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Broadband Dispersion Extraction of Borehole Acoustic Modes via Sparse Bayesian Learning

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***Abstract*—This paper considers broadband extraction of multiple strong/weak borehole acoustic modes in acoustic array waveforms by processing the data from multiple frequency points. We first formulate it as basis selection in a multiple measurement vector (MMV) model with varying overcomplete dictionaries and then, propose a generalized sparse Bayesian learning (SBL) method for the application-specified MMV model. The SBL method results in an iterative, hyperparameter-free algorithm to estimate the mode spectrum and update prior parameters. Specifically, the iteration can be implemented in either the fixed-point or expectation-maximization mechanism. Numerical validation with synthetic and field datasets confirms the effectiveness of the proposed method and its advantages over the narrowband (modified matrix pencil) approach.**

I. Introduction

Acoustic waves in fluid-filled boreholes are excited and recorded with sonic logging tools. These are dominated by waveguided borehole modes whose propagation is characterized by dispersion, i.e., frequency dependence of slowness, that in turn is affected by the surrounding rock properties. Therefore, analysis and estimation of the dispersion provide important information for geophysical and geomechanical interpretation. In this paper, we present a novel broadband approach to estimate the dispersion of various borehole modes.

Over the past two decades, both *narrowband* and *broadband* approaches have been proposed to estimate the slowness for the non-dispersive and dispersive modes. These frequency-based methods first convert the discrete time waveforms to the frequency domain, and then process the frequency-domain waveforms for the estimation of slowness and its dispersion. Within the narrowband framework, the slowness is estimated independently from one frequency to another, i.e., it processes the frequency-domain array waveforms to estimate the wavenumber at each frequency point. The Prony's method [1], the modified matrix pencil (MP, also referred to as the TKO method in the industry) method [2], and the narrowband maximum-likelihood (ML) method [3] belong to this category. These methods do not exploit the continuity of dispersion with frequency and outputs scattered estimates that need labeling. Moreover, they are sensitive to model number and can either suppress weak modes or output spurious ones.

On the other hand, the *broadband* approach collects the array data from a chosen frequency band, instead of a single frequency point, and simultaneously estimates the phase and group slownesses from the broadband array data [4], [5]. As the first attempt along this direction, the broadband ML method was proposed in [4] but it has to solve a $2P$-dimensional nonlinear optimization for P acoustic modes. The two-dimensional broadband Capon method was introduced in [6], but has robustness issues with small receiver arrays. A broadband approach exploits the time-frequency localization was proposed in [7], but cannot deal with overlapping modes. A promising broadband technique applicable to multiple overlapping modes was recently proposed in [5], which uses the ℓ_1 norm, together with the ℓ_2 fitting criterion, to regularize the slowness estimates towards a sparse solution. Specifically, the group LASSO method was used to solve the hybrid ℓ_1/ℓ_2 optimization problem. However, its use for the acoustic mode extraction may be limited due to the demanding computational complexity, compounded by the need to select a proper regularization parameter.

In this paper, we reformulate the broadband dispersion extraction from a Bayesian framework where a multiple measurement vector (MMV) model with *varying* overcomplete dictionaries is introduced, and then propose a *generalized* sparse Bayesian learning (SBL) method exploiting the mode sparsity for our application-specified MMV model. For a given broad frequency band, the resulting SBL method is able to extract the phase and group slownesses in a fully automatic, user-parameter free fashion. To obtain the global dispersion analysis, an integrated workflow is proposed with configuration parameters such as the number of frequency points in a frequency band, the overlapping ratio between two consecutive frequency bands and the peak number. It is seen that, by exploiting the mode sparsity, the proposed workflow produces smoother slowness estimates with less fluctuation, less spurious estimates, and better capability to extract weak modes. The proposed SBL method is significantly better than the broadband ML method and the the group LASSO method in terms of the computational complexity. For example, with the same set of configuration parameter, the proposed SBL method is about ten times faster than the group LASSO method with a given regularization parameter. The need to search for an optimal regularization parameter in the latter method makes the comparison even more favorable.

