accuracy within a certain desired 2D spatial sector and the sidelobe levels. The performance of the root-MUSIC for the mapped transmit array is shown to achieve the performance of the root-MUSIC associated with a perfect uniform array of the same shape as the mapped one. Simulation examples are used to validate the effectiveness of the proposed method.**

***Keywords*—Arbitrary arrays, array interpolation, direction finding, MIMO radar, root-MUSIC.**

I. Introduction

The emerging concept of multiple-input multiple-output (MIMO) radar has been recently the focus of intensive research [1]– [6]. The essence of MIMO radar is to transmit multiple orthogonal waveforms using multiple transmit colocated (or widely separated) antennas and to jointly process the received echoes due to all transmitted waveforms. It has been pointed out in the literature that MIMO radar with collocated transmit antennas suffers from the loss of coherent transmit processing gain as a result of omnidirectional transmission of orthogonal waveforms [6]. The concepts of phased-MIMO radar and transmit energy focussing have been developed to address the latter problem [6], [7]. Other methods for achieving coherent transmit processing gain in MIMO radar have also been reported in the literature [8]– [12]. However, the aforementioned methods are primarily developed for the case of MIMO radar with one dimensional (1D) transmit array.

Motivated by the great practical interest in two dimensional (2D) transmit arrays, the idea of 2D transmit beamforming has been reported in [13] and [14]. The number of transmit elements used in 2D arrays is typically large and can range from several dozens to few thousands. To reduce the cost, the use of sparse 2D transmit arrays in practice is not uncommon. Sparse 2D arrays are also used to realize dual-band radar systems where antennas that operate within one frequency band are not allowed to co-exist with antennas that operate within the other frequency band. However, because sparse arrays do not enjoy uniform structure they do not straightforwardly enable the use of computationally efficient direction finding algorithms for source localization. In [14], a method has been proposed for transmit array interpolation that maps a non-uniform (possibly sparse) transmit array into two pairs of virtual elements where one pair depends on the elevation angle and the other pair depends on the azimuth angle. It enables the use of ESPRIT-based direction-of-arrival (DOA) estimation to independently estimate the elevation and azimuth parameters of the targets.

In this paper, we consider a MIMO radar with arbitrary 2D arrays and develop a generalization of the aforementioned transmit array interpolation method that enables the use of root-MUSIC for joint elevation-azimuth DOA estimation. In particular, we develop a method for designing a mapping matrix that maps an arbitrary transmit array into a uniform rectangular array (URA) or a uniform L-shaped array. The transmit array mapping matrix design problem is cast as a minmax convex optimization problem. The deviation of the mapped array from the desired array is upper bounded by a positive value that can be controlled by the user. The root-MUSIC DOA estimation algorithm is used to jointly estimate the elevation and azimuth parameters of the targets. We show by simulation examples that the DOA estimation performance of the root-MUSIC applied to the mapped array can achieve the performance of the root-MUSIC applied to a perfect uniform array that has the same shape of the mapped one.

II. Signal Model

Consider the case of a mono-static MIMO radar with transmit and receive arrays of M and N elements, respectively. Both the transmit and receive arrays are assumed to be planar arrays with arbitrary geometries. In a Cartesian two-dimensional space, the transmit antennas are assumed to be located at $\mathbf{p}_m \triangleq [x_m\ y_m]^T,\ m = 1, \ldots, M$ where $(\cdot)^T$ stands for the transpose operator. The antenna locations are measured in wavelength. The $M \times 1$ steering vector of the transmit array is defined as

$$\mathbf{a}(\theta,\phi) = \left[e^{-j2\pi\boldsymbol{\mu}^T(\theta,\phi)\mathbf{p}_1}, \ldots, e^{-j2\pi\boldsymbol{\mu}^T(\theta,\phi)\mathbf{p}_M}\right]^T \quad (1)$$

where θ and ϕ denote the elevation and azimuth spatial angles, respectively, and $\boldsymbol{\mu}(\theta,\phi) = [\sin\theta\cos\phi,\ \sin\theta\sin\phi]^T$ denotes the propagation vector.

