# Modeling the Aggregator problem: the economic dispatch and dynamic scheduling of flexible electrical loads

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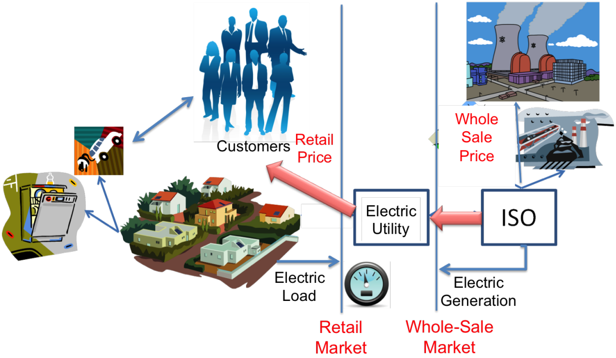
Premise

# Balancing the Grid

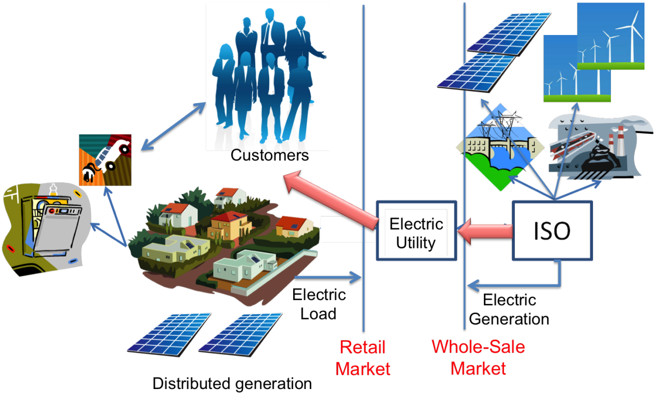
# How can generators know how much to produce?




1. Retail electricity market → public utility, serves/tracks demand
   - Customers do not see and do not respond to the real prices
2. Wholesale electricity market ≈ perfect competition for generators
   - A centralized optimization (run by an Independent System Operator) provides prices
   - Multiple settlements: Day Ahead (DA) → Hour Ahead (HA) → Real Time (RT) → Regulation . . . to manage load uncertainty

# Why are we not using more green electricity?

We are scheduling for Net consumption = Load - Renewable power




- Advantage: inelastic net consumption is back compatible with current electricity market
- Problem: unsustainable. *Large generator ramps + reserves for dealing with uncertainty blow up costs and pollution*

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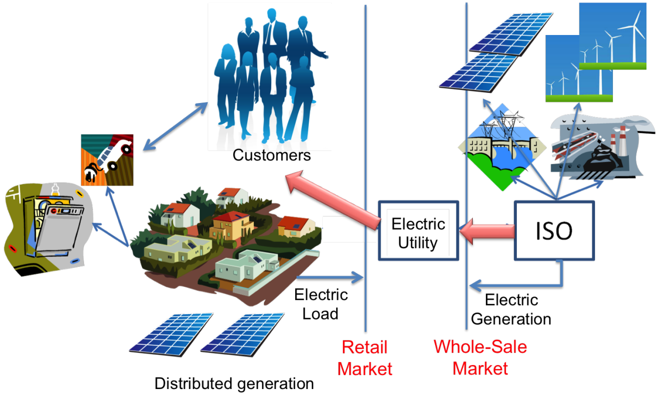


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# Electric Consumption Flexibility

- Demand is random but not truly inflexible, but today there is no standard appliance interface to modulate it

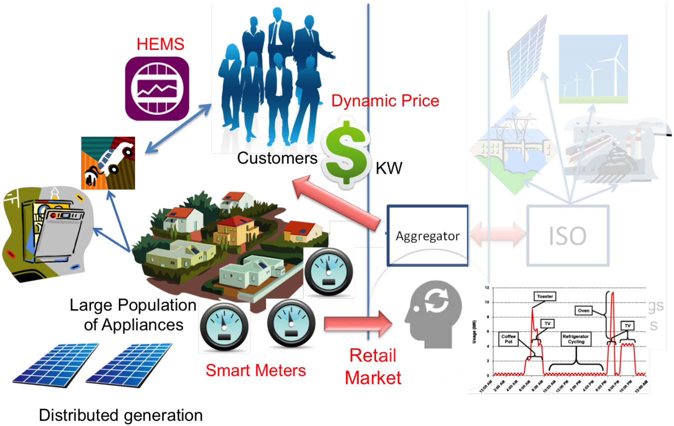


- Demand Response (DR) programs tap into the flexibility of end-use demand for multiple purposes
- But how much **intrinsic flexibility** does the aggergate demand of a large appliance population have?