II. Problem Formulation

To begin with our signal model, we use the conventional space-frequency model of the recorded array waveform from L receivers in an acoustic logging tool, i.e., [1]:

$$y_l(\omega) = \sum_{p=1}^{P} \bar{a}_p(\omega) e^{j\bar{k}_p(\omega) z_l} + v_n(\omega), \quad l = 1, 2, \cdots, L \quad (1)$$

where $y_l(\omega)$ is the frequency spectrum of the recorded waveform at the l-th receiver at a given angular frequency ω, P is the number of modes at frequency ω, $\bar{a}_p(\omega)$ and $\bar{k}_p(\omega)$ are the *true* but *unknown* spectrum amplitude and wavenumber of the p-th mode, z_l is the distance from the source (or a reference point) to the l-th receiver, and $v_n(\omega)$ is the complex white Gaussian noise with zero mean and variance σ^2.

The broadband approach usually divides the whole frequency domain into successive (overlapping and/or non-overlapping) frequency bands and estimates the slowness for one frequency band by jointly processing the frequency-domain waveforms within that frequency band. In the following, we reformulate the slowness estimation over a particular frequency band as the MMV model with *varying* overcomplete dictionaries; see (6).

For any given frequency band Ω centered at ω_0 with bandwidth ω_B, e.g., $\Omega = \{\omega_0 - \omega_B \leq \omega \leq \omega_0 + \omega_B\}$, a local representation of the wavenumber dispersion for jointly processing the array waveforms $\{y_l(\omega)\}_{l=1}^{L}$, $\omega \in \Omega$, is given by a first-order Taylor series expansion

$$\bar{k}_p(\omega) \approx \bar{k}_p(\omega_0) + \bar{k}'_p(\omega_0)(\omega - \omega_0), \quad \text{for all } \omega \in \Omega, \tag{2}$$

where $\bar{k}_p(\omega_0)$ and $\bar{k}'_p(\omega_0)$ denote the wavenumber and, respectively, its first-order derivative of the p-th mode at the center frequency ω_0. Taking (2) to (1) yields

$$y_l(\omega) = \sum_{p=1}^{P} \bar{a}_p(\omega) e^{j\bar{s}_p(\omega_0)\omega_0 z_l} e^{j\bar{g}_p(\omega_0)(\omega-\omega_0)z_l} + v_n(\omega), \tag{3}$$

where, by convention, $\bar{k}_p(\omega_0)$ and $\bar{k}'_p(\omega_0)$ are one-to-one mapped to the pair of the phase and group slownesses, $(\bar{s}_p(\omega_0), \bar{g}_p(\omega_0))$, at the center frequency ω_0: $\bar{s}_p = \bar{k}_p/\omega_0$ and $\bar{g}_p = \bar{k}'_p$. [1] With this mapping, (2) and (3) imply that each dispersion curve can be locally approximated by the phase and group slownesses $(\bar{s}_p, \bar{g}_p)$ in the *slowness-frequency* domain.

Stacking the frequency data from all L receivers at a given frequency $\omega \in \Omega$ into a column vector gives

$$\mathbf{y}(\omega) \in \mathbb{C}^{L\times 1} = [y_1(\omega), y_2(\omega), \cdots, y_L(\omega)]^T, \quad \omega \in \Omega. \tag{4}$$

Defining an overcomplete dictionary of N bases spanning a grid of N_p phase slownesses and N_g group slownesses, each parametrized by a *pre-determined* pair $\{s_i\}_{i=1}^{N_p}, \{g_j\}_{j=1}^{N_g}$, where $N = Np \cdot Ng \gg P$, and assuming the true $\bar{s}_p$ and $\bar{g}_p$ of P acoustic modes align with some pre-determined $\{s_i, g_j\}$ pairs, (3) can be expanded as an MMV model with overcomplete dictionaries

$$\mathbf{y}(\omega) = \mathbf{\Phi}(\omega)\mathbf{a}(\omega) + \mathbf{v}(\omega), \quad \omega \in \Omega \tag{5}$$

where the varying dictionaries $\mathbf{\Phi}(\omega)$ are *frequency-dependent*:

$$\mathbf{\Phi}(\omega) \in \mathbb{C}^{L\times N} = [\boldsymbol{\phi}_1(\omega), \boldsymbol{\phi}_2(\omega), \cdots, \boldsymbol{\phi}_N(\omega)], \tag{6}$$

