One-Bit Quantization in Massive MIMO Systems

Lee Swindlehurst

Center for Pervasive Communications and Computing

University of California Irvine

Hans Fischer Senior Fellow, Institute for Advanced Study

Technical University of Munich

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Collaborators:

Yongzhi Li, Amodh Saxena, Amine Mezghani (UCI)

Fabian Steiner, Hela Jedda, Josef Nossek, Wolfgang Utschick (TUM)

Inbar Fijalkow (Univ. Cergy-Pontoise, France)

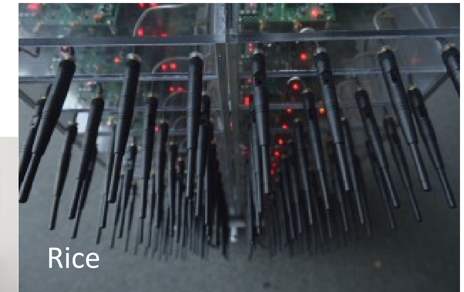
Gonzalo Seco Granados (UAB, Spain)

Outline

- **Background and Motivation**
- **Massive MIMO Uplink with One-Bit ADCs**
  - Model
  - Channel estimation
  - Bussgang decomposition
  - Optimized training
  - Achievable rate analysis
  - Energy efficiency
  - How many more antennas are needed?
- **Massive MIMO Downlink with One-Bit DACs**
  - Model
  - ML Encoding
  - Bussgang analysis
  - Quantized precoders
- **Conclusions**

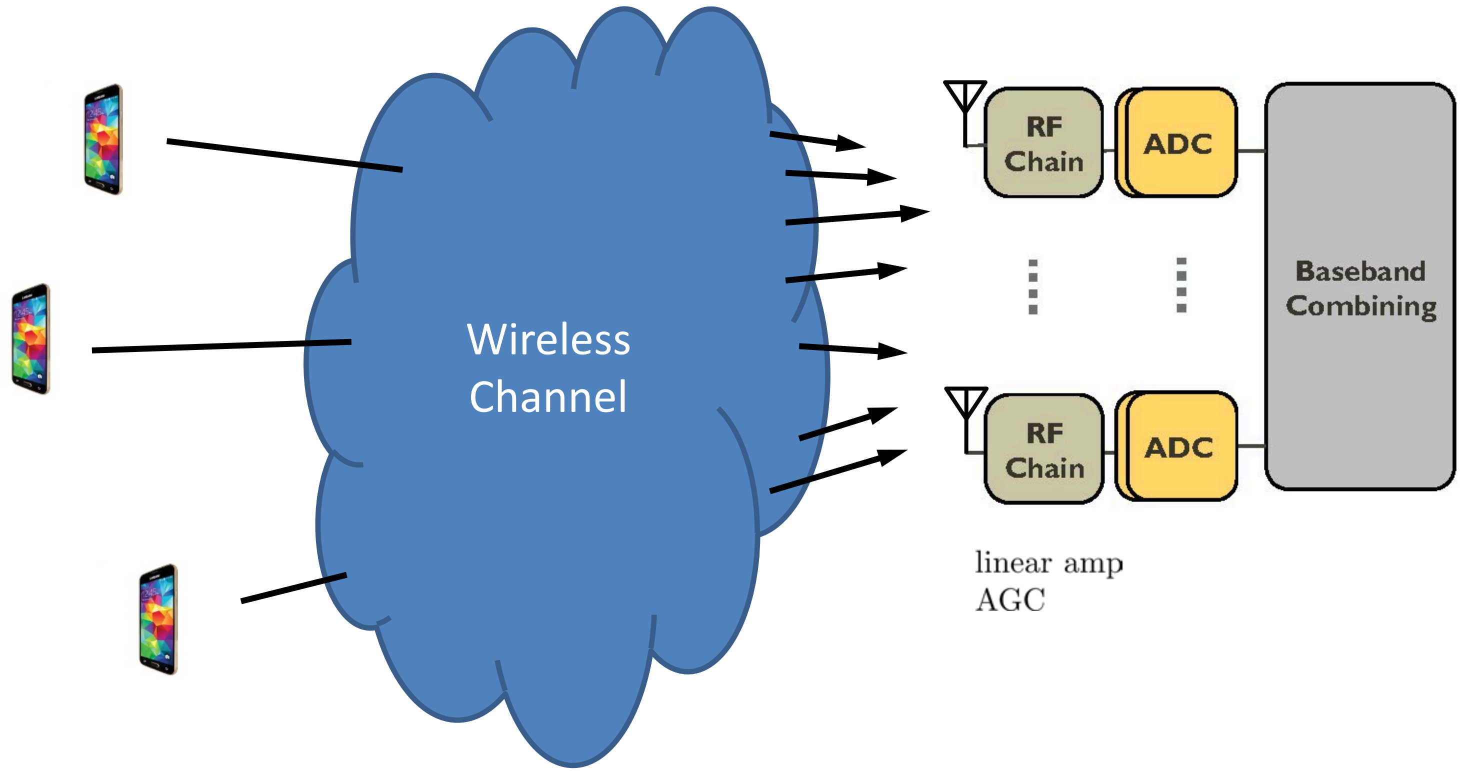
The Road to Gigabit Wireless

- How do we get to Gb/s wireless links?



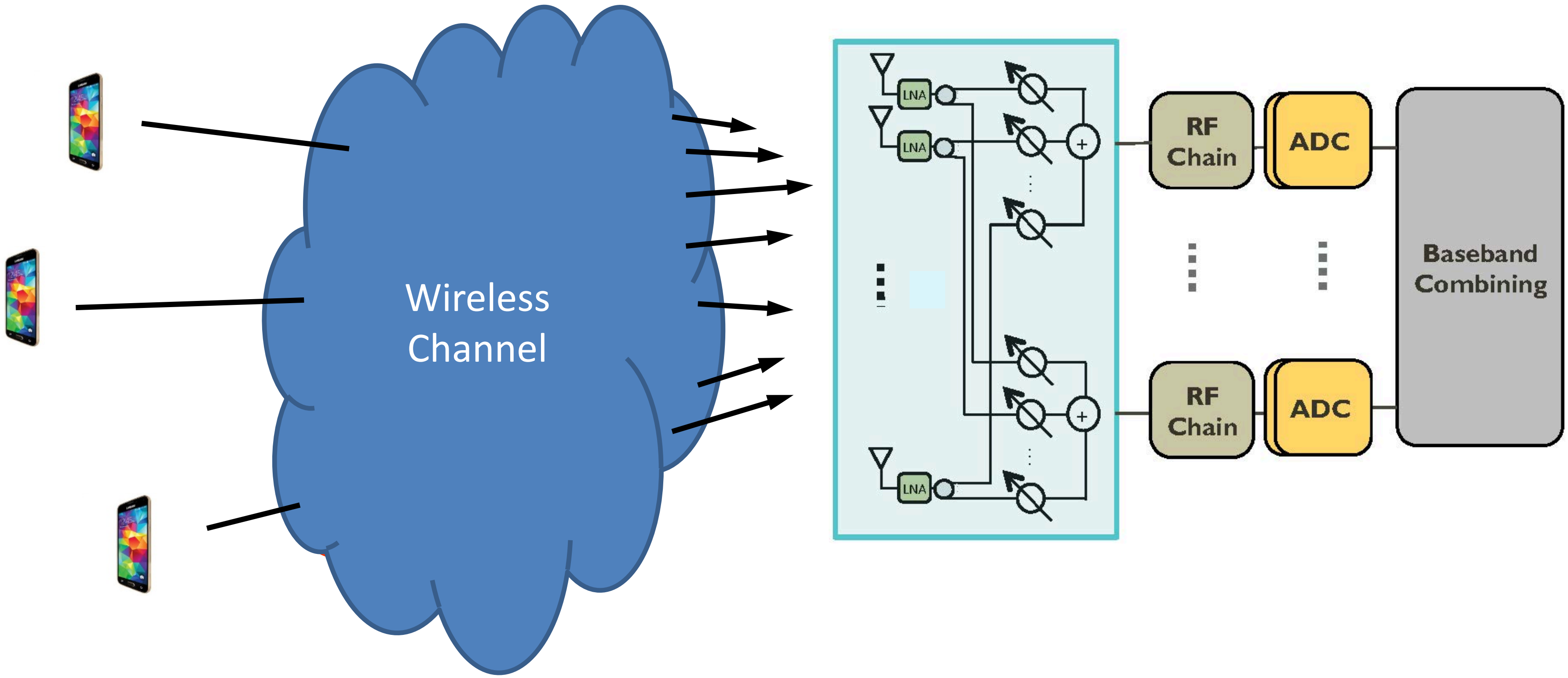
- Incremental gains from
  - standard MIMO
  - cooperative comm
  - cognitive radios
- What are the next steps?
- Three symbiotic trends emerging:
  - Deployment of pico- and femto-cells (OoM decrease in cell size)
  - Millimeter wave frequencies (OoM increase in bandwidth)
  - Massive MIMO (OoM increase in antennas)

mmWave Ma$$ive MIMO



- Full precision ADC requires linear, low-noise amplifiers and AGC
- ADC power consumption grows exponentially with sampling rate
- A commercial TI 1 Gs/s 12-bit ADC requires 4W
- Not practical for ideal massive MIMO