Let $\mathbf{C} = [\mathbf{c}_1, \ldots, \mathbf{c}_Q]$ be the $M \times Q$ transmit array mapping/interpolation matrix, where $Q \leq M$ is the number of elements in the desired virtual transmit array and the mth column of $\mathbf{C}$ is denoted as $\mathbf{c}_m$. The relationship between the actual and the mapped/interpolated transmit arrays is given by

$$\mathbf{C}^H\mathbf{a}(\theta,\phi) \simeq \mathbf{d}(\theta,\phi) \qquad \theta \in \Theta,\ \phi \in \Phi \quad (2)$$

where $\mathbf{d}(\theta,\phi)$ is the $Q \times 1$ steering vector associated with the desired virtual transmit array, Θ and Φ are predefined sectors in the elevation and azimuth domains, respectively, and $(\cdot)^H$ stands for the Hermitian transpose. Let $\mathbf{s}(t) = [s_1(t), \ldots, s_Q(t)]$ be the $Q \times 1$ vector of predesigned independent waveforms which satisfy the orthogonality condition $\int_T \mathbf{s}(t)\mathbf{s}^H(t) = \mathbf{I}_Q$, where T is the radar pulse duration and $\mathbf{I}_Q$ is the $Q \times Q$ identity matrix. Each of the orthogonal waveforms is radiated via one element of the virtual transmit array. Therefore, the signal radiated towards a hypothetical spatial location (θ,ϕ) is given by

$$\xi(t,\theta,\phi) = \mathbf{d}^T(\theta,\phi)\mathbf{s}(t) = \sum_{i=1}^{Q}\left(\mathbf{c}_i^H\mathbf{a}(\theta,\phi)\right)s_i(t). \quad (3)$$

It is worth noting that the power radiation pattern of the ith orthogonal waveform $s_i(t)$ is given by $|\mathbf{c}_i^H\mathbf{a}(\theta,\phi)|^2$. Therefore, the vector $\mathbf{c}_i$ can be used to concentrate the transmit power within the 2D spatial sector defined by Θ and Φ. In other words, the transmit array mapping/interpolation matrix $\mathbf{C}$ can be properly designed to jointly achieve transmit array mapping and transmit coherent processing gain.

Assuming that L targets are present in the far-field of the array, the $N \times 1$ receive array observation vector can be written as

$$\mathbf{x}(t,\tau) = \sum_{l=1}^{L}\beta_l(\tau)\left(\mathbf{d}^T(\theta_l,\phi_l)\mathbf{s}(t)\right)\mathbf{b}(\theta_l,\phi_l) + \mathbf{z}(t,\tau) \quad (4)$$

where t and τ are the fast and slow time indexes respectively, $\mathbf{b}(\theta,\phi)$ is the $N \times 1$ steering vector of the receive array, $\beta_l(\tau)$ is the reflection coefficient associated with the lth target with variance σ_β^2, and $\mathbf{z}(t,\tau)$ is the $N \times 1$ vector of zero-mean white Gaussian noise. We assume that the reflection coefficients obey the Swerling II target model, i.e., they remain constant within the whole duration of the radar pulse but change from pulse to pulse. The receive array observation vector $\mathbf{x}(t,\tau)$ is matched-filtered to each of the orthogonal basis waveforms $\mathbf{s}_i(t), i = 1, \ldots, Q$, producing the $N \times 1$ virtual data vectors

$$\begin{aligned}\mathbf{y}_i(\tau) &= \int_{\mathrm{T}}\mathbf{x}(t,\tau)s_i^*(t)dt \\ &= \sum_{l=1}^{L}\beta_l(\tau)\left(\mathbf{c}_i^H\mathbf{a}(\theta_l,\phi_l)\right)\mathbf{b}(\theta_l,\phi_l) + \mathbf{z}_i(\tau) \quad (5)\end{aligned}$$