**Definition: Plasticity**

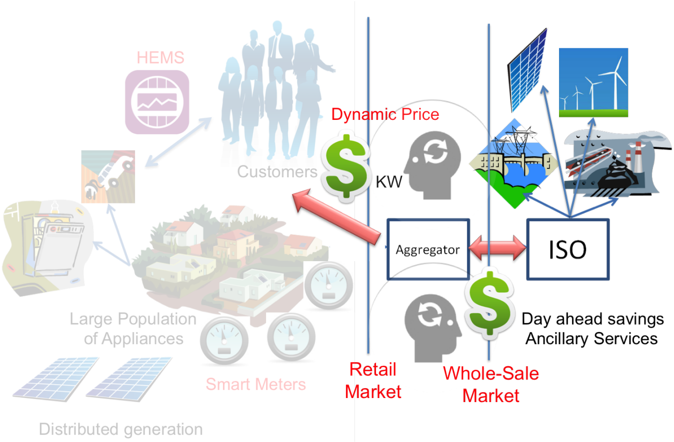
The potential shapes that the load of an appliance or a popoulation of appliances can take

# The Smart Grid vision

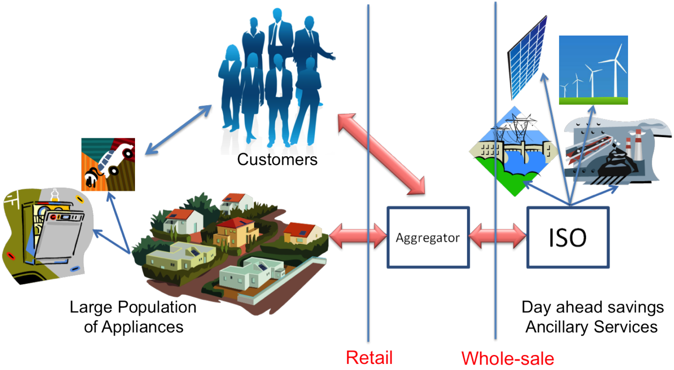



- Most of the work is on the home price response side

# The Smart Grid System Challenge



# Demand Response: The Aggregator Problem




Heterogenous population (...it is "The Internet of Things")

- Challenge 1: Modeling the flexibility ex-ante in the market
- Challenge 2: Real time control of the appliances
- Challenge 3: Economics: Convincing the customers to participate

# Outline

PART I - Modeling Electric Load Flexibility ("Plasticity")

- The plasticity of a canonical battery
- Population models for a very large number of canonical batteries
- Generalizing results to real appliances
- Planning and control

PART II - Retail Markets with Plasticity

- Designing retail prices and incentives

Part I

# Load Flexibility Models

# Various research camps

- Tank model: Fill the flexible demand tank by the end of the day *[Lambert, Gilman, Lilienthal,'06], [Lamadrid, Mount, Zimmerman, Murillo-Sanchez, '11],[Papavasiliou, Oren '10]*
  - For the market, to set prices
- Detailed model: Model each individual appliance constraints *[Joo,Ilic,'10], [Huang, Walrand, Ramchandran,'11], [Foster,Caramanis,'13]*
  - For local controllers that respond to dynamic prices
- Quantized Population Models: Cluster appliances and derive an aggregate model *[Chong85],[Mathieu,Koch, Callaway,'13],[Alizadeh, Scaglione, Thomas,'12]...*
  - Good for both! What we discuss next....

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# Example of Load Plasticity: Ideal Battery

One ideal battery indexed by $i$

- Arrives at $t_i$ and remains on indefinitely
- No rate constraint
- Initial charge of $S_i$
- Capacity $E_i$

The plasticity of battery $i$ is defined as

$$\mathcal{L}_\mathrm{i}(\mathrm{t}) = \{\mathrm{L_i(t)}|\mathrm{L}_i(\mathrm{t}) = \mathrm{dx_i(t)/dt}, \mathrm{x_i(t_i)} = \mathrm{S_i}, 0 \leq \mathrm{x_i(t)} \leq \mathrm{E_i}, \mathrm{t} \geq \mathrm{t_i}\}.$$

In English:

Load (power) = rate of change in state of charge $x(t)$ (energy)

- Set $\mathcal{L}_i(\mathrm{t})$ characterized by appliance category $v$ (ideal battery) and 3 continuous parameters:

$$\boldsymbol{\theta}_i = (t_i, S_i, E_i)$$

But how can we capture the plasticity of thousands of these batteries?

# Aggregate Plasticity

We define the following operations on plasticities $\mathcal{L}_1(\mathrm{t})$, $\mathcal{L}_2(\mathrm{t})$:

$$\mathcal{L}_1(t) + \mathcal{L}_2(t) = \left\{ L(t) | L(t) = L_1(t) + L_2(t), (L_1(t), L_2(t)) \in \mathcal{L}_1(t) \times \mathcal{L}_2(t) \right\}$$

$$n\mathcal{L}(t) = \left\{ L(t) | L(t) = \sum_{k=1}^{n} L_k(t),\ (L_1(t), .., L_n(t)) \in \mathcal{L}^n(t) \right\},$$

where $n \in \mathbb{N}$ and $0\mathcal{L}_1(t) \equiv \{0\}$.