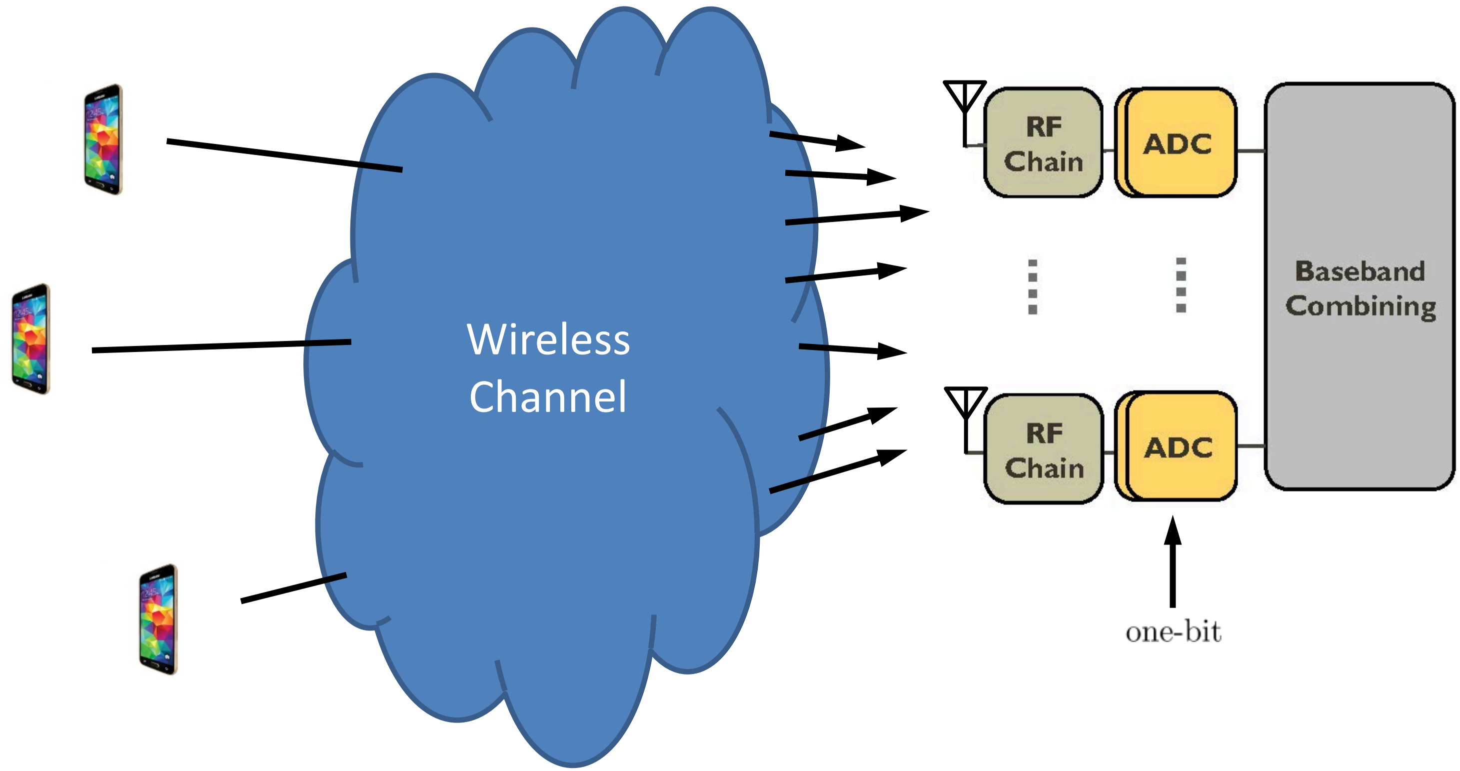
Hybrid Analog-Digital Beamforming



- Reduce dimensionality with RF beamforming network
- Complicates receiver design, scalability issues for wideband operation
- Phase shifters are typically quantized
- Power/cost still an issue

*graphic courtesy of Robert Heath

Alternative: Low-Resolution (1-bit) ADC



- One-bit ADC $\Rightarrow$ simple RF, no AGC or high cost LNA
- Operates at a fraction of the power
- Low SNR loss (typical operating point for mmWave massive MIMO) only 2dB
- Compensate for quantization error with signal processing

Signal Processing Issues for One-Bit Quantization

- **Channel Estimation**
  - training-based methods
  - channel models? Rayleigh, sparse, DOA-based, etc.
  - price of ignoring 1-bit ADCs?
- **Uplink Decoding**
  - joint decoding & channel estimation
  - high quantization noise $\Rightarrow$ less dense constellations
  - high SNR error floor $\Rightarrow$ gains from power control
- **Downlink Precoding**
  - 1-bit DAC $\Rightarrow$ finite alphabet, non-linear precoding
  - ML encoder too expensive and over constrains problem
  - antenna selection?
- **Information Theoretic Analyses**
  - what spectral efficiencies are achievable?
  - how many more antennas do we need?
  - exploit Bussgang decomposition

Single Antenna Analysis – Mezghani & Nossek

<u>AWGN Channel Capacity</u>

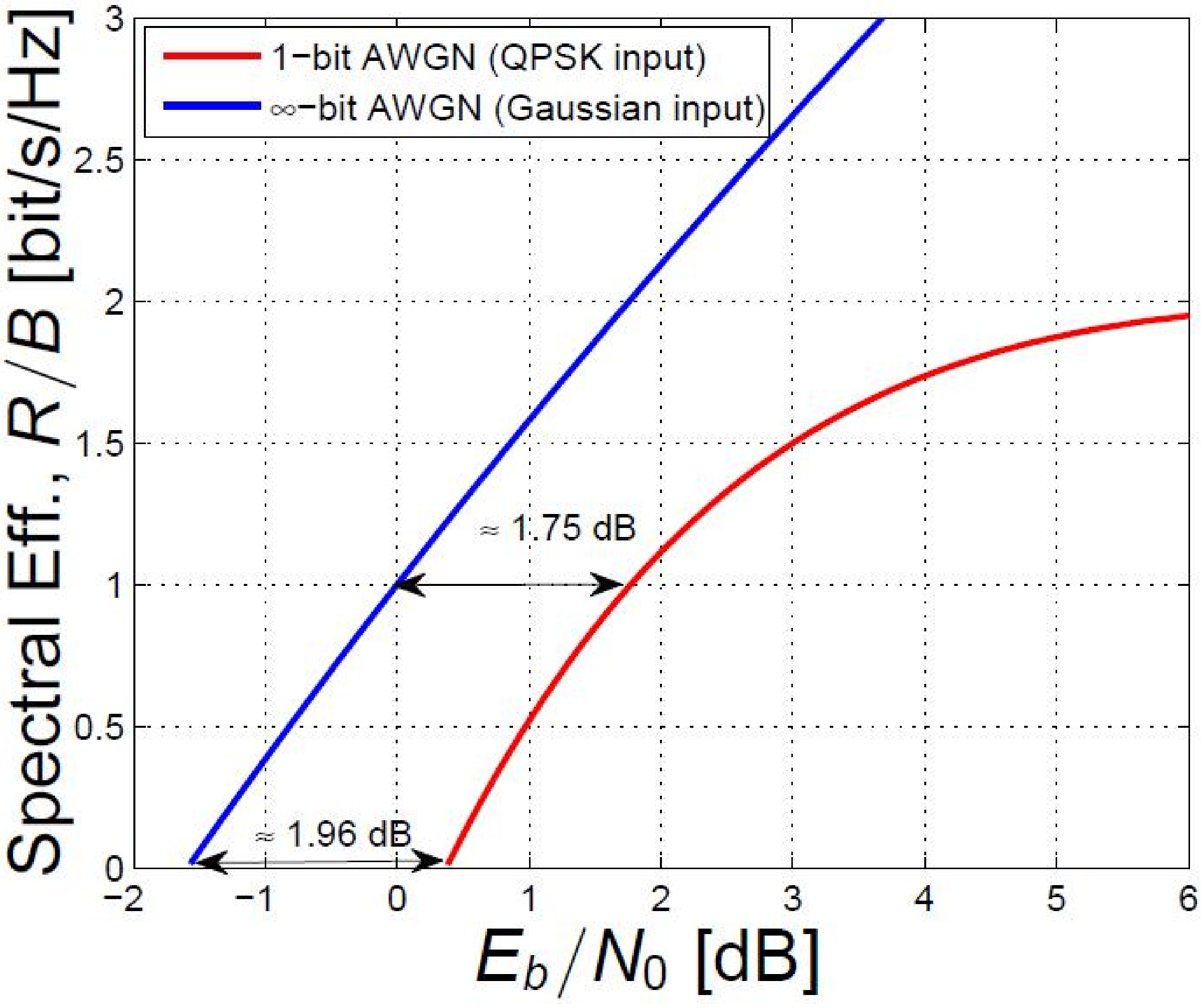
$$B\log_2(1+\mathrm{SNR})$$

<u>1-Bit AWGN Capacity</u>

$$2B\left(1 - H_b(\Phi(\sqrt{\mathrm{SNR}}))\right)$$

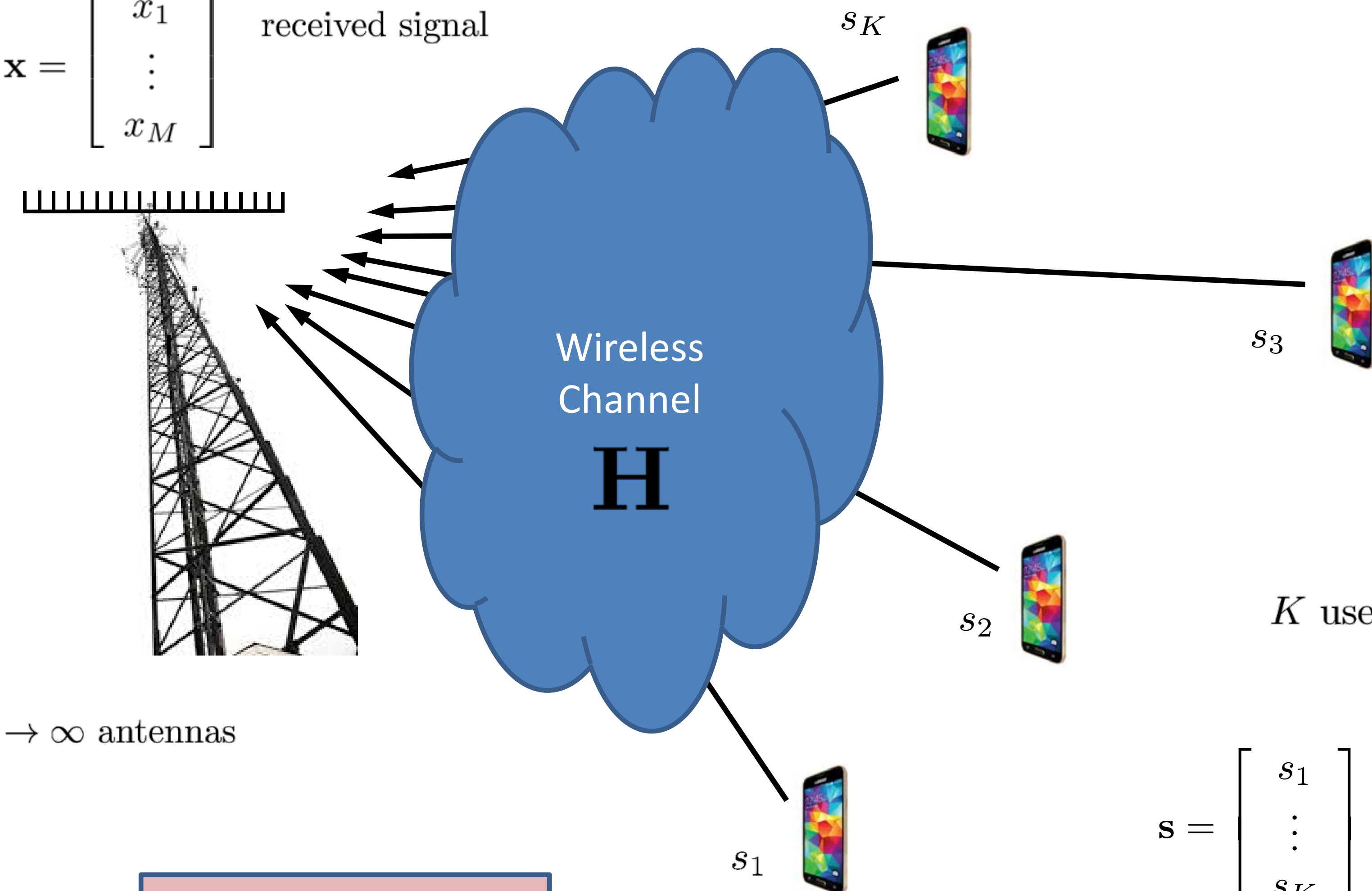
loss in power
efficiency < 2dB
when SE < 1.4 bpcu

trade-off between power
and energy efficiency is
less apparent for 1-bit
systems



Massive MIMO Uplink

$$\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix}$$ received signal



$M \to \infty$ antennas

K users

$$\mathbf{s} = \begin{bmatrix} s_1 \\ \vdots \\ s_K \end{bmatrix}$$

transmitted symbols

$$\mathbf{x} = \sqrt{\rho}\mathbf{H}\mathbf{s} + \mathbf{n}$$

Optimal Signal Input Distribution (Mezghani etal)