where $\mathbf{z}_i(\tau) \triangleq \int_{\mathrm{T}}\mathbf{z}(t,\tau)s_i^*(t)dt$ is the $N \times 1$ noise term whose covariance is $\sigma_z^2\mathbf{I}_N$. Note that $\mathbf{z}_i(\tau)$ and $\mathbf{z}_{i'}(\tau)$ $(i \neq i')$ are independent due to the orthogonality between $\mathbf{s}_i(t)$ and $\mathbf{s}_{i'}(t)$. It is worth noting that for a certain Doppler-range bin, the received data is matched-filtered to time-delayed Doppler-shifted version of the transmitted waveforms.

III. 2D Transmit Array Mapping

In this section, we develop a method for designing the transmit array mapping matrix $\mathbf{C}$ that maps a non-uniform and sparse array into a virtual uniform array. One meaningful approach that enables controlling the sidelobe levels is to use the minmax criterion to minimize the maximum difference over a grid in azimuth and elevation between the mapped array steering vector and the desired one while keeping the sidelobe level bounded by some constant. Therefore, the mapping matrix design problem can be formulated as the following optimization problem

$$\min_{\mathbf{C}}\max_{\theta_k,\phi_{k'}}\left\|\mathbf{C}^H\mathbf{a}(\theta_k,\phi_{k'}) - \mathbf{d}(\theta_k,\phi_{k'})\right\|_1 \quad (6)$$
$$\theta_k \in \Theta,\ k = 1,\ldots,K_\theta, \quad \phi_{k'} \in \Phi,\ k' = 1,\ldots,K_\phi$$
$$\text{subject to}\left\|\mathbf{C}^H\mathbf{a}(\theta_n,\phi_{n'})\right\|_1 \leq \gamma, \quad (7)$$
$$\theta_n \in \bar{\Theta},\ n = 1,\ldots,N_\theta, \quad \phi_{n'} \in \bar{\Phi},\ n' = 1,\ldots,N_\phi$$

where $\|\cdot\|_1$ stands for the L_1 norm, $\{\theta_k \in \Theta, k = 1,\ldots,K_\theta\}$ is the angular grid that properly approximates the desired elevation sector Θ by a finite number K_θ of directions, $\{\phi_k \in \Phi, k = 1,\ldots,K_\phi\}$ represents an angular grid that approximates the desired azimuth sector Φ by a finite number K_ϕ of directions, $\bar{\Theta}$ and $\bar{\Phi}$ are the out-of-sector regions in the elevation and azimuth domains, respectively, $\{\theta_n \in \bar{\Theta},\ n = 1,\ldots,N_\theta\}$ and $\{\phi_{n'} \in \bar{\Phi},\ n' = 1,\ldots,N_\phi\}$ are angular grids which properly approximate the out-of-sector regions $\bar{\Theta}$ and $\bar{\Phi}$, respectively, and γ is a positive number of user choice used to upper-bound the worst-case sidelobe level. The optimization problem (6)–(7) is convex and can be efficiently solved using the interior-point methods.

Alternatively, it is possible to minimize the worst-case out-of-sector sidelobe level while upper-bounding the norm of the difference between the mapped array steering vector and the desired one. This can be formulated as the following optimization problem

$$\min_{\mathbf{C}}\max_{\theta_n,\phi_{n'}}\left\|\mathbf{C}^H\mathbf{a}(\theta_n,\phi_{n'})\right\|_1 \quad (8)$$
$$\theta_n \in \bar{\Theta},\ n = 1,\ldots,N_\theta, \quad \phi_{n'} \in \bar{\Phi},\ n' = 1,\ldots,N_\phi$$
$$\text{subject to}\left\|\mathbf{C}^H\mathbf{a}(\theta_k,\phi_{k'}) - \mathbf{d}(\theta_k,\phi_{k'})\right\|_1 \leq \Delta \quad (9)$$
$$\theta_k \in \Theta,\ k = 1,\ldots,K_\theta, \quad \phi_{k'} \in \Phi,\ k' = 1,\ldots,K_\phi$$

where Δ is a positive number of user choice needed to control the deviation of the mapped array from the desired one. It is worth noting that decreasing the value of Δ enhances the mapping within the desired sector at the price of higher sidelobe levels.