with the n-th basis defined as

$$\boldsymbol{\phi}_n(\omega) \in \mathbb{C}^{L\times 1} = \begin{bmatrix} e^{j[\omega_0 s_i + (\omega-\omega_0)g_j]z_1} \\ e^{j[\omega_0 s_i + (\omega-\omega_0)g_j]z_2} \\ \vdots \\ e^{j[\omega_0 s_i + (\omega-\omega_0)g_j]z_L} \end{bmatrix}. \tag{7}$$

with $i = \lceil n/N_g \rceil$ and $j = n - (i-1)N_g$. The (spectrum) coefficient vector $\mathbf{a}(\omega) = [a_1(\omega), a_2(\omega), \cdots, a_N(\omega)]^T$ has a few non-zero values only when the pre-determined $\{s_i, g_j\}$ coincides with the true $\{\bar{s}_p, \bar{g}_p\}$. Denoting F as the number of frequency points in Ω, the coefficient matrix is formed by F coefficient vectors $\mathbf{a}(\omega)$ in Ω

$$\mathbf{A} \in \mathbb{C}^{N\times F} = [\mathbf{a}(\omega_1), \mathbf{a}(\omega_2), \cdots, \mathbf{a}(\omega_F)], \{\omega_f\}_{f=1}^{F} \in \Omega. \tag{8}$$

Considering the fact that each row of $\mathbf{A}$ determines the presence/absence of the corresponding mode in the recorded array waveforms and there are only a few excited acoustic modes, the *mode sparsity* naturally appears as the row sparsity in the MMV model of (5) with varying overcomplete dictionaries $\mathbf{\Phi}(\omega)$ over a chosen frequency band.

[1] Henceforth the dependence of $\bar{s}_p$ and $\bar{g}_p$ on ω_0 is skipped for notation simplicity.

III. Proposed Sparse Bayesian Learning for Broadband Dispersion Extraction

In this section, instead of treating the coefficient matrix $\mathbf{A}$ as an unknown deterministic parameter, we consider $\mathbf{A}$ as a random matrix and propose to recover the sparse coefficient vectors from the SBL framework [8]–[10].

A. Sparse Bayesian Model

Specifically, we assume the n-th row of $\mathbf{A}$ is complex Gaussian distributed with zero mean and covariance matrix $\gamma_n \mathbf{I}_F$, i.e.,

$$\mathbf{A}^n; \gamma_n \sim \mathcal{CN}(\mathbf{0}, \gamma_n \mathbf{I}_F) \tag{9}$$

where γ_n denotes the prior variance and $\mathbf{I}_F$ is the identity matrix of dimension F. It is noted that, if $\gamma_n = 0$, we have $\mathbf{A}^n = \mathbf{0}$. In addition, the noise vector $\mathbf{v}(\omega)$ is distributed as

$$\mathbf{v}(\omega) \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_L). \tag{10}$$

with an *unknown* noise variance σ^2. Given the above sparse Bayesian model and conditioned on $\mathbf{A}$ and σ^2, the likelihood function is shown as

$$\mathbf{y}(\omega)|\mathbf{a}(\omega); \sigma^2 \quad \sim \quad \mathcal{CN}(\mathbf{\Phi}(\omega)\mathbf{a}(\omega), \sigma^2 \mathbf{I}_L). \tag{11}$$

By applying the Bayes' rule, the posterior distribution of $\mathbf{a}(\omega)$ and the marginal distribution of $\mathbf{y}(\omega)$ are obtained as

$$\mathbf{a}(\omega)|\mathbf{y}(\omega); \boldsymbol{\gamma}, \sigma^2 \sim \mathcal{CN}\left(\mathbf{u}(\omega), \mathbf{\Sigma}_{\mathbf{a}(\omega)}\right) \tag{12}$$

$$\mathbf{y}(\omega); \boldsymbol{\gamma}, \sigma^2 \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{\Sigma}_{\mathbf{y}(\omega)}\right), \quad \omega \in \Omega. \tag{13}$$

where

$$\mathbf{u}(\omega) = \sigma^{-2}\mathbf{\Sigma}_{\mathbf{a}(\omega)}\mathbf{\Phi}^H(\omega)\mathbf{y}(\omega) \tag{14}$$

$$\mathbf{\Sigma}_{\mathbf{a}(\omega)} = \left(\sigma^{-2}\mathbf{\Phi}^H(\omega)\mathbf{\Phi}(\omega) + \mathbf{\Gamma}^{-1}\right)^{-1} \tag{15}$$