<u>Theorem</u>: Ergodic capacity of one-bit quantized i.i.d. MIMO channel with $\mathbf{H}_{ij} \sim \mathcal{CN}(0,1)$ is achieved asymptotically at low SNR by QPSK signals:

$$C_{1-bit}^{erg} \simeq \frac{2}{\pi} M \cdot \mathrm{SNR} - \frac{M(M+(\pi-1)K-1)}{2K} \left(\frac{2}{\pi}\mathrm{SNR}\right)^2$$

Unquantized channel with QPSK signals achieves

$$C^{erg} \simeq M \cdot \mathrm{SNR} - \frac{M(M+K)}{2K} (\mathrm{SNR})^2$$

Channel Estimation with One-Bit ADC

Use $K \times \tau$ uplink training data $\mathbf{\Phi}$

$$\mathbf{X} = \sqrt{\rho}\mathbf{H}\mathbf{\Phi} + \mathbf{N}$$

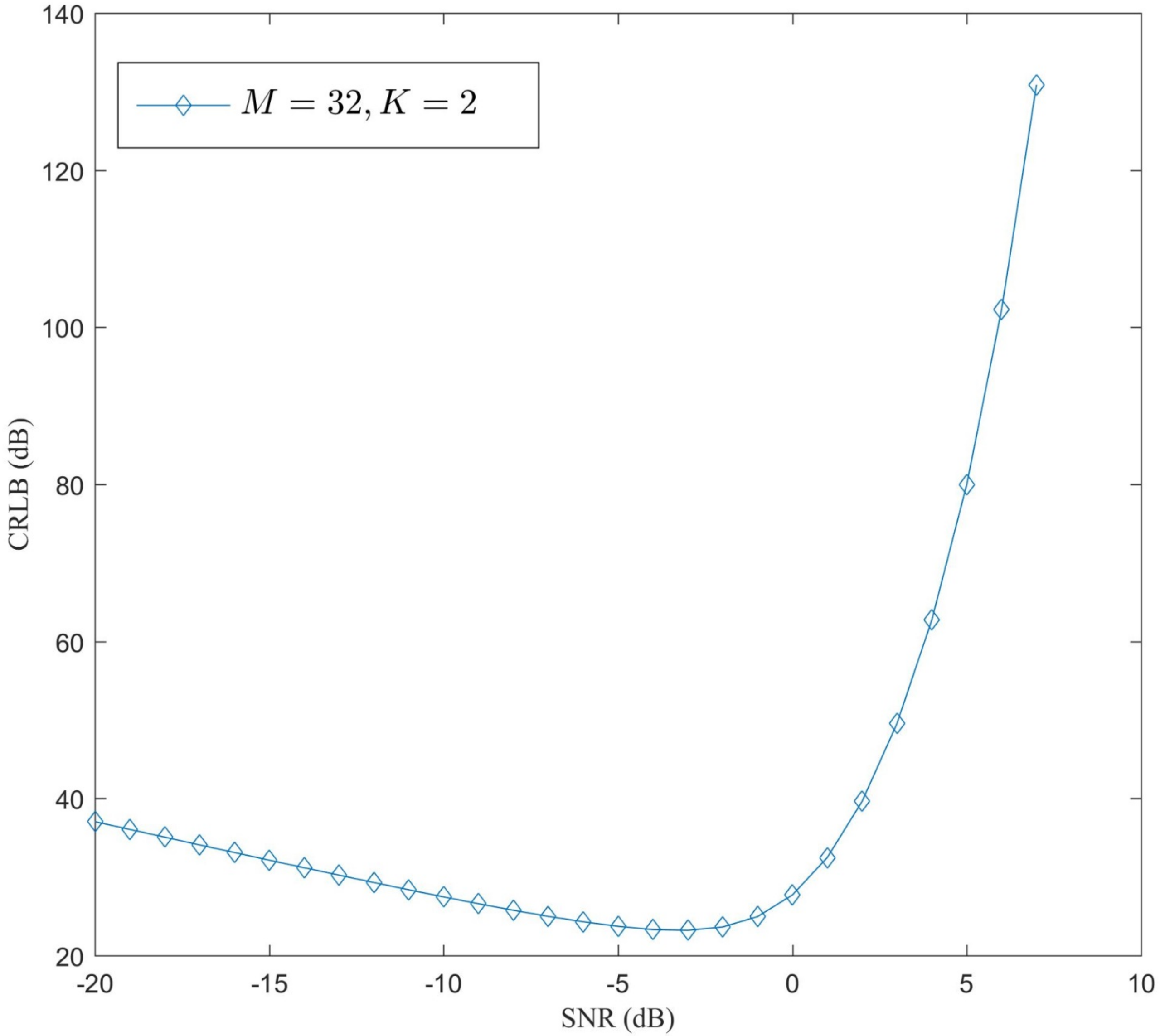
Vectorized model

$$\begin{aligned}\mathbf{x} = \mathrm{vec}(\mathbf{X}) &= \sqrt{\rho}\left(\mathbf{\Phi}^T \otimes \mathbf{I}\right)\mathrm{vec}(\mathbf{H}) + \mathrm{vec}(\mathbf{N}) \\ &= \tilde{\mathbf{\Phi}}\mathbf{h} + \mathbf{n}\end{aligned}$$

1-bit quantization $\mathcal{Q}(\cdot)$ maps complex data to $\pm 1 \pm j$

$$\mathbf{r} = \mathcal{Q}(\mathbf{x}) = \mathcal{Q}\left(\tilde{\mathbf{\Phi}}\mathbf{h} + \mathbf{n}\right)$$

The CRB: Do We Really Want an Unbiased Estimator?



The Bussgang Theorem

Let $x(t)$ be a Gaussian random process, and $r(t) = \mathcal{Q}(x(t))$ be the output of some nonlinear function. Then for a certain constant a, we have

$$r_{xx}(\tau) = a r_{xr}(\tau)$$

for the auto-correlation and cross-correlation functions $r_{xx}(\tau)$ and $r_{xr}(\tau)$, respectively.

Bussgang Theorem: Implications for Channel Estimation

Represent nonlinear quantization by "equivalent" linear operator:

$$\begin{aligned}\mathbf{r} &= \mathcal{Q}(\mathbf{x}) = \mathcal{Q}\left(\tilde{\mathbf{\Phi}}\mathbf{h} + \mathbf{n}\right) \\ &= \mathbf{A}\mathbf{x} + \mathbf{q}\end{aligned}$$

where

$$\mathbf{A}\mathbf{C}_{xx} = \mathbf{C}_{xr}^{H}$$

Under this model $\mathbf{x}$ and $\mathbf{q}$ are uncorrelated, and $\mathbf{A}$ minimizes the equivalent quantization noise:

$$\mathbf{A} = \arg\min_{\mathbf{A}} \|\mathbf{r} - \mathbf{A}\mathbf{x}\|^2$$

Linear model simplifies algorithm design and analysis

Bussgang Channel Estimator

With $\mathbf{r} = \mathbf{A}\mathbf{x} + \mathbf{q}$ and $\mathbf{A} = \mathbf{C}_{xr}^{H}\mathbf{C}_{xx}^{-1}$

$$\mathbf{C}_{xr} = \sqrt{\frac{2}{\pi}}\mathbf{C}_{xx}\mathrm{diag}(\mathbf{C}_{xx})^{-\frac{1}{2}}$$

$$\mathbf{A} = \sqrt{\frac{2}{\pi}}\mathrm{diag}((\mathbf{\Phi}^{T}\mathbf{\Phi}^{*} \otimes \rho\mathbf{I}) + \mathbf{I})^{-\frac{1}{2}}$$

Channel estimates:

$$\hat{\mathbf{h}}^{\mathtt{BLS}} = \left(\tilde{\mathbf{\Phi}}^{H}\tilde{\mathbf{\Phi}}\right)^{-1}\tilde{\mathbf{\Phi}}^{H}\mathbf{r}$$

$$\hat{\mathbf{h}}^{\mathtt{BLM}} = \mathbf{C}_{hr}\mathbf{C}_{rr}^{-1}\mathbf{r}_{p} = \left(\tilde{\mathbf{\Phi}}^{H} + \mathbf{C}_{hq}\right)\mathbf{C}_{rr}^{-1}\mathbf{r}$$

Bussgang Channel Estimator Performance

Assume special case of $\tau = K$ and $\mathbf{\Phi}\mathbf{\Phi}^H = \tau\mathbf{I}$

$$\mathrm{MSE}^{\texttt{BLS}} = \frac{\pi(1+\rho K)}{2\rho K} - 1 \quad \underset{\rho\to\infty}{\longrightarrow} \quad \frac{\pi}{2} - 1 \qquad (-2.43\mathrm{dB})$$

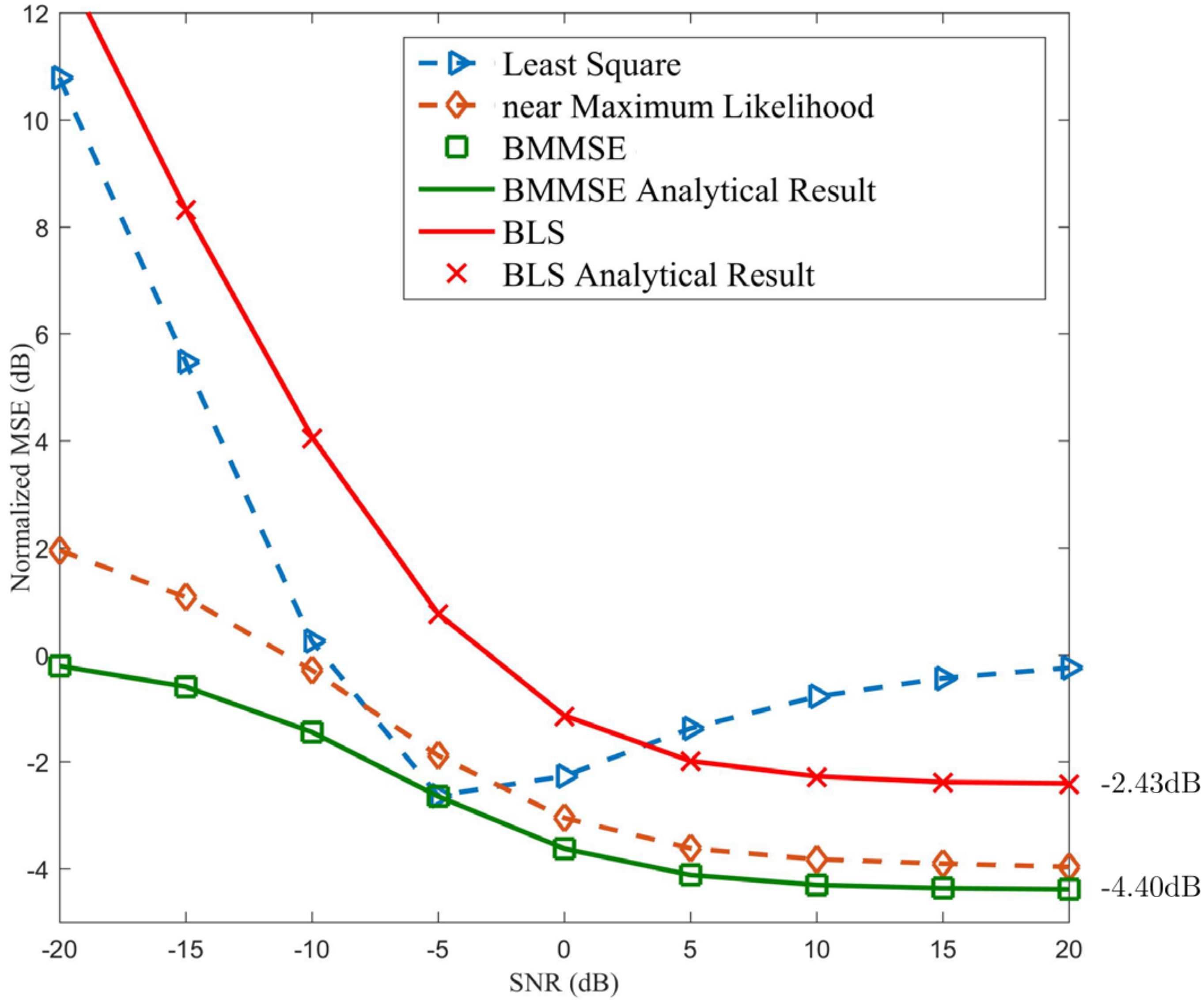
$$\mathrm{MSE}^{\texttt{BLM}} = 1 - \frac{2\rho K}{\pi(1+\rho K)} \quad \underset{\rho\to\infty}{\longrightarrow} \quad 1 - \frac{2}{\pi} \qquad (-4.40\mathrm{dB})$$

What if $\tau > K$? What is trade-off in MSE vs. throughput?