- Then, the plasticity of a population $\mathcal{P}^v$ of ideal batteries is

$$\mathcal{L}^v(\mathrm{t}) = \sum_{\mathrm{i} \in \mathcal{P}^{\mathrm{v}}} \mathcal{L}_{\mathrm{i}}(\mathrm{t}) \tag{1}$$

Plasticity of population = sum of individual plasticities

What if we have a very large population?

# Quantizing Plasticity

- Natural step → quantize the paramaters: $\boldsymbol{\theta}_i = (t_i, S_i, E_i)$

$$\boldsymbol{\theta} \;\mapsto\; \boldsymbol{\vartheta} \in \text{Finite set } \mathcal{T}^v$$

- Quantize state and time uniformly with step $\delta t = 1$ and $\delta x = 1$
- Discrete version (after sampling + quantization) of plasticity:

$$\mathcal{L}_i(t) = \{L_i(t) | L_i(t) = \partial x_i(t), x_i(t_i) = S_i, x_i(t) \in \{0, 1, \ldots, E_i\}, t \geq t_i\}.$$

- Plasticity of all batteries with discrete parameters $\boldsymbol{\vartheta} = \mathcal{L}^v_{\boldsymbol{\vartheta}}(t)$

# Lowering the Complexity of $\mathcal{L}^v(\mathrm{t})$?

- Let $a^v_{\boldsymbol{\vartheta}}(t) \triangleq$ number of batteries with discrete parameters $\boldsymbol{\vartheta}$

$$\mathcal{L}^v(t) = \sum_{\boldsymbol{\vartheta}\in\mathcal{T}^v} a^v_{\boldsymbol{\vartheta}}(t)\mathcal{L}^v_{\boldsymbol{\vartheta}}(t), \qquad \sum_{\boldsymbol{\vartheta}\in\mathcal{T}^v} a^v_{\boldsymbol{\vartheta}}(t) = |\mathcal{P}_v|. \tag{2}$$

- $v = 1, \ldots, V$ different categories of appliances

$$\mathcal{L}(\mathrm{t}) = \mathcal{L}^{\mathrm{I}}(\mathrm{t}) + \sum_{\mathrm{v}=1}^{\mathrm{V}} \mathcal{L}^{\mathrm{v}}(\mathrm{t}), \qquad \mathcal{L}^{\mathrm{I}}(\mathrm{t}) = \{\mathrm{L}^{\mathrm{I}}(\mathrm{t})\} \;\; \text{inelastic load} \tag{3}$$

- Still redundant for aggregate load modeling
- The set $\sum_{\boldsymbol{\vartheta}} a^v_{\boldsymbol{\vartheta}}\mathcal{L}^v_{\boldsymbol{\vartheta}}(t)$ can be combined for some $\boldsymbol{\vartheta}$ and represented by fewer variables

# Bundling Appliances with Similar Constraints

- Population $\mathcal{P}_E^v$ with homogenous $E$ but different $(t_i, S_i)$
- Define arrival process for battery $i$

$$a_i(t) = u(t - t_i) \rightarrow \text{indicator that battery } i \text{ is plugged in}$$

- We prefer not to keep track of individual appliances
- Random state arrival process on aggregate

$$a_x(t) = \sum_{i \in \mathcal{P}_E^v} \delta(S_i - x) a_i(t), \quad x = 1, \ldots, E$$

- Aggregate state occupancy

$$n_x(t) = \sum_{i \in \mathcal{P}_E^v} \delta(x_i(t) - x) a_i(t), \quad x = 1, \ldots, E$$

# Aggregate load plasticity

**Lemma**

*The relationship between load and occupancy is:*

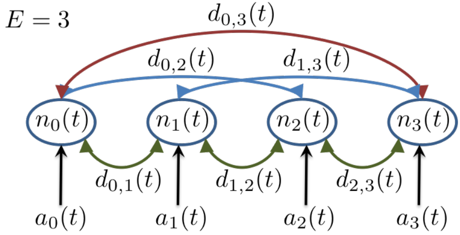
$$L(t) = \sum_{x=0}^{E} \left[ \left( \sum_{x'=x}^{E} \partial n_{x'}(t) \right) - (x+1)\partial a_x(t) \right].$$

- Can we say more when the change in state is the result of a control action?

**Activation process from state $x'$ to $x$ :**

$d_{x,x'}(t) = \#$ **batteries that go from state $x$ to state $x'$ up to time $t$**

Naturally, $\partial d_{x,x'}(t) \leq n_x(t)$.