$$\begin{aligned}\mathbf{\Sigma}_{\mathbf{y}(\omega)} &= \left[\sigma^{-2}\mathbf{I}_L - \sigma^{-4}\mathbf{\Phi}(\omega)\mathbf{\Sigma}_{\mathbf{a}(\omega)}\mathbf{\Phi}^H(\omega)\right]^{-1} \\ &= \sigma^2\mathbf{I}_L + \mathbf{\Phi}(\omega)\mathbf{\Gamma}\mathbf{\Phi}^H(\omega)\end{aligned} \tag{16}$$

with new notations defined as $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \cdots, \gamma_N]^T$ and $\mathbf{\Gamma} = \operatorname{diag}\{\boldsymbol{\gamma}\}$. We note that the model order P

B. The MMSE Estimate of The Coefficient $\mathbf{a}(\omega)$

The optimal Bayesian minimum mean-square error (MMSE) estimate of the coefficient vector $\mathbf{a}(\omega), \omega \in \Omega$ can be obtained by minimizing the Bayesian MSE

$$\begin{aligned}& E\left(\|\mathbf{a}(\omega) - \hat{\mathbf{a}}(\omega)\|^2 |\boldsymbol{\gamma}, \sigma^2\right) \\ =& \int\int \|\mathbf{a}(\omega) - \hat{\mathbf{a}}(\omega)\|^2 p(\mathbf{a}(\omega), \mathbf{y}(\omega)|\boldsymbol{\gamma}, \sigma^2) d\mathbf{a}(\omega) d\mathbf{y}(\omega).\end{aligned} \tag{17}$$

Given the Gaussian posterior of $\mathbf{a}(\omega)$ in (12) and conditioned on σ^2 and $\mathbf{\Gamma}$, it is known that the MMSE estimate of $\mathbf{a}(\omega)$ is given by the posterior mean of $\mathbf{a}(\omega)$, i.e., $\mathbf{u}(\omega)$ in (14),

$$\begin{aligned}\hat{\mathbf{a}}(\omega) &= \mathbf{u}(\omega) = \sigma^{-2}\mathbf{\Sigma}_{\mathbf{a}(\omega)}\mathbf{\Phi}^H(\omega)\mathbf{y}(\omega), \\ \hat{\mathbf{A}} &= [\hat{\mathbf{a}}(\omega_1), \hat{\mathbf{a}}(\omega_2), \cdots, \hat{\mathbf{a}}(\omega_F)]\end{aligned} \tag{18}$$

Several comments on the MMSE estimate of $\mathbf{a}(\omega)$ are in order. First, the proposed MMSE estimate generalizes existing Bayesian estimates in the single measurement vector (SMV) case and the MMV case with a fixed dictionary. In the former case, $F = 1$, (18) reduces to

$$\hat{\mathbf{a}} = \left(\mathbf{\Phi}^H\mathbf{\Phi} + \sigma^2\mathbf{\Gamma}^{-1}\right)^{-1}\mathbf{\Phi}^H\mathbf{y}, \tag{19}$$

which is the same as (11) in [9]. In the latter case, $F > 1$ and the dictionaries are identical over different frequency points, i.e., $\mathbf{\Phi}(\omega_1) = \cdots = \mathbf{\Phi}(\omega_F) = \mathbf{\Phi}$, (18) reduces to

$$\hat{\mathbf{A}} = \left(\mathbf{\Phi}^H\mathbf{\Phi} + \sigma^2\mathbf{\Gamma}^{-1}\right)^{-1}\mathbf{\Phi}^H\mathbf{Y} \tag{20}$$

which is given by (16) of [10]. Second, the row sparsity is enforced by pushing the hyperparameters $\gamma_i, i = 1, \cdots, N$ into zeros. As $\gamma_i \to 0$, from (18), we have the i-th element of $\hat{\mathbf{a}}(\omega)$ approaches to zeros as well. Combining all coefficient vectors $\hat{\mathbf{a}}(\omega), \omega \in \Omega$, we have the i-th row of $\hat{\mathbf{A}}$ is zeros, i.e., $\hat{\mathbf{A}}^i = \mathbf{0}$ if $\gamma_i = 0$ with probability one. Therefore, the row sparsity of the coefficient matrix $\mathbf{A}$ is naturally transferred to the sparsity in the hyperparameter vector $\boldsymbol{\gamma}$. Once the sparse hyperparameter $\boldsymbol{\gamma}$ and the noise variance σ^2 are estimated, we can then recover the amplitude $\hat{\mathbf{a}}(\omega)$ from (18).