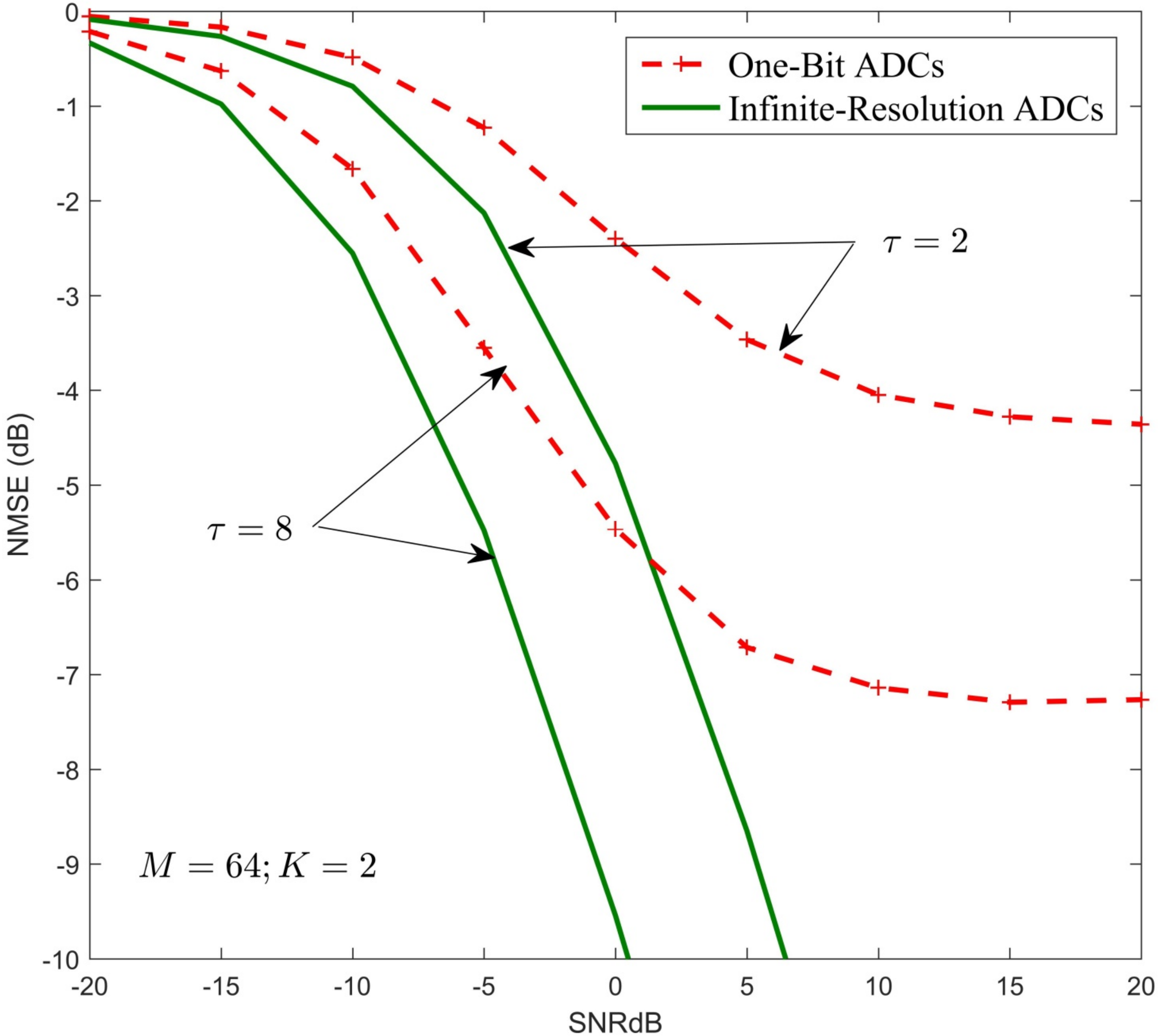
Look at MSE first, save spectral efficiency for later ...

Channel Estimation Simulation Results

- $M = 128$
- $K = 8$
- $\tau = K$
- Rayleigh fading



Channel Estimation Simulation Results (cont.)

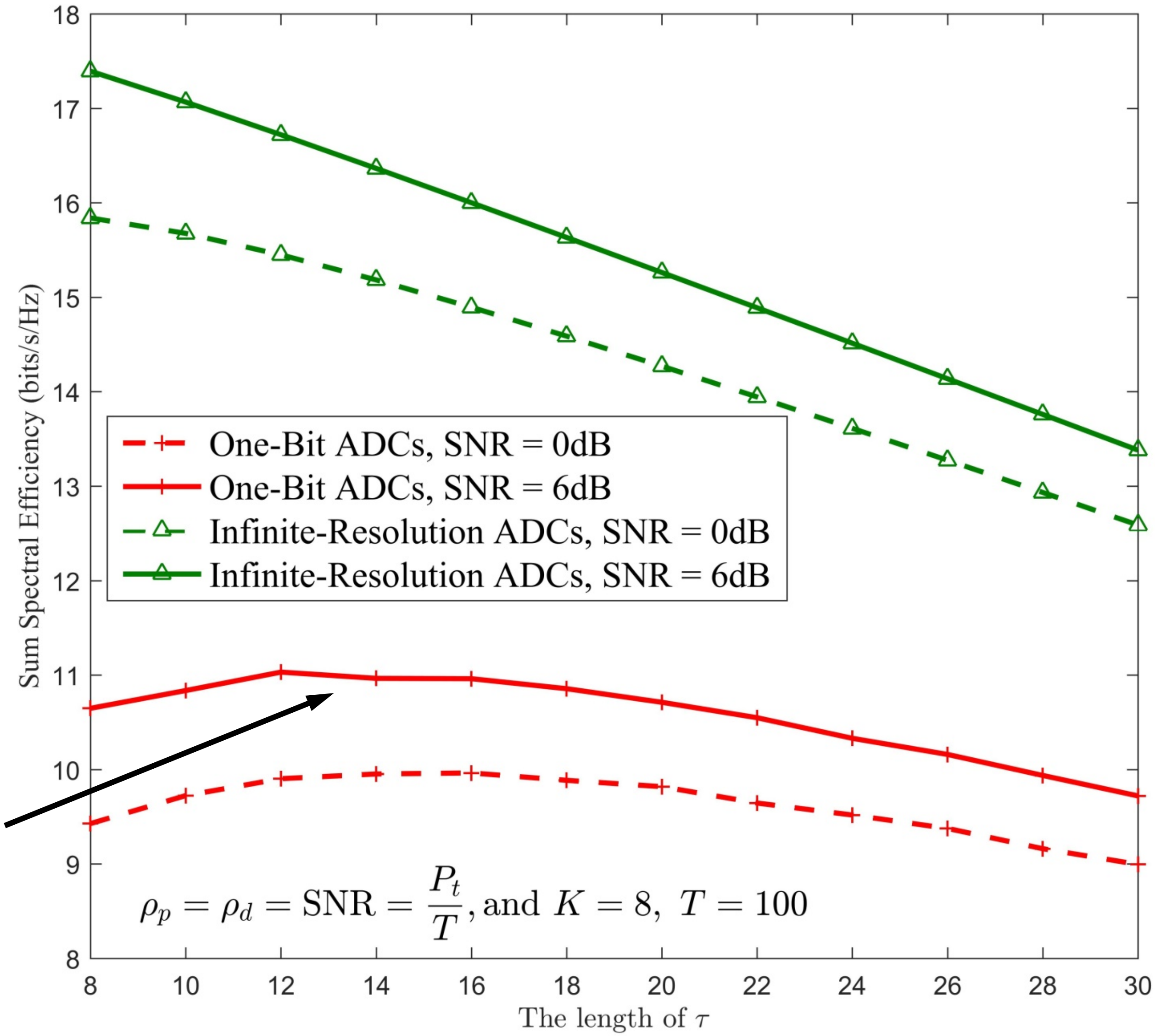


Example: Optimum Training Interval

- $M = 32$
- coherence interval $T = 100$
- sum spec. eff.

$$\mathcal{S} = \frac{T-K}{T} KR$$

To find optimal τ and analyze trade-off with throughput, we need analysis of spectral efficiency



Analysis of Achievable Rate

- Assume special case of $\tau = K$ and $\mathbf{\Phi}\mathbf{\Phi}^H = \tau \mathbf{I}$
- Allows derivation of lower bound on rate assuming quantization noise is Gaussian
- Allows for possibly different SNR for training ρ_p and data ρ_d
- Quantifies effect of using channel estimates to form MRC and ZF receivers
- Provides simple closed-form expressions

<u>MRC Receiver</u>:
$$R_{\mathtt{MRC}} \geq \log_2 \left(1 + \frac{\rho_d \alpha_d^2 \alpha_p^2 \rho_p K M}{\rho_d \alpha_d^2 K + \alpha_d^2 + (1 - 2/\pi)} \right)$$