IV. Root-MUSIC Based Target Localization

Let the desired virtual transmit array be a URA of size $M_d \times N_d$. Assuming that the desired inter-element spacing is half a wavelength, the $M_dN_d \times 1$ steering vector of the desired virtual transmit array is modeled as

$$\mathbf{d}(\theta,\phi) = \mathrm{vec}\left(\mathbf{u}(\theta,\phi)\mathbf{v}^T(\theta,\phi)\right) \quad (10)$$

where $\mathrm{vec}(\cdot)$ is the operator that stacks the columns of a matrix in one column vector and $\mathbf{u}(\theta,\phi)$ and $\mathbf{v}(\theta,\phi)$ are vectors of dimensions $M_d \times 1$ and $N_d \times 1$, respectively, which are defined as follows

$$\mathbf{u}(\theta,\phi)=\Big[1, e^{-j\pi \sin\theta\cos\phi}, \ldots, e^{-j\pi(M_d-1)\sin\theta\cos\phi}\Big]^T \quad (11)$$

$$\mathbf{v}(\theta,\phi)=\Big[1, e^{-j\pi \sin\theta\sin\phi}, \ldots, e^{-j\pi(N_d-1)\sin\theta\sin\phi}\Big]^T. \quad (12)$$

As a case of particular interest, the desired virtual transmit array can be an L-shaped array. In other words, such virtual array contains only the first row and first column on the $M_d \times N_d$ URA, i.e., $\mathbf{d}(\theta,\phi) = [\mathbf{u}^T(\theta,\phi)\ \mathbf{v}^T(\theta,\phi)]^T$. In this case, solving the mapping matrix design problem (8)–(9) yields

$$\mathbf{C}^H\mathbf{a}(\theta,\phi) \approx [\mathbf{u}^T(\theta,\phi)\ \mathbf{v}^T(\theta,\phi)]^T, \quad \theta \in \Theta,\ \phi \in \Phi. \quad (13)$$

Substituting (13) in (5), we can construct the following two sets of data

$$\begin{aligned}\mathbf{Y}_u(\tau) &= [\mathbf{y}_1(\tau), \ldots, \mathbf{y}_{M_d}(\tau)]^T \\ &\approx \sum_{l=1}^{L} \beta_l(\tau)\mathbf{u}(\theta,\phi)\mathbf{b}^T(\theta_l,\phi_l) + \mathbf{Z}_u(\tau)\end{aligned} \quad (14)$$

and

$$\begin{aligned}\mathbf{Y}_v(\tau) &= [\mathbf{y}_{M_d+1}(\tau), \ldots, \mathbf{y}_{M_d+N_d}(\tau)]^T \\ &\approx \sum_{l=1}^{L} \beta_l(\tau)\mathbf{v}(\theta,\phi)\mathbf{b}^T(\theta_l,\phi_l) + \mathbf{Z}_v(\tau)\end{aligned} \quad (15)$$

where $\mathbf{Y}_u$ and $\mathbf{Y}_v$ have the sizes $M_d \times N$ and $N_d \times N$, respectively, $\mathbf{Z}_u(\tau) = [\mathbf{z}_1(\tau), \ldots, \mathbf{z}_{M_d}(\tau)]^T$, and $\mathbf{Z}_v(\tau) = [\mathbf{z}_{M_d+1}(\tau), \ldots, \mathbf{z}_{M_d+N_d}(\tau)]^T$.