C. Estimation of Hyperparameter $\boldsymbol{\gamma}$ and Noise Variance σ^2

The above MMSE estimate of $\mathbf{a}(\omega)$ is derived based on a given γ_i and σ^2. In the following, the hyperparameters and the noise variance are estimated and updated from the type-II ML estimates [8] by maximizing the joint *marginal* likelihood function of $\mathbf{y}(\omega), \omega \in \Omega$ with respect to $(\boldsymbol{\gamma}, \sigma^2)$. Using (13), this is equivalent to minimizing the negative logarithm of the joint marginal likelihood function which is proportional to

$$\begin{aligned}\mathcal{L}(\boldsymbol{\gamma}, \sigma^2) &= \sum_{f=1}^{F}\mathcal{L}(\omega_f), \quad \{\omega_f\}_{f=1}^F \in \Omega \\ &= \sum_{f=1}^{F}\left[\ln|\mathbf{\Sigma}_{\mathbf{y}(\omega_f)}| + \mathbf{y}^H(\omega_f)\mathbf{\Sigma}^{-1}_{\mathbf{y}(\omega_f)}\mathbf{y}(\omega_f)\right], \end{aligned} \tag{21}$$

where $\mathcal{L}(\omega_f)$ is the negative log-likelihood function at ω_f, and the marginal covariance matrix $\mathbf{\Sigma}_{\mathbf{y}(\omega)}$ is given by (16). Using the second line of (16) for $\ln\left|\mathbf{\Sigma}_{\mathbf{y}(\omega)}\right|$ and the first line of of (16) for $\mathbf{y}^H(\omega)\mathbf{\Sigma}^{-1}_{\mathbf{y}(\omega)}\mathbf{y}(\omega)$, $\mathcal{L}(\omega_f)$ can be expressed as

$$\begin{aligned}\mathcal{L}(\omega) = &\left(L\ln\sigma^2 + \ln|\mathbf{\Gamma}| + \ln\left|\mathbf{\Sigma}^{-1}_{\mathbf{a}(\omega)}\right|\right) + \sigma^{-2}\mathbf{y}^H(\omega)\mathbf{y}(\omega) \\ &- \sigma^{-4}\mathbf{y}^H(\omega)\mathbf{\Phi}(\omega)\mathbf{\Sigma}_{\mathbf{a}(\omega)}\mathbf{\Phi}^H(\omega)\mathbf{y}(\omega).\end{aligned} \tag{22}$$

Unfortunately, the direct maximization of $\mathcal{L}(\boldsymbol{\gamma}, \sigma^2)$ in (21) with respect to $(\boldsymbol{\gamma}, \sigma^2)$ yields no closed-form solution to the type-II ML estimates. Alternatively, we resort to iterative implementation to update $(\boldsymbol{\gamma}, \sigma^2)$.

1) The Fixed-Point Algorithm: The fixed-point algorithm takes the derivative of the joint marginal distribution $\mathcal{L}(\boldsymbol{\gamma}, \sigma^2)$ with respect to $(\boldsymbol{\gamma}, \sigma^2)$ and, then, forms a fixed-point equation by setting the derivatives to zeros. The fixed-point updates for $\boldsymbol{\gamma}$ and σ^2 can be found as (the detailed derivation can be found in [11])

$$\hat{\gamma}_i^{\text{new}} = \frac{\sum_{f=1}^{F}|\mathbf{u}_i(\omega_f)|^2}{\sum_{\omega_f}\zeta_i(\omega_f)} \tag{23}$$

$$\left(\hat{\sigma}^2\right)^{\text{new}} = \frac{\sum_{f=1}^{F}\|\mathbf{y}(\omega_f) - \mathbf{\Phi}(\omega_f)\mathbf{u}(\omega_f)\|^2}{FL - \sum_{\omega_f}\sum_{i=1}^{N}\zeta_i(\omega_f)}. \tag{24}$$

where $\zeta_i(\omega) = 1 - \gamma_i^{-1}\mathbf{\Sigma}^{ii}_{\mathbf{a}(\omega)}$, and the posterior mean $\mathbf{u}(\omega)$ and the posterior covariance matrix $\mathbf{\Sigma}_{\mathbf{a}(\omega)}$ are obtained from (14) and (15) with $\hat{\boldsymbol{\gamma}}^{\text{old}}$ and $\left(\hat{\sigma}^2\right)^{\text{old}}$ in the previous iteration.