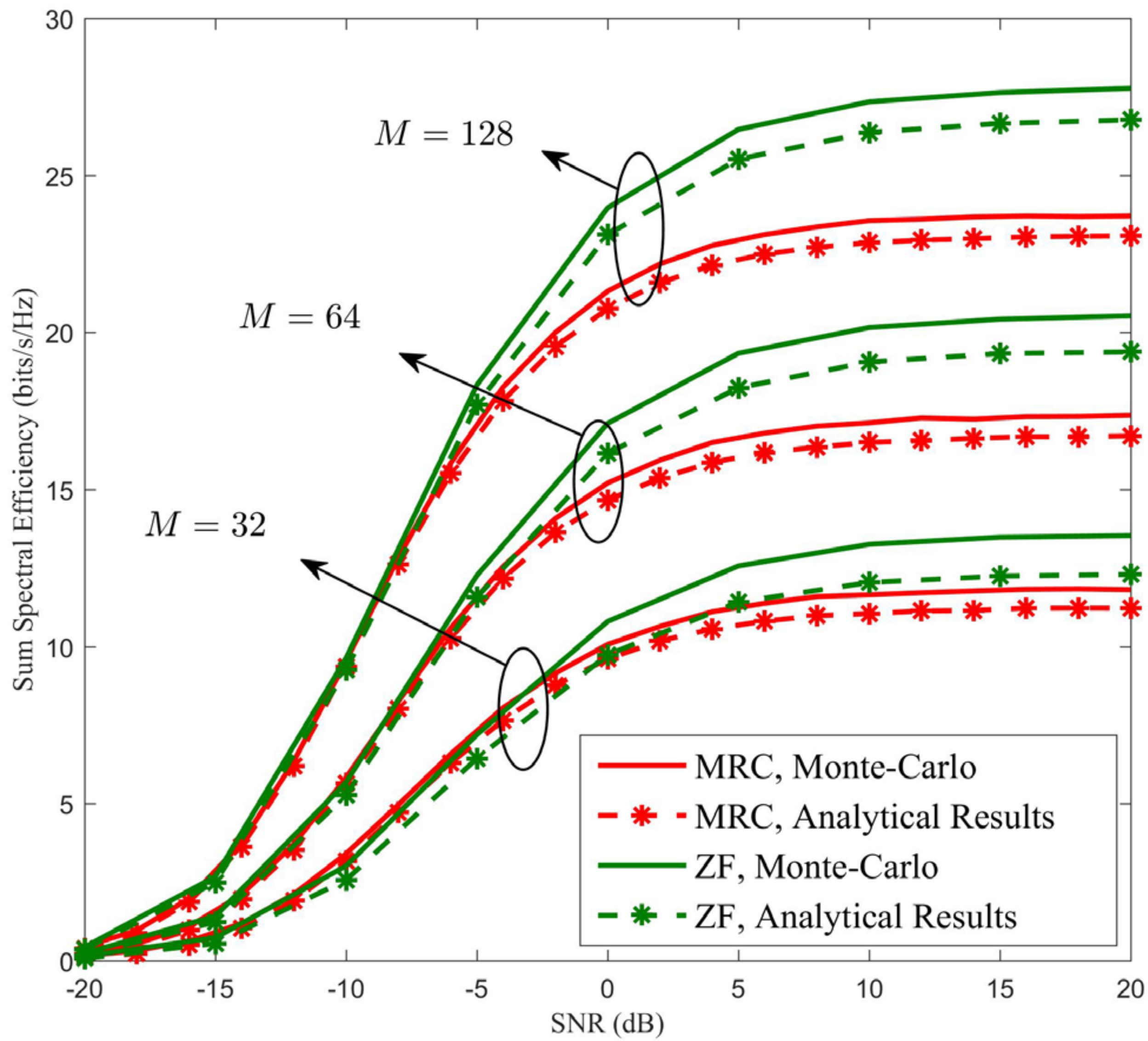
<u>ZF Receiver</u>:
$$R_{\mathtt{ZF}} \geq \log_2 \left(1 + \frac{\rho_d \alpha_d^2 \alpha_p^2 \rho_p K (M - K)}{\rho_d \alpha_d^2 K \eta + \alpha_d^2 + (1 - 2/\pi)} \right)$$

$$\alpha_i = \sqrt{\frac{2}{\pi(1 + \rho_i K)}} \qquad \eta = 1 - \alpha_p^2 \rho_p K$$

Example: Sum Spectral Efficiency

- $K = 8$ users
- equal power
- coherence interval $T = 200$
- sum spec. eff.

$$\mathcal{S} = \frac{T-K}{T} KR$$



How Many More Antennas Are Needed with One-Bit ADCs?

	Typical Massive MIMO	One-Bit Massive MIMO
MRC	$C\left(\frac{\rho^2 K M_{typ}}{K\rho(K\rho+1)+K\rho+1}\right)$	$C\left(\frac{\rho^2\alpha^4 K M_{one}}{\rho\alpha^2 K+\alpha^2+1-2/\pi}\right)$
ZF	$C\left(\frac{\rho^2 K(M_{typ}-K)}{2K\rho+1}\right)$	$C\left(\frac{\rho^2\alpha^4 K(M_{one}-K)}{\rho\alpha^2 K\eta+\alpha^2+1-2/\pi}\right)$

$$C(x) = \frac{T-K}{T} K \log_2(1+x) \qquad \rho_d = \rho_p = \rho \qquad \alpha = \sqrt{\frac{2}{\pi(1+\rho K)}}$$

- $C_{typ} = C_{one}$ for MRC when

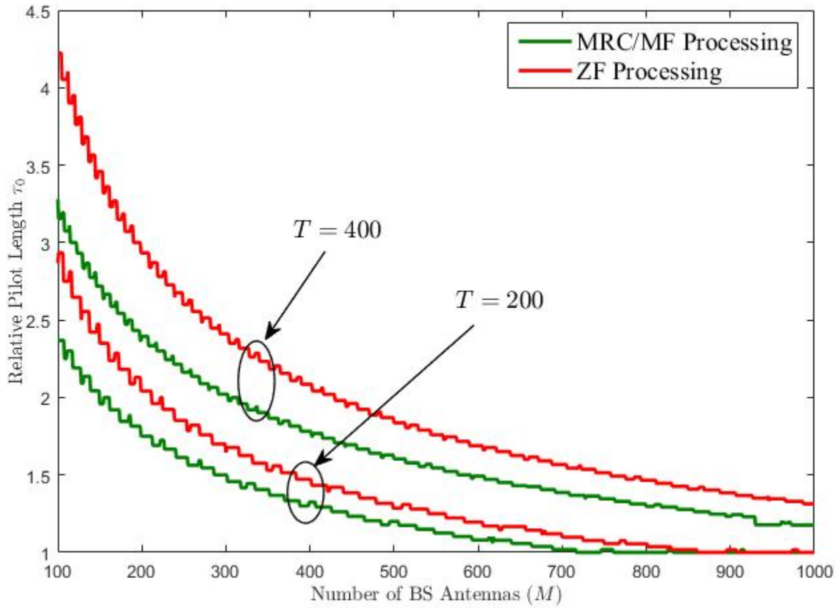
$$\frac{M_{one}}{M_{typ}} = \frac{\pi^2}{4} \simeq 2.5$$

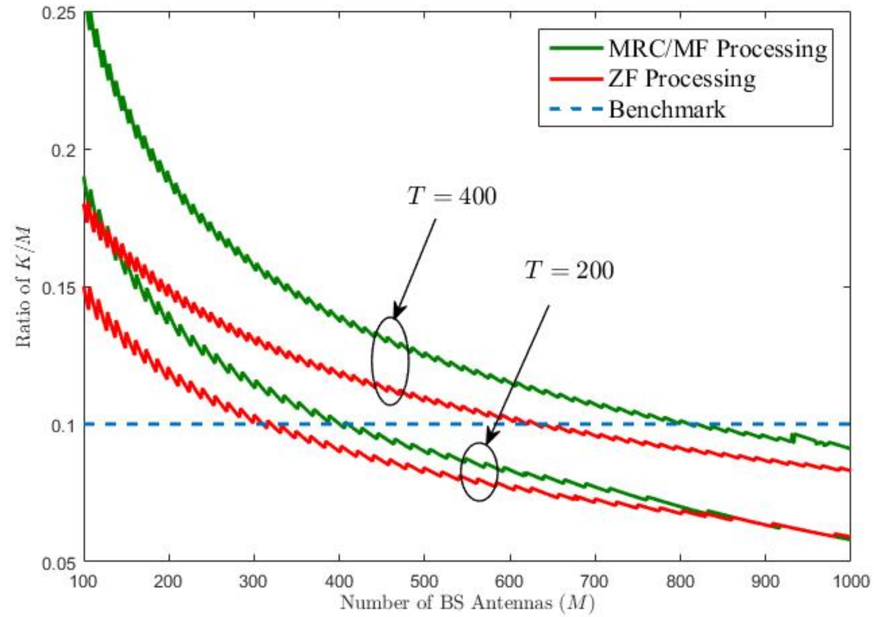
- $C_{typ} = C_{one}$ for ZF when

$$\frac{M_{one}}{M_{typ}} = \frac{\pi^2(1+\rho K)^2 - 4\rho^2 K^2}{4+8\rho K}$$

$$\longrightarrow \frac{\pi^2}{4} \qquad \rho \to 0$$

Example: Joint Optimization of # of Users and Training Interval

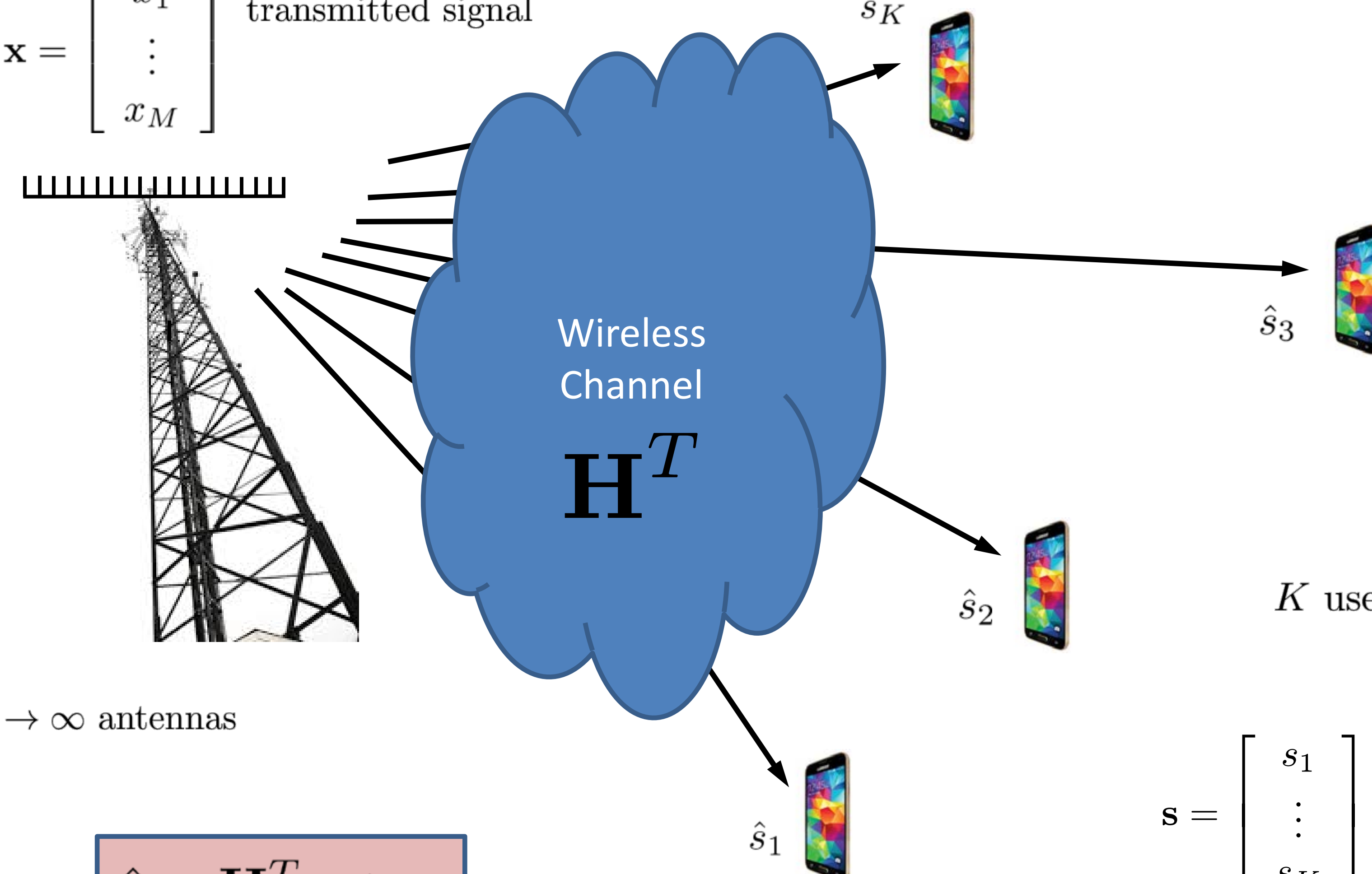




- Assumes operating point chosen to achieve SINR duality for uplink and downlink
- Transmit power optimized as well, and decreases with M
- Optimization goal is to minimize weighted product of spectral and energy efficiency
- One-bit quantization benefits from additional training beyond minimum
- But optimal user loading decreases with M for fixed T