# Controlled Aggregate Load Plasticity

## Corollary

*The relationship between occupancy, control and load are:*

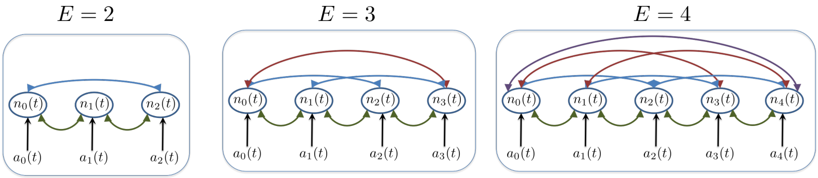
$$n_x(t+1) = a_x(t+1) + \sum_{x'=0}^{E} [d_{x',x}(t) - d_{x,x'}(t)]$$

$$L(t) = \sum_{x=0}^{E} \sum_{x'=0}^{E} (x' - x) \partial d_{x,x'}(t)$$

Notice the linear and simple nature of $L(t)$ in terms of $d_{x,x'}(t)$

# Bundling Batteries with Non-homogeneous Capacity

- Results up to now are valid for batteries with homogenous capacity $E$
- The capacity changes the underlying structure of plasticity
- We divide appliances into **clusters** $q = 1, \ldots, Q^v$ based on the quantized value of $E_i$



# Quantized Linear Load Model

## Load plasticity of heterogenous ideal battery population

$$\mathcal{L}^v(t) = \left\{ L(t) | L(t) = \sum_{q=1}^{Q} \sum_{x=0}^{E^q} \sum_{x'=0}^{E^q} (x' - x) \partial d_{x,x'}^q(t) \right.$$

$$\left. \partial d_{x,x'}^q(t) \in \mathbb{Z}^+, \sum_{x'=1}^{E^q} \partial d_{x,x'}^q(t) \leq n_x^q(t) \right\}$$

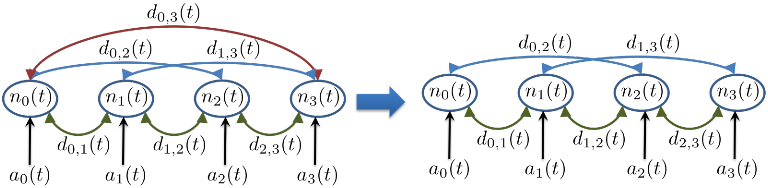
$$n_x^q(t) = a_x^q(t) + \sum_{x'=0}^{E^q} [d_{x',x}^q(t-1) - d_{x,x'}^q(t-1)]$$

Linear, and scalable at large-scale by removing integrality constraints

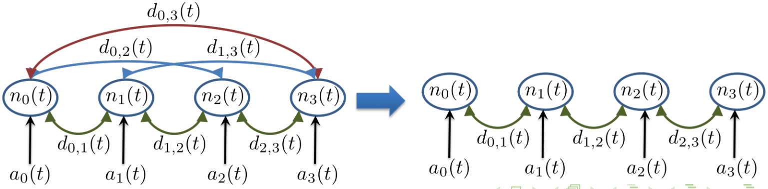
Aggregate model= Tank Model *[Lambert, Gilman, Lilienthal,'06]*

# More constrained models for load plasticity

- The canonical battery can go from any state to any state and has no deadline or other constraints.
- What about real appliances? Some are simple extensions
- Rate-constrained battery chage, e.g., V2G




- Interruptible consumption at a constant rate, e.g., pool pump, EV 1.1kW charge



# Deadlines

- You can add deadlines using the same principle: cluster appliances with the same deadline $\chi^q$
- Then, you simply express the constraint inside the plasticity set

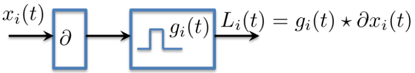
$$\mathcal{L}^v(t) = \Big\{ L(t) | L(t) = \sum_{q=1}^{Q^v} \sum_{x=0}^{E^q} \sum_{x'=0}^{E^q} (x' - x) \partial d^q_{x,x'}(t)$$
$$\partial d^q_{x,x'}(t) \in \mathbb{Z}^+, \forall x, x' \in \{0, 1, \ldots, E^q\}$$
$$\sum_{x'=1}^{E^q} \partial d^q_{x,x'}(t) \leq n^q_x(t), \forall x < E^q \rightarrow n_x(\chi^q) = 0 \Big\} \quad (4)$$