2) The Expectation-Maximization Algorithm: The EM algorithm, on the other hand, is an alternative to maximize the marginal likelihood function in (21) by treating $\mathbf{a}(\omega), \omega \in \Omega$, as the hidden (unobservable) variable [12]. In particular, the EM algorithm is a two-step iterative algorithm by first, in the E-step, computing the expectation of log-likelihood of the complete data (the observation $\mathbf{y}(\omega)$ and the hidden variable $\mathbf{a}(\omega)$), given the old parameter estimates and then, in the M-step, updating the estimates of unknown parameters by maximizing this expectation with respect to unknown parameters. The EM updates for $(\boldsymbol{\gamma}, \sigma^2)$ can be obtained as (the detailed derivation can be found in [11])

$$\hat{\gamma}_i^{\text{new}} = \frac{\sum_f\left(\mathbf{\Sigma}^{ii}_{\mathbf{a}(\omega_f)} + |\mathbf{u}_i(\omega_f)|^2\right)}{F}, \tag{25}$$

$$(\hat{\sigma}^2)^{\text{new}} = \frac{\sum_f\left[(\sigma^2)^{\text{old}}\sum_{i=1}^{N}\zeta_i(\omega_f) + \|\mathbf{y}(\omega) - \mathbf{\Phi}(\omega_f)\mathbf{u}(\omega_f)\|^2\right]}{FL}, \tag{26}$$

where $\mathbf{u}(\omega)$ and $\mathbf{\Sigma}_{\mathbf{a}(\omega)}$ are computed from (14) and (15) with $\hat{\boldsymbol{\gamma}}^{\text{old}}$ and $(\hat{\sigma}^2)^{\text{old}}$ in the previous iteration. Compared with the FP iteration, it is observed that the EM iteration converges at a slower rate.

D. The Overall Workflow

The above procedure can be applied for dispersion extraction for a given frequency band. For a global dispersion extraction of borehole acoustic modes, one needs to split the array measurements into successive blocks and apply the SBL algorithm to each block. For each data block over the chosen frequency band, the workflow first uses the generalized SBL method to estimate the coefficient matrix $\hat{\mathbf{A}}$, and then forms the one-dimensional spectrum $\mathcal{A}(s_i)$ over the phase slowness by the following step

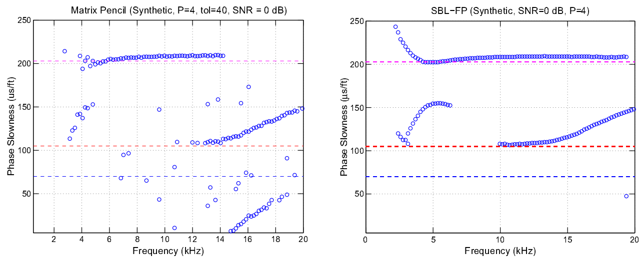
$$\mathcal{A}(s_i) = \sum_{j=1}^{N_g}\|\hat{\mathbf{A}}^{(i-1)N_g+j}\|_2 \tag{27}$$

where $\mathbf{A}^n$ denotes the n-th row of $\mathbf{A}$ and $\|\cdot\|_2$ is the Euclidean norm of a (row) vector. This step essentially sums the (squared root) coefficient energy along the group slowness domain. Applying the one-dimensional peak finding algorithm to the one-dimensional spectrum $\mathcal{A}(s_i)$ gives the phase slowness estimates at the center frequency. By stacking the estimated phase slowness from all frequency bands together, the dispersion of slowness for various modes can be obtained.

Even though the overall workflow involves several configuration parameters such as the frequency bandwidth, the overlapping ratio between successive frequency bands, and the number of peaks extracted, we note that the proposed broadband SBL approach is much less sensitive to the preset number of peaks; see Figures. 1 and 2.