Massive MIMO Downlink

$$\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix}$$ transmitted signal



$M \to \infty$ antennas

$$\hat{\mathbf{s}} = \mathbf{H}^T \mathbf{x} + \mathbf{n}$$

symbols desired at users

A “Natural” Approach: ML Encoding

Since $\mathbf{x}$ is constrained to QPSK alphabet due to 1-bit quantization, suggests ML encoding:

$$\mathbf{x} = \arg \min_{x_i \in \pm 1 \pm j} \|\mathbf{s} - \mathbf{H}^T \mathbf{x}\|$$

- Probhibitively complex, especially for a massive antenna array ($\mathbf{H}^T$ is $K \times M$)
- Requires special “handling” since $\mathbf{H}^T$ is a fat matrix
- Even a sphere encoding approach is too costly
- We will see the ML encoding is outperformed by something much simpler ...

Quantized Linear Precoding

Output of linear precoder $\mathbf{x}_P = \mathbf{P}\mathbf{s}$ is 1-bit quantized prior to transmission:

$$\mathbf{x} = \frac{1}{\sqrt{M}} \mathcal{Q}(\mathbf{x}_P)$$

$$\hat{\mathbf{s}} = \frac{1}{\sqrt{M}} \mathbf{H}^T \mathcal{Q}(\mathbf{P}\mathbf{s}) + \mathbf{n}$$

Example: Quantized MRT precoder with BLMMSE channel estimate:

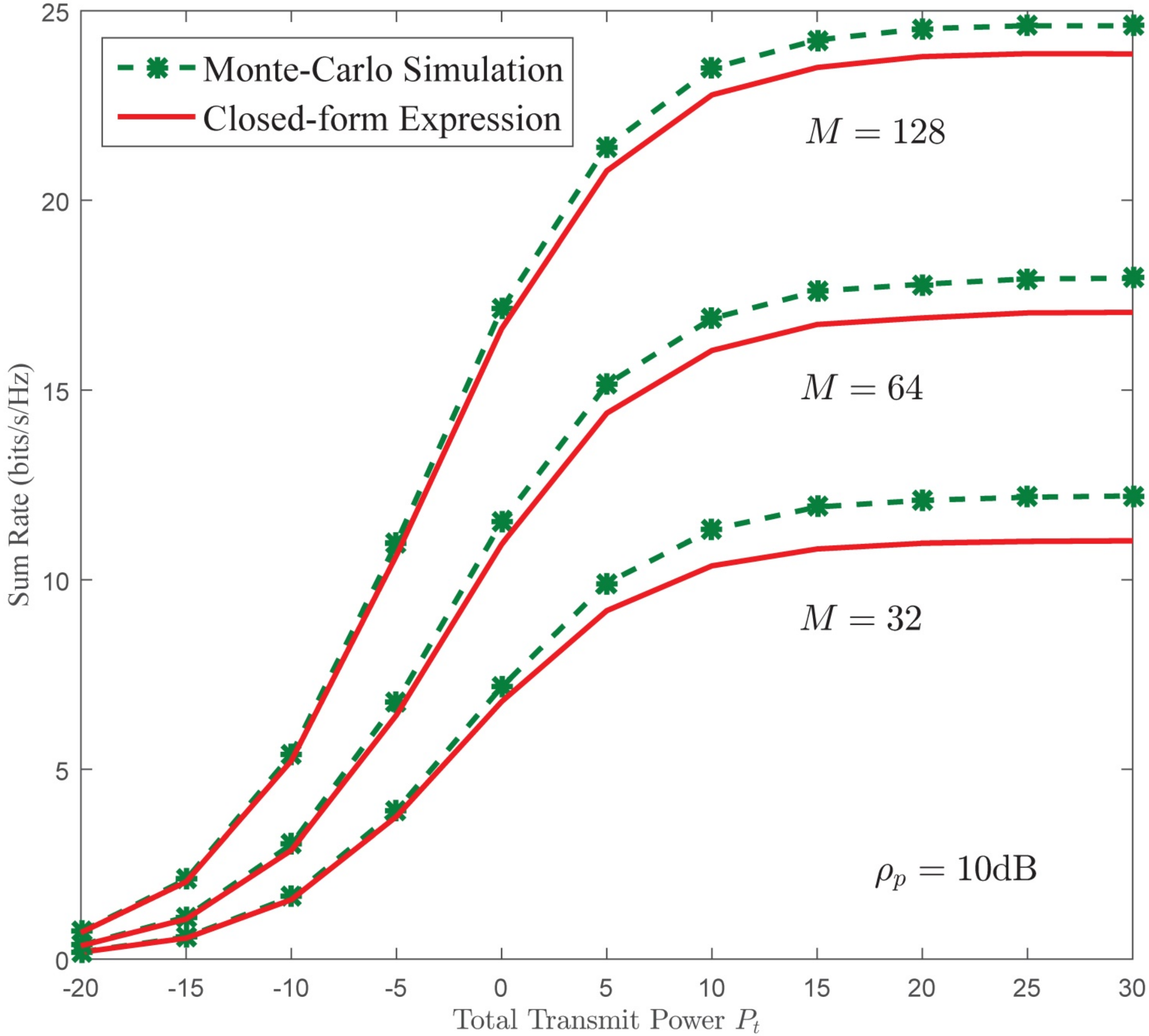
$$\hat{\mathbf{s}} = \sqrt{\frac{P_t}{M}} \mathbf{H}^T \mathbf{x} + \mathbf{n} = \sqrt{\frac{P_t}{M}} \mathbf{H}^T \left(\mathbf{A}\hat{\mathbf{H}}^* \mathbf{s} + \mathbf{q} \right) + \mathbf{n}$$

For BLMMSE channel estimate with $\tau = K$ and training SNR ρ_p, downlink rate for user k is lower bounded by

$$R_k \geq \log_2 \left(1 + \frac{4\rho_p M P_t}{\pi^2 (1 + \rho_p K)(1 + P_t)} \right)$$

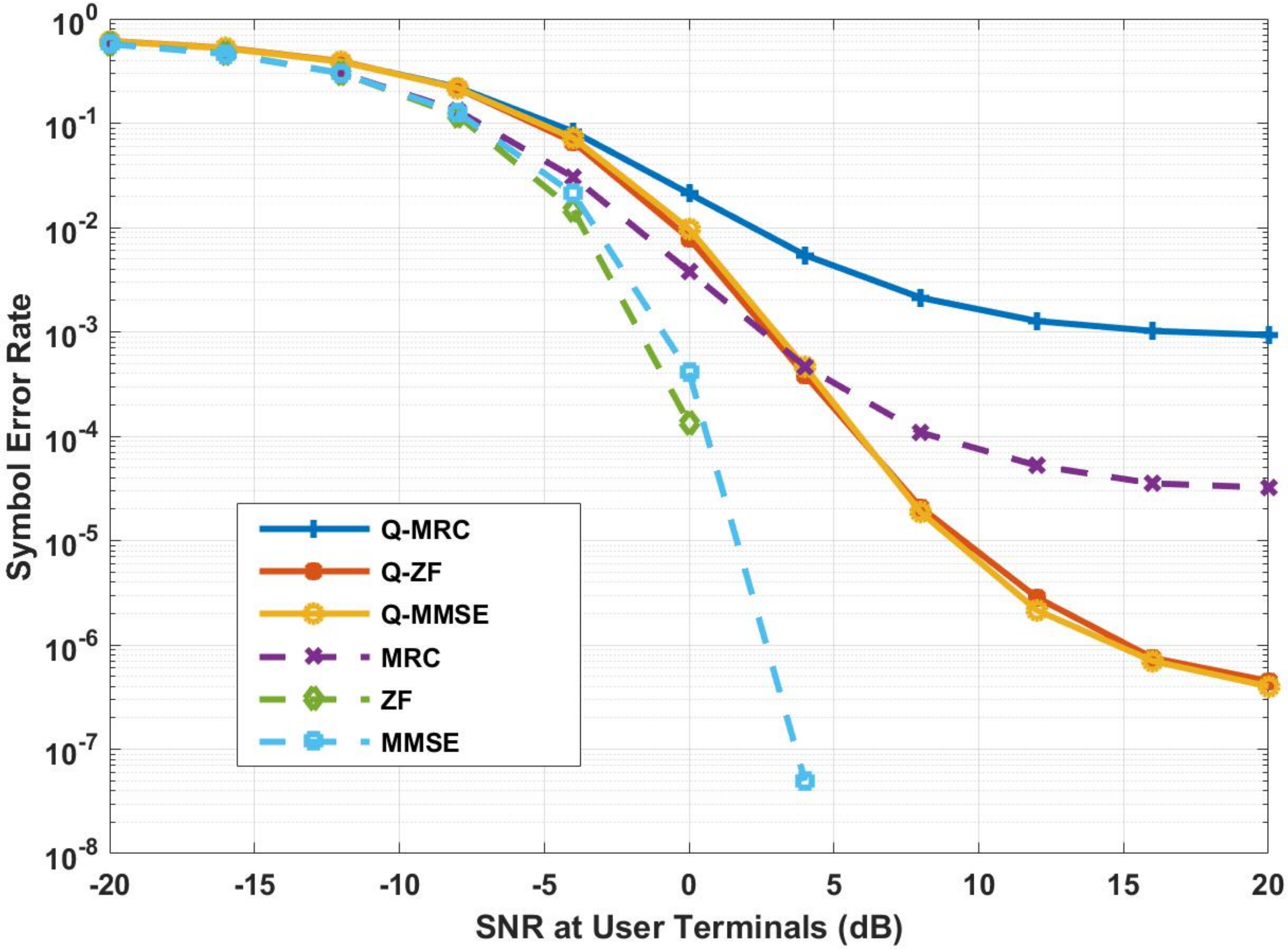
Sum Downlink Rate for Estimated MRT Precoder

- $K = 8$ users
- Rayleigh fading
- BLMMSE channel estimate
- MRT precoder



Comparison with Unquantized Case

- $M = 128$
- $K = 8$ users
- Rayleigh fading
- Perfect CSI



Special Case: Quantized ZF Precoder

Assume $\mathbf{H}^T = \sigma\tilde{\mathbf{H}}^T$, where elements of $\tilde{\mathbf{H}}^T$ are iid $\mathcal{CN}(0,1)$
Assume $M \gg K \gg 1$, for asymptotic SER:

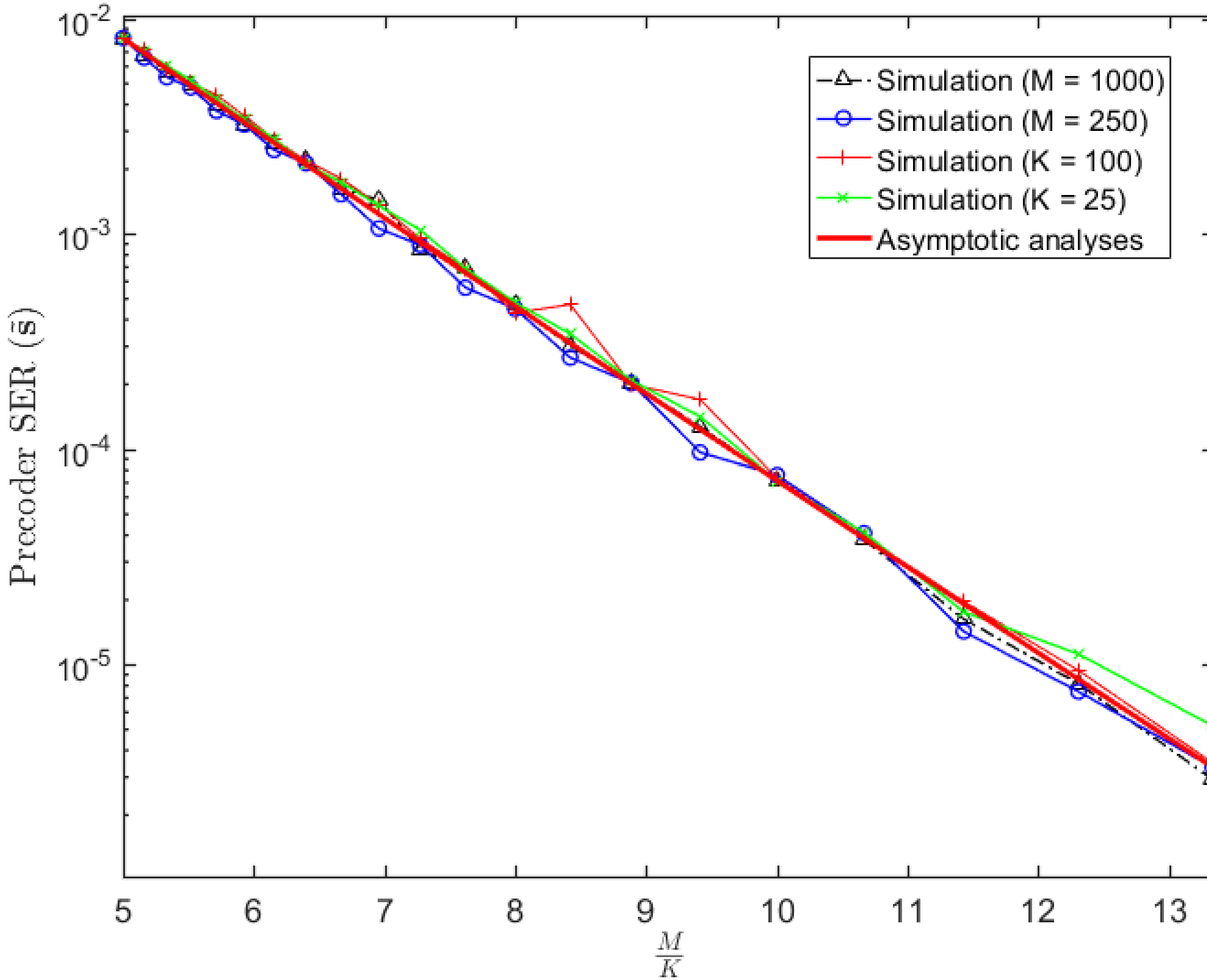
$$P_e = 2Q\left(\sqrt{\frac{\frac{4\sigma^2(M-K)^2}{MK\pi}}{\frac{2\sigma^2}{M}\left(1-\frac{2}{\pi}\right)(M-K)+\sigma_n^2}}\right)$$

High SNR error floor:

$$P_e \longrightarrow 2Q\left(\sqrt{\frac{\frac{2}{\pi}}{1-\frac{2}{\pi}}\left(\frac{M}{K}-1\right)}\right)$$

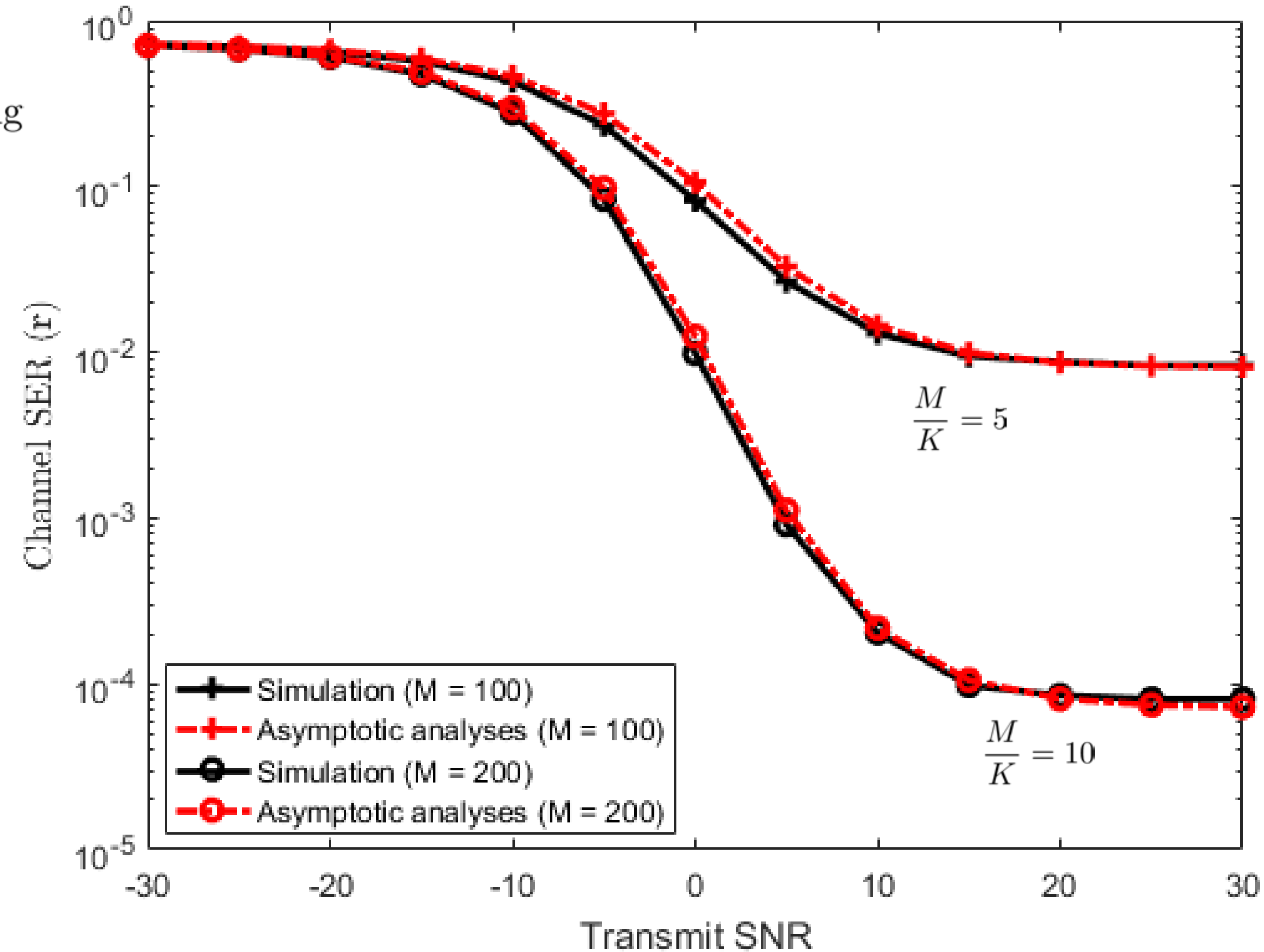
Noiseless Case, Impact of Quantization Errors Only

- Rayleigh fading
- $\sigma = 1$
- noiseless case



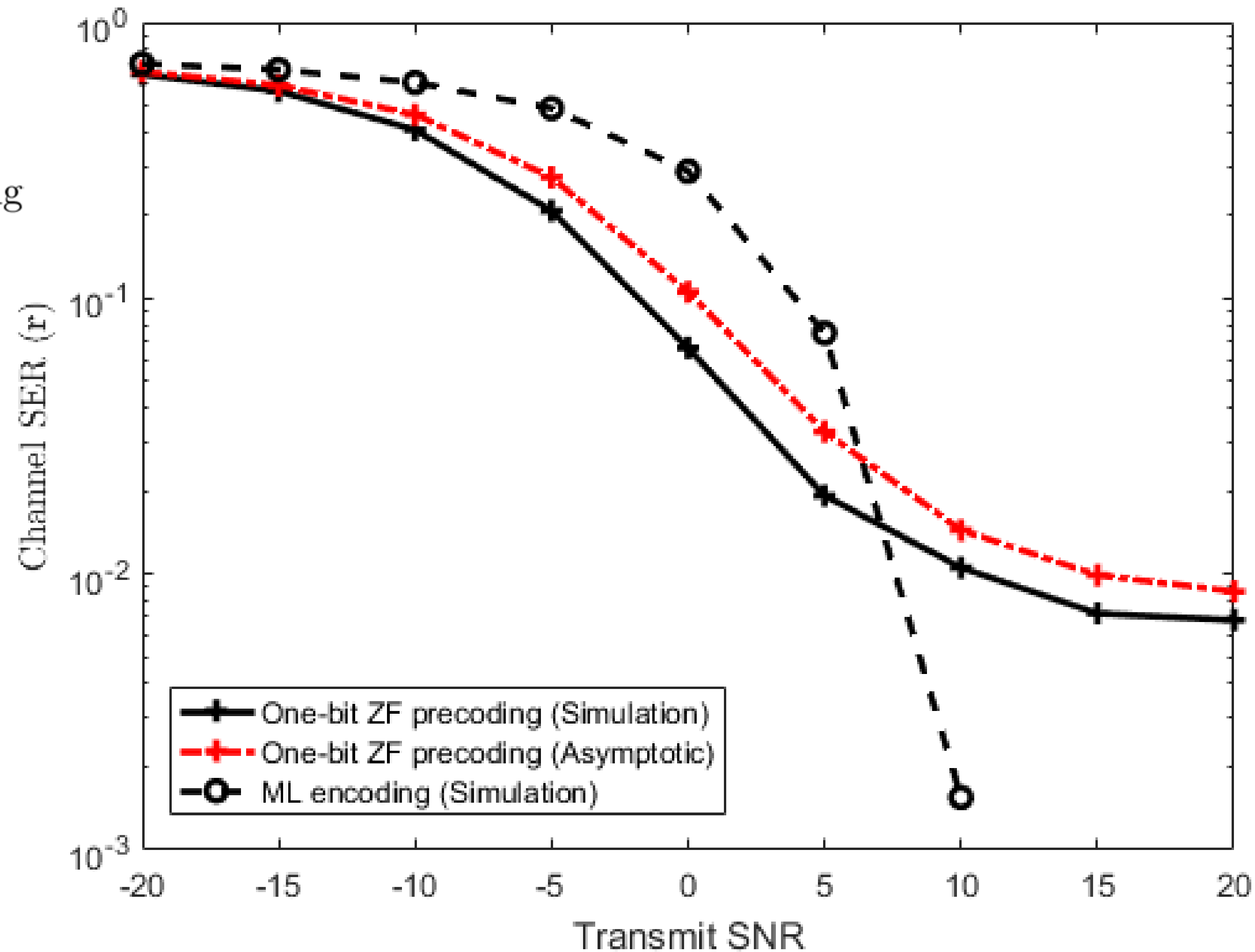
Quantized ZF Precoder Performance with Noise

- $K = 20$ users
- $\sigma = 1$
- Rayleigh fading
- Perfect CSI



Comparison with ML Encoding

- $M = 10$
- $K = 2$ users
- $\sigma = 1$
- Rayleigh fading
- Perfect CSI



Why Does Quantized ZF Precoding Perform So Well?