# Non-interruptible Appliances - Individual Plasticity

- Loads that can be shifted within a time frame but cannot be modified after activation, e.g., washer/dryers
- $x_i(t) \in \{0, 1\}$ = state of appliance $i$ (wainting/activated)
- Impluse response of appliance $i$ if activated at time $0 = g_i(t)$
- Laxity (slack time) of $\chi_i$

$$\mathcal{L}_i(t) = \{L_i(t) | L_i(t) = g_i(t) \star \partial x_i(t), x_i(t) \in \{0, 1\}, \quad x_i(t) \geq a_i(t - \chi_i), \ x_i(t-1) \leq x_i(t) \leq a_i(t)\}. \tag{5}$$

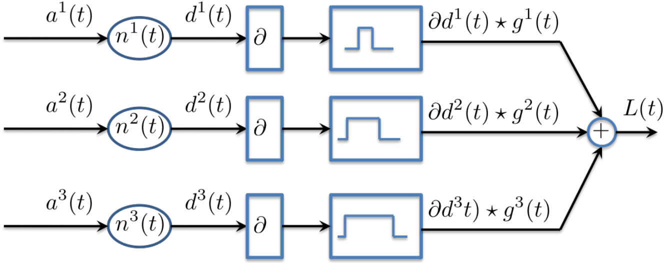
Load = change in state convolved with the load shape $g_i(t)$




**Note**: $d^q_{0,1}(t) \equiv d^q(t) \equiv x^q(t)$

# Non-interruptible Appliances - Aggregate Plasticity

- We assign appliances to cluster $q$ based on quantized pulses $g^q(t)$
- $a^q(t)$ = total number of arrivals in cluster $q$ up to time $t$
- $d^q(t)$ = total number of activations from cluster $q$ up to time $t$




$$\mathcal{L}^v(t) = \Big\{ L(t) | L(t) = \sum_{q=1}^{Q^v} g^q(t) \star \partial d^q(t), \, d^q(t) \in \mathbb{Z}^+ \qquad (6)$$

$$d^q(t) \geq a^q(t - \chi^q), \;\; d^q(t-1) \leq d^q(t) \leq a^q(t) \Big\}$$

# Other cases....

- Dimmable Lighting, like Hybrid system, but you control $g_i(t)$ instead of the switch state
- Thermostatically Controlled Loads (TCL) require a bit more effort but one can follow the same constructs
- ....you can soon get a pretty complete family of models
- If it can shift demand, the Aggregator can hedge the electricity market settlements.
- The Aggregator needs to control the appliances. How?

# How can the Aggregator harness plasticity?

Two options to harness the population plasticity $\mathcal{L}(t)$

- Dynamic Pricing: The Aggregator sends a price signal, the customers respond with a local Home/Building Energy Management System
- Direct Load Scheduling: The Aggregator provides different pricing incentives, to control directly electric loads

- In both cases, due to limited degrees of control on heterogenous demand:

$$\mathcal{L}^{DR}(t) \subseteq \mathcal{L}(t)$$

- The price signal or incentive affects the arrival processes $a_x^q(t)$

# Steps for the Aggregator Direct Load Scheduling (DLS)

Pricing Incentive design:

- Design incentives to recruit appliances - - will discuss in part II

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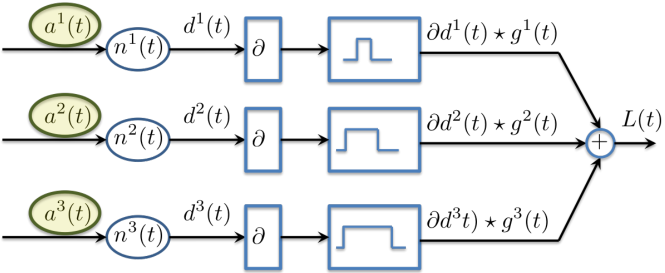
- Forecast arrivals in clusters for different categories
- Make optimal market decisions based on forecasted plasticity



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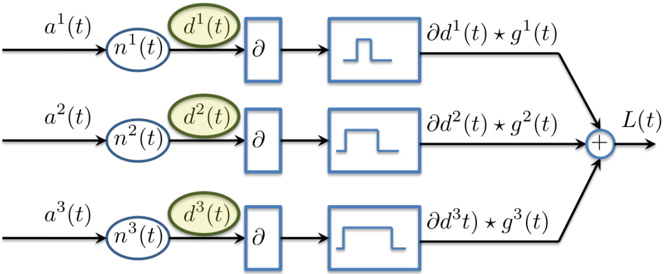
Real-time:

- Observe arrivals in clusters
- Decide appliance schedules $d^q(t)$ to optimize load

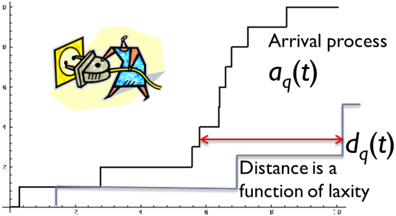


# Real-time: How do we activating appliances?

**Arrival and Activation Processes**

$a_q(t)$ and $d_q(t)$ $\rightarrow$ total recruited appliances and activations before time $t$ in the $q$-th queue