Using $\mathbf{Y}_u(\tau)$, we build the $M_d \times M_d$ covariance matrix

$$\hat{\mathbf{R}}_u = \sum_{\tau=1}^{T_s} \mathbf{Y}_u(\tau)\mathbf{Y}_u^H(\tau) \quad (16)$$

where T_s is the number of radar pulses. Since $\mathbf{u}(\theta,\phi)$ corresponds to a uniform linear array, the root-MUSIC algorithm can be straightforwardly applied to $\hat{\mathbf{R}}_u$ to obtain estimates of $\zeta_l = \sin\theta_l\cos\phi_l,\ l = 1,\ldots,L$. Similarly, $\mathbf{Y}_v(\tau)$ can be used to build the $N_d \times N_d$ covariance matrix

$$\hat{\mathbf{R}}_v = \sum_{\tau=1}^{T_s} \mathbf{Y}_v(\tau)\mathbf{Y}_v^H(\tau). \quad (17)$$

Then the root-MUSIC DOA estimation method can be used to obtain the estimates $\nu_l = \sin\theta_l\sin\phi_l,\ l = 1,\ldots,L$. The so-obtained estimates can be further arranged in the form

$$\chi_l = \zeta_l + j\nu_l, \qquad l = 1,\ldots,L. \quad (18)$$

Thus, the estimates of the elevation angles can be obtained as $\hat{\theta}_l = \sin^{-1}(|\chi_l|),\ l = 1,\ldots,L$ and the estimates of the azimuth angles can be obtained as $\hat{\phi}_l = \angle(\chi_l),\ l = 1,\ldots,L$.

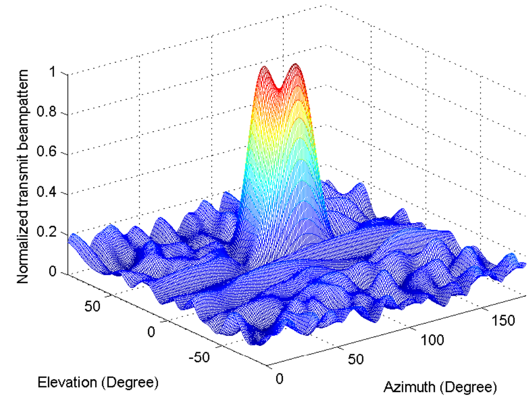

Fig. 1. Normalized transmit beampattern of a 10-element L-shaped virtual transmit array for $\Delta = 0.1$.

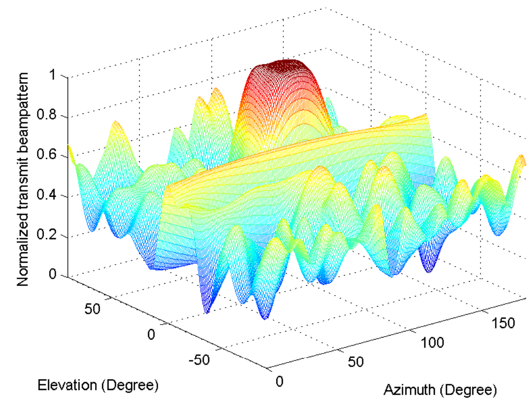

Fig. 2. Normalized transmit beampattern of a 10-element L-shaped virtual transmit array for $\Delta = 0.001$

V. Simulation Results

We assume a transmit array of $M = 64$ elements with the x- and y-components of their positions be drawn randomly from the interval [0, 8] measured in wavelength. The desired sector is defined as $\Theta = [30^\circ,\ 40^\circ]$ and $\Phi = [100^\circ,\ 110^\circ]$. We allow for a transition zone of width 15° at each side of the mainlobe in the elevation domain and 20° at each side of the mainlobe in the azimuth domain. The remaining areas of the elevation and azimuth domains are assumed to be a stopband region. The desired virtual transmit array is assumed to be an L-shaped array with 5 equally spaced elements located on the x-axis and 5 equally spaced elements located on the y-axis. The mapping matrix $\mathbf{C}$ is designed by solving the problem (8)–(9) for $\Delta = 0.1,\ 0.02,$ and 0.001.