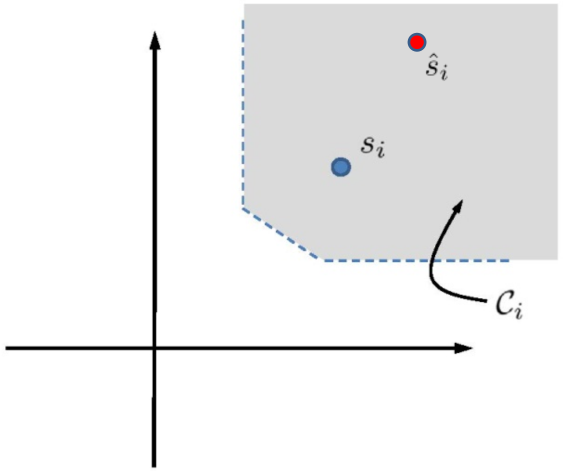
We can show that $\mathbf{R}_{\hat{s}s} \simeq \frac{2\sigma}{\sqrt{\pi}}\sqrt{\frac{M}{K}}\mathbf{I}_K$

As M/K grows, symbols move farther from decision boundaries

ML encoding overconstrains the problem if the desired signal at the receiver is digital. For example, if the elements of $\mathbf{s}$ should be QPSK (e.g., due to one-bit quantization), then all we need is that s_i lie in the right decision region $\mathcal{C}_i$:

Why not find $\mathbf{x} \in \{\pm 1 \pm j\}^M$ such that

$$\hat{s}_i = (\mathbf{H}^T)_{i:}\mathbf{x} \in \mathcal{C}_i \;\; \forall\, i$$



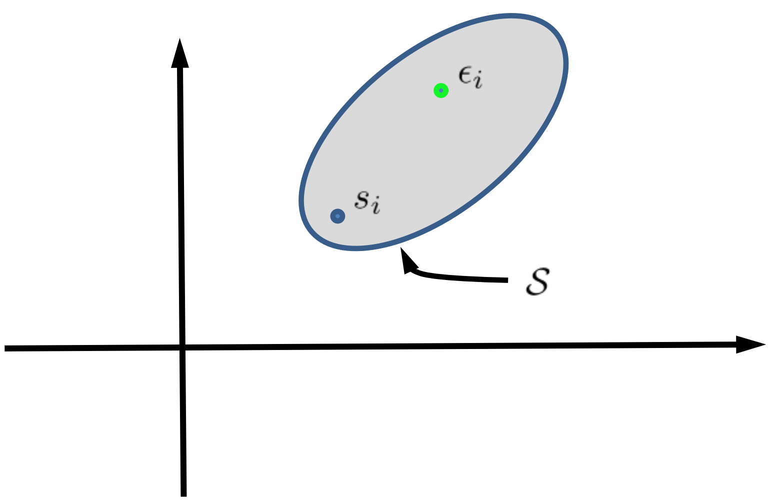
Quantization ZF precoder $\mathcal{Q}(\mathbf{Ps}) = \mathcal{Q}(\mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{s})$ eliminates scaling due to $(\mathbf{H}^T\mathbf{H})^{-1} \to \beta\mathbf{I}$ scaling which "props up" weak channels and scales down strong channels in order to enforce $\mathbf{s} \simeq \mathbf{H}^T\mathbf{x}$.

Perturbed Quantized ZF Precoding

$$\text{1-bit Quantized ZF:} \qquad \mathbf{x} = \mathcal{Q}\left(\mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{s}\right)$$

- Idea: All we need is a correct detection at the receivers . . . we don't necessarily need $\mathbf{s} \simeq \mathbf{H}^T\mathbf{x}$
- Instead of zero-forcing to $\mathbf{s}$, try to zero-force to a better point in the correct detection region
- "better" not defined by distance to $\mathbf{s}$, but rather by distance to avoid probability of error

$$\min_{\boldsymbol{\epsilon}} \; d\left(\mathbf{s}, \mathbf{H}^T\hat{\mathbf{x}}\right) \quad \text{s.t.} \quad \hat{\mathbf{x}} = \mathcal{Q}\left(\mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}(\mathbf{s}+\boldsymbol{\epsilon})\right) \;,\; \boldsymbol{\epsilon} \in \mathcal{S}$$



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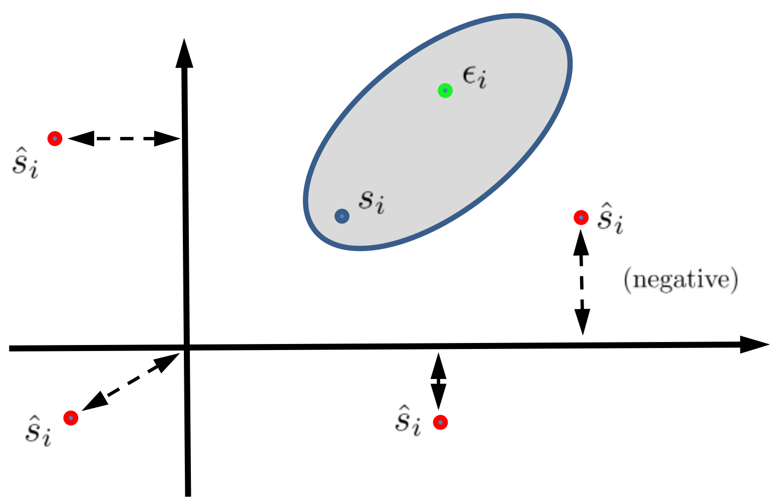
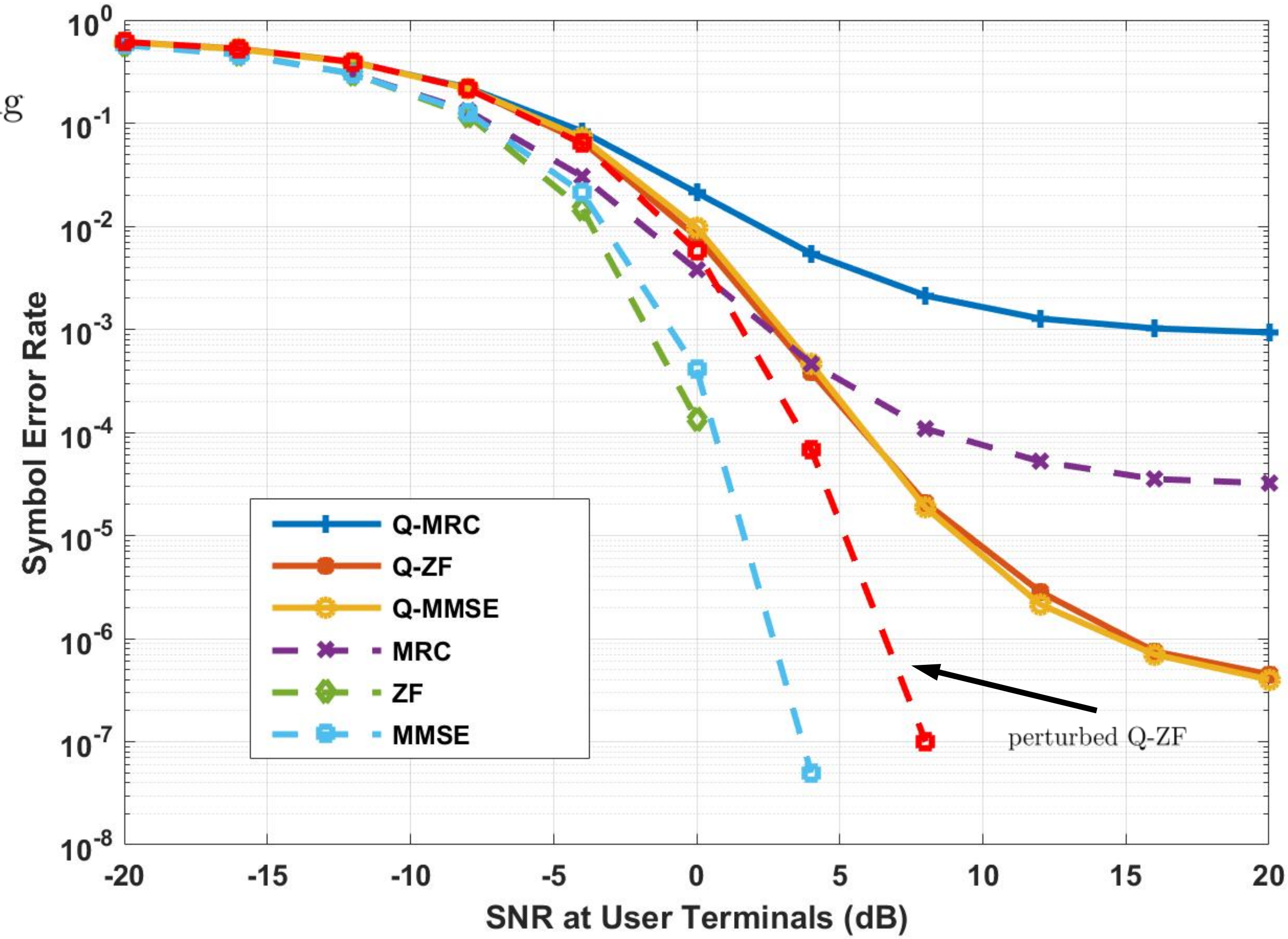


$$\hat{s}_i = \left(\mathbf{H}^T\mathbf{x}\right)_i$$

Example: Performance of Perturbed Quantized ZF Precoder

- $M = 128$
- $K = 8$ users
- Rayleigh fading
- Perfect CSI



Conclusions

- **Significant advantages in energy and cost for one-bit ADCs & DACs in massive MIMO systems**
- **Low SNR loss is tolerable, high SNR loss unavoidable but not necessarily critical**
- **Bussgang decomposition provides framework for tractable one-bit algorithm designs and system performance analyses**
  - Channel estimation
  - Optimized training
  - Achievable rates
  - Energy efficiency
  - Number of antennas
- **For the downlink, simply quantizing standard linear precoders provides reasonable performance, without enormous ML encoding cost. But there are gains for perturbation precoding!**
- **We've just scratched the surface, there are many interesting open problems that remain …**