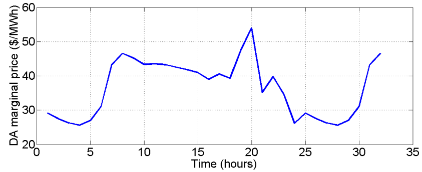
- Easy communications: Broadcast time stamp $T_{act}$: $a_q(t - T_{act}) = d_q(t)$

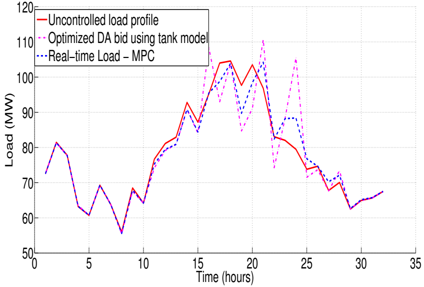



- Appliance whose arrival is prior than $T_{act.}$ initiate to draw power based on the broadcast control message

# Population modeling with the Tank Model

- Population of 40000 PHEVs + 1.1 kW non-interruptible charging
- Tank model = PHEVs effectively modeled as canonical batteries

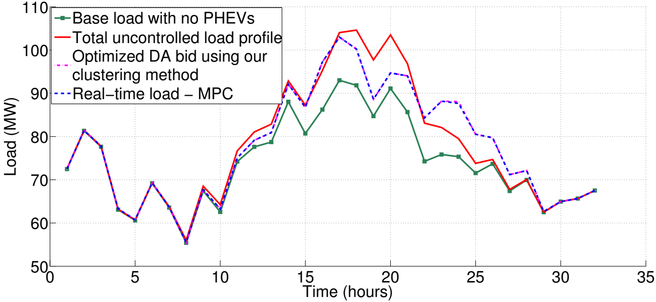



• Real-world plug-in times and charge lengths
• 15 clusters (1-5 hours charge + 1-3 hours laxity)
• PHEV demand = 10% of peak load
• DA= Day Ahead
• PJM market prices DA 10/22/2013 • Real time prices = adjustments cost 20% more than DA
• DA = LP + SAA with 50 random scenarios + tank model
• RT = ILP + Certainty equivalence + clustering

# Population modeling with proposed quantizion scheme

- Quantized Deferrable EV model
- Load following dispatch very closely when using our model




- Same setting
- DA = LP + Sample Average $\approx \mathbb{E}\{a^q(t)\}$ (50 random scenarios) + clustering
- Real Time Control = ILP + Certainty equivalence + clustering

# Part II
# Pricing Incentive Design

# DR #1: Dynamic Pricing

- Dynamic retail prices $\mathbf{x}(t) = [\pi^r(t), \ldots, \pi^r(t+T)] \in \mathcal{Z}(t)$ (set of regulated prices)
- Possible load shapes:

$$\mathcal{L}^{DR}(t) = \left\{ L(t) | L(t) = f(t; \mathbf{x}(t)), \mathbf{x}(t) \in \mathcal{Z}(t) \right\} \tag{7}$$

- Here $f(.)$ is the price-response of the population

$$f(t; \mathbf{x}(t)) = L^I(t) + \sum_{v=1}^{V} \sum_{\boldsymbol{\vartheta} \in \mathcal{T}^v} \Bigg\{ \underbrace{a^v_{\boldsymbol{\vartheta}}(\mathbf{x}(t))}_{\text{unobservable}} \overbrace{\arg\min_{L(t) \in \mathcal{L}^v_{\boldsymbol{\vartheta}}(t)} \sum_{t=1}^{T} \pi^r(t) L(t)}^{\text{quantized price response - known}} \Bigg\}$$

- Price response only observable in aggregate and not for different clusters $\rightarrow$ learning $a^v_{\boldsymbol{\vartheta}}(\mathbf{x}(t))$ from limited observations

# DR #2: Pricing for Direct Load Scheduling (DLS)

- An aggregator hires appliances and directly schedules their load
- Set of differentiated prices based on plasticity

$$\boldsymbol{x}^v(t) = \{x^v_\vartheta(t), \forall \boldsymbol{\vartheta} \in \mathcal{T}^v\}$$

But how can we have voluntary participation in DLS?