The normalized overall transmit beampattern for $\Delta = 0.1$ is shown in Fig. 1. The figure shows that the transmit power is concentrated in the desired sector. Moreover, the achived worst sidelobe level is low at the price of non-uniform transmit power distribution within the desired sector. As a result, the phase variations between the virtual mapped array and the desired one are expected to be large and may cause deterioration in the root-MUSIC DOA estimation performance. The overall

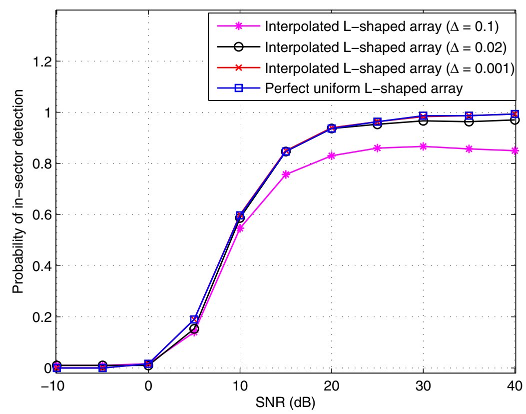

Fig. 3. Probability of in-sector detection versus SNR.

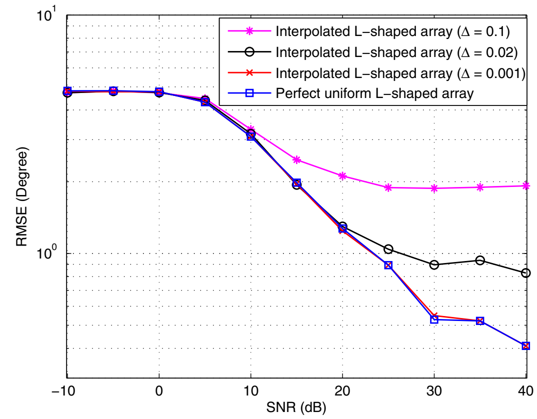

Fig. 4. RMSE versus SNR.

beampattern for the case of $\Delta = 0.001$ is shown in Fig. 2. The figure shows that the transmit power distribution is uniform within the desired sector. It means that the virtual mapped array is very close to the desired one. However, this comes at the price of higher sidelobe levels as shown in the figure.

In the second example, a single target is assumed to be located at $\theta = 35°$ and $\phi = 105°$. The root-MUSIC is tested for the mapping matrices obtained in the first example. Moreover, the case of a 10-element perfect actual L-shaped array is also considered for comparison. The total transmit energy is fixed to M and a total number of 100 radar pulses are used for all methods tested.

The probability of in-sector source detection (i.e., the probability that the estimated elevation-azimuth parameters of the target are located within the desired sector) versus the signal-to-noise ration (SNR) is shown in Fig. 3. The root mean square error (RMSE) versus SNR is shown in Fig. 4 for all methods tested. It can be observed from both figures that the performance of the mapped array with $\Delta = 0.1$ is the worst. As the value of Δ decreases, the performance improves. The performance associated with the case of $\Delta = 0.001$ is almost the same as the performance of the perfect L-shaped array. It is worth nothing that the case of a perfect L-shaped array requires that each antenna radiates more than six times the power rating of the actual non-uniform (sparse) array, and, therefore, is difficult to realize in practice at reasonable cost.