IV. Simulation Results

In this section, we present simulation results to verify the effectiveness of the proposed SBL method for dispersion extraction of borehole acoustic modes. The performance is numerically evaluated by using one synthetic dataset and one field dataset. We compare the proposed SBL method with the industry benchmark MP method [2].



(a) the MP method (b) the SBL-FP method

Fig. 1. Extracted dispersion curves from a synthetic dataset.

A. Synthetic Dataset

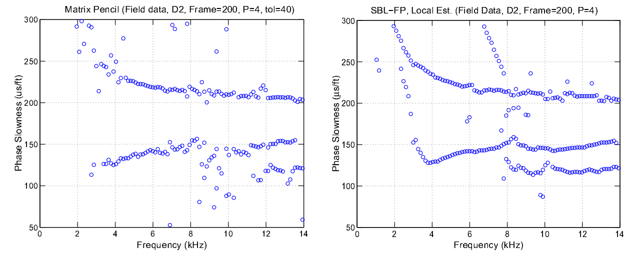
The synthetic borehole acoustic array data are generated according to a semi-analytic (RZX) method with $L = 20$ receivers at SNR $= 0$ dB. In Fig. 1 (a), the MP method is applied to the noisy synthetic RZX data with a preset mode number $P = 4$ and a tolerance level $tol = 40\%$. It is seen that the MP method can estimate the slowness for stronger modes (i.e., the tool flexural mode) but is unable to deliver the slowness estimates when the signal energy is small (i.e., the formation flexural mode). Fig. 1 (b) gives the slowness estimates by the proposed SBL-FP workflow with a preset mode number $P = 4$ [2]. The algorithm parameters for the SBL-FP method include the number of frequency points included in a data block $F = 7$ and the overlapping ratio $\eta = 0.8333$. These parameters give the bandwidth of each frequency band $2\omega_B = 1.1070$ kHz since the frequency resolution for this dataset is $\Delta f = 0.1845$ kHz. The result in Fig. 1 (b) shows a better capability to estimate the slowness for the flexural mode at a broader frequency range, better recovery of the low-frequency slowness of the higher-order formation flexural mode, robustness against the spurious estimates (even though the preset peak number $P = 4$ is larger than the detected modes) due to the exploiting of sparsity, and the smooth dispersion extraction with less variation. Moreover, the extracted slownesses of the flexural mode and higher-order flexural mode at the low frequency range agree well with the true shear slowness at 105 us/ft.

B. Field Dataset

In this section we present the dispersion extraction results for a logging-while-drilling (LWD) field dataset. The frame we extracted from the field dataset corresponds to the depth $12,9647$ ft. For this experiment, we process the dipole inline (DIIN) waveforms recorded from 12 receivers during the second dipole firing (D2).

The MP method with $P = 4$ and $tol = 40\%$ is first applied to the dipole inline field data and the result is shown in Fig. 2 (a). For the proposed SBL-FP method, the configuration parameters are set to $F = 9$, $\eta = 0.8889$ and $P = 4$. These parameters give the bandwidth of each frequency band $2\omega_B = 1.0417$ kHz and the extracted dispersion curves are shown in Fig. 2 (b). Comparing Fig. 2 (a) with Fig. 2 (b) reveals that the proposed SBL method deliveries more robust slowness estimates of the two formation flexural modes with less fluctuation from one frequency to another. Particularly, the SBL method provides consistent slowness estimates of the higher-order flexural mode at the high frequency region.

[2]The SBL-EM method gives similar performance with more computation time and, hence, it is skipped here. Results are superior with respect to the Capon and LASSO methods. Details will be reported in [11].



(a) the MP method (b) the SBL-FP method

Fig. 2. Extracted dispersion curves from a field dataset (dipole inline)

V. Conclusion

In this paper, we reformulated the broadband dispersion extraction of borehole acoustic modes as the sparse signal recovery under a unique multiple measurement vectors model with varying overcomplete dictionary matrices. Accordingly, the generalized SBL method has been proposed to take into account the dictionary propagation over the frequency. Two iterative implementation of the generalized SBL method have also been developed based on the Type-II maximum likelihood estimation. Finally, simulation results are provided to show the effectiveness of the proposed SBL method for the dispersion extraction of borehole acoustic modes.

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