- Differentiated discounts $\boldsymbol{x}^v(t)$ from a high flat rate → incentives
- Appliances choose to participate based on incentives → $a^v_{\boldsymbol{\vartheta}}(\boldsymbol{x}^v(t))$

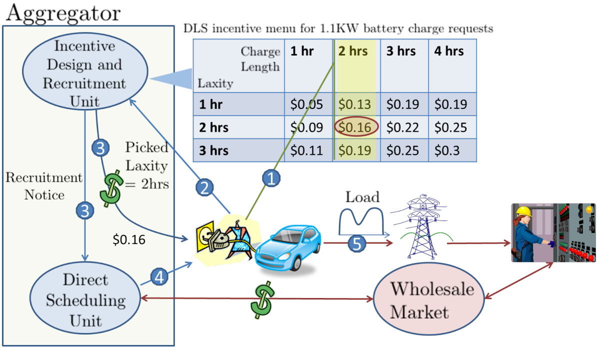
$$\mathcal{L}^{DR}(t) = \mathcal{L}^I(t; \boldsymbol{x}^v) + \sum_{v=1}^{V} \sum_{\boldsymbol{\vartheta} \in \mathcal{T}^v} a^v_{\boldsymbol{\vartheta}}(\boldsymbol{x}^v(t)) \mathcal{L}^v_{\boldsymbol{\vartheta}}(t). \quad (8)$$

- Reliable: aggregator observes $a^v_{\boldsymbol{\vartheta}}(\boldsymbol{x}^v(t))$ after posting incentives and before control - no uncertainty in control

# Dynamically Designed Cluster-specific Incentives

- Characteristics in $\vartheta$ have 2 types: intrinsic and customer chosen
- We cluster appliances based on intrinstic characterics, e.g. $g^q(t)$
- Customer picks operation mode $m$, e.g., laxity $\chi$

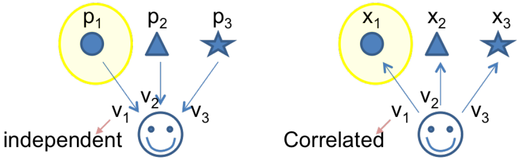
We design a set of incentives $x_m^{v,q}(t), m = 1, \ldots, M^{v,q}$ for each cluster




| Laxity \ Charge Length | 1 hr | 2 hrs | 3 hrs | 4 hrs |
|---|---|---|---|---|
| 1 hr | $0.05 | $0.13 | $0.19 | $0.19 |
| 2 hrs | $0.09 | $0.16 | $0.22 | $0.25 |
| 3 hrs | $0.11 | $0.19 | $0.25 | $0.3 |

*[Alizadeh, Xiao, Scaglione, Van Der Schaar 2013], see also [Bitar, Xu 2013], [Kefayati, Baldick, 2011]*

- Category $v$ and cluster $q \rightarrow$ intrinsic properties of loads
- Aggregator posts incentives for each mode of loads in cluster $q$ and category $v$
- Optimal posted prices? The closest approximation is the "optimal unit demand pricing"
- Customers valuation for different modes correlated (value of EV charge with 1 hr laxity vs. value of EV charge with 2 hrs laxity)



# The Incentive Design Problem

- Independent incentive design problem for different categories $v$ and clusters $q$ $\rightarrow$ Let's drop $q, v$ for brevity
- Aggregator designs

$$\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_M(t)]^T, \tag{9}$$

- From recruitment of flexible appliances, the aggregator saves money in the wholesale market (utility):

$$\mathbf{u}(t) = [U_1(t), \ldots, U_M(t)]^T \tag{10}$$

- Aggregator payoff when interacting with a specific cluster population:

$$Y(\mathbf{x}(t); t) = \sum_{m \in \mathcal{M}} \overbrace{(U_m(t) - x_m(t))}^{\text{Payoff of mode } m} \sum_{i \in \mathcal{P}(t)} \overbrace{a_{i,m}(\mathbf{x}(t); t)}^{\text{indicator of mode } m \text{ selection}} \quad . \tag{11}$$

$a_{i,m}(\mathbf{x}(t); t) = 1$ if load $i$ picks mode $m$ given incentives $\mathbf{x}(t)$

- Goal: maximize payoff $Y(\mathbf{x}(t); t)$
- Problem: we don't know how customers pick modes

# Probabilistic Model for Incentive Design Problem

- At best we have statistics → Maximize expected payoff
- Probability of load $i$ picking mode $m$:

$$P_{i,m}(\mathbf{x}(t);t) = \mathbb{E}\{a_{i,m}(\mathbf{x}(t);t)\}. \quad (12)$$

- Incentives posted publically - Individual customers not important
- Define the *mode selection average probability* across population:

$$P_m(\mathbf{x}(t);t) = \frac{\sum_{i\in\mathcal{P}(t)} P_{i,m}(\mathbf{x}(t);t)}{|\mathcal{P}(t)|} \quad (13)$$

$$\mathbf{p}(\mathbf{x}(t);t) = [P_0(\mathbf{x}(t);t),\ldots,P_M(\mathbf{x}(t);t)]^T \rightarrow \text{what we need} \quad (14)$$