VI. Conclusions

The problem of target localization in MIMO radar with arbitrary planar arrays has been considered. A method for mapping an arbitrary 2D transmit array into a URA or a uniform L-shaped array has been developed. The uniform structure of the mapped array enables applying the computationally efficient 2D root-MUSIC algorithm for estimating the elevation and azimuth parameters of the targets. The transmit array mapping problem is formulated as a minmax convex optimization problem. It has been shown that there exists a tradeoff between the mapping accuracy within a certain desired 2D spatial sector and the sidelobe levels of the transmit beampattern. The performance of the root-MUSIC algorithm has been validated by simulation examples which demonstrate the effectiveness of the proposed method.

References

[1] J. Li and P. Stoica, *MIMO Radar Signal Processing*. New Jersy: Wiley, 2009.

[2] A. Haimovich, R. Blum, and L. Cimini, "MIMO radar with widely separated antennas," *IEEE Signal Processing Magaz.*, vol. 25, pp. 116–129, Jan. 2008.

[3] J. Li and P. Stoica, "MIMO radar with colocated antennas," *IEEE Signal Processing Magaz.*, vol. 24, pp. 106–114, Sept. 2007.

[4] A. Hassanien, S. A. Vorobyov, and A. B. Gershman, "Moving target parameters estimation in non-coherent MIMO radar systems," *IEEE Trans. Signal Processing*, vol. 60, no. 5, pp. 2354–2361, May 2012.

[5] A. Hassanien, S. A. Vorobyov, A. B. Gershman, and M. Ruebsamen, "Estimating the parameters of a moving target in MIMO radar with widely separated antennas," invited paper, in *Proc. 6th IEEE Workshop Sensor Array and Multichannel Signal Processing, SAM'10*, Oct. 2010, pp. 57–60.

[6] A. Hassanien and S. A. Vorobyov, "Phased-MIMO radar: A tradeoff between phased-array and MIMO radars," *IEEE Trans. Signal Processing*, vol. 58, no. 6, pp. 3137–3151, June 2010.

[7] A. Hassanien and S. A. Vorobyov, "Transmit energy focusing for DOA estimation in MIMO radar with colocated antennas," *IEEE Trans. Signal Processing*, vol. 59, no. 6, pp. 2669–2682, June 2011.

[8] T. Aittomaki and V. Koivunen, "Beampattern optimization by minimization of quartic polynomial," in *Proc. 15th IEEE/SP Statist. Signal Processing Workshop*, Cardiff, U.K., Sep. 2009, pp. 437–440.

[9] D. Fuhrmann and G. San Antonio, "Transmit beamforming for MIMO radar systems using signal cross-correlation," *IEEE Trans. Aerospace and Electronic Systems*, vol. 44, no. 1, pp. 1–16, Jan. 2008.

[10] P. Stoica, J. Li, and Y. Xie, "On probing signal design for MIMO radar," *IEEE Trans. Signal Processin*, vol. 55, no. 8, pp. 4151–4161, Aug. 2007.

[11] A. Hassanien and S. A. Vorobyov, "Direction finding for MIMO radar with colocated antennas using transmit beamspace preprocessing," in *Proc. 3rd IEEE CAMSAP*, Aruba, Dutch Antilles, Dec. 2009, pp. 181–184.

[12] A. Khabbazibasmenj, S. A. Vorobyov, and A. Hassanien, "Transmit beamspace design for direction finding in colocated MIMO radar with arbitrary receive array," in *Proc. 36th IEEE ICASSP*, Prague, Czech Republic, May 2011, pp. 2784–2787.

[13] A. Hassanien, M. W. Morency, A. Khabbazibasmenj, S. A. Vorobyov, J.-Y. Park, and S.-J. Kim, "Two-dimensional transmit beamforming for MIMO radar with sparse symmetric arrays," *IEEE Radar Conf.*, Ottawa, ON, Canada, Apr.-May 2013.

[14] A. Hassanien, S. A. Vorobyov, and J.-Y. Park, "Joint transmit array interpolation and transmit beamforming for source localization in MIMO radar with arbitrary arrays," in *Proc. 38th IEEE ICASSP*, Vancouver, BC, Canada, May 2013, pp. 4139–4143.