- Maximize expected payoff across cluster population

$$\max_{\mathbf{x}(t)\succeq\mathbf{0}} \mathbb{E}\left\{\sum_{m\in\mathcal{M}}(U_m(t)-x_m(t))\sum_{i\in\mathcal{P}(t)} a_{i,m}(\mathbf{x}(t);t)\right\} =$$

$$\max_{\mathbf{x}(t)\succeq\mathbf{0}} \overbrace{(\mathbf{u}(t)-\mathbf{x}(t))^T}^{\text{known}} \overbrace{\mathbf{p}(\mathbf{x}(t);t)}^{\text{unknown}} \quad (15)$$

# Modeling the customer's decision

Approaches to model $\mathbf{p}(\mathbf{x}(t); t)$? (average probability that the aggregator posts $\mathbf{x}(t)$ and a customer picks each mode $m$)

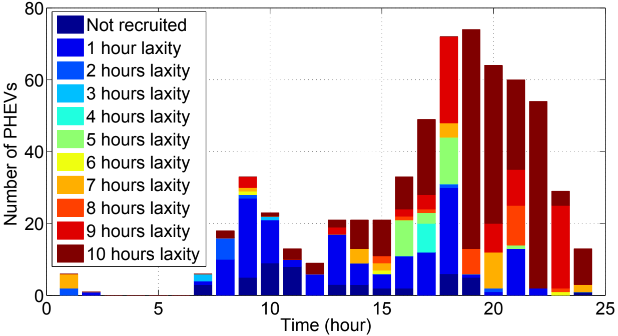
1. Bayesian model-based method: rational customer - $\max(V_i(t))$
   Risk-averseness captured by *types*

$$\text{customer utility} \quad V_i(t) = \sum_{v,q} x_m^{v,q}(t) - R_{i,m}^{q,v}(t)$$

   $R_{i,m}^{q,v}(t) = \gamma_i^{v,q} r_m^{v,q}(t)$, $\gamma_i$ random variable drawn from one PDF
2. Model-free learning method: customers may only be boundedly rational. We need to learn their response to prices

# How do we recruit? Residential charging...

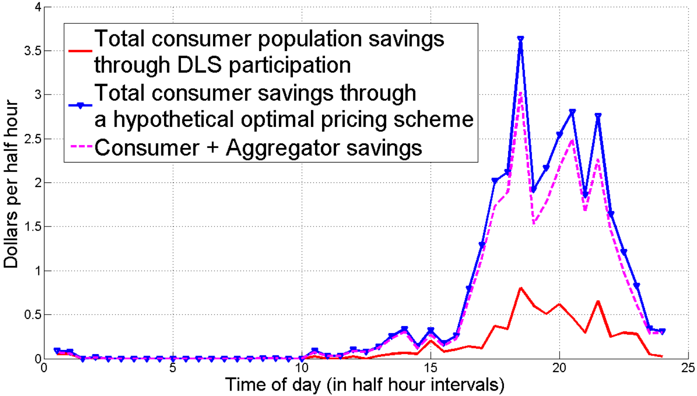
- Aggregator schedules 620 uninterruptible PHEV charging events
- Prices from New England ISO DA market - Maine load zone on Sept 1st 2013
- How many do we recruit (out of 620) and with what flexibility?




- More savings in the evening...

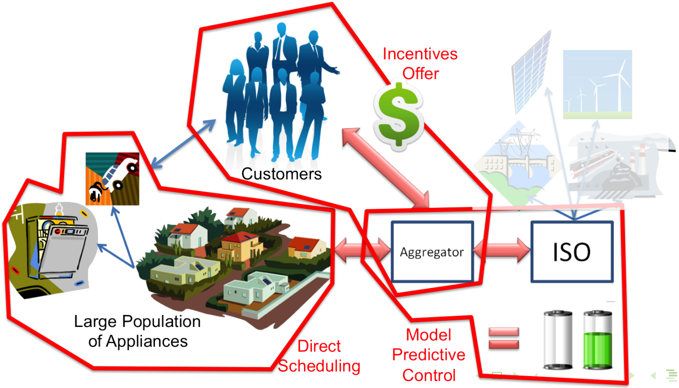
# Welfare Effects in Retail Market

- Welfare generate via Direct Load Scheduling (DLS) vs. idealized Dynamic Pricing (marginal price passed directly to customer - no aggregator)
- Savings summed up across the 620 events (shown as a function of time of plug-in)



# Conclusion

- We have discussed an information, decision, control and market models for responsive loads
- We left out how to sell renewables power as a result of this *See work on Risk Limiting Dispatch (RLD) [Varaiya, Wu, Bialek,2011],[He, Murugesan, Zhang 2011], [Rajagopal, Bitar, Varaiya, Wu, 2013],...*
- How much risk can one hedge in generation with load flexibility?...many questions left



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