Unified Stochastic Reverberation Modeling

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September 6, 2018

Why this research work?

- Applications of reverberation models:
  - Dereverberation (Belhomme et al., 2017),
  - Source separation (Leglaive et al., 2018),
  - Source localization, denoising, audio inpainting...

A. Belhomme, R. Badeau, Y. Grenier, and E. Humbert. Amplitude and phase dereverberation of harmonic signals. In *Proc. of IEEE WASPAA*, New Paltz, New York, USA, October 2017

S. Leglaive, R. Badeau, and G. Richard. Student's t source and mixing models for multichannel audio source separation. *IEEE Trans. Audio, Speech, Language Process.*, 26(5):1–15, May 2018

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- Applications of reverberation models:
  - Dereverberation (Belhomme et al., 2017),
  - Source separation (Leglaive et al., 2018),
  - Source localization, denoising, audio inpainting. . .
- Existing stochastic models of late reverberation:
  - Time domain (Schroeder, 1962; Moorer, 1979)
  - Frequency domain (Schroeder, 1962)
  - Space-frequency domain (Cook et al., 1955)
  - Time-frequency domain (Polack, 1988)

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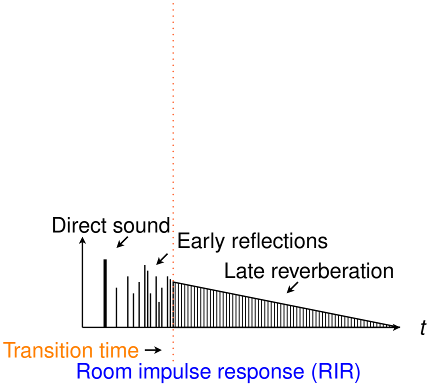
Outline

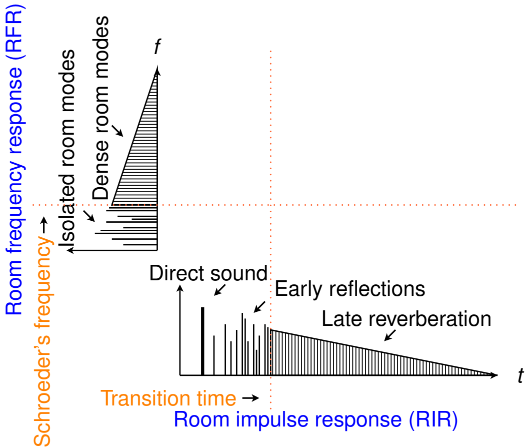
Part II

Properties of reverberation

Time-frequency profile of reverberation



Time-frequency profile of reverberation



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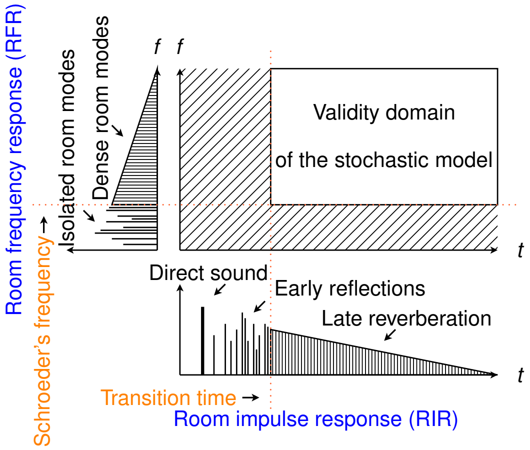
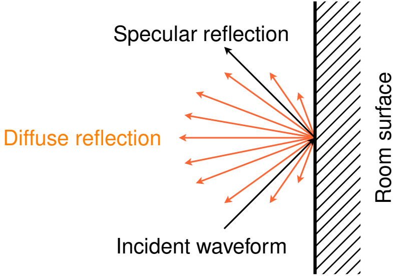


Space domain: diffuse sound field

- *Diffusion*: reflections on the room surfaces are not *specular* (mirror-like), but rather *scattered* in various directions



T.J. Schultz. Diffusion in reverberation rooms. *Journal of Sound and Vibration*, 16(1):17 – 28, 1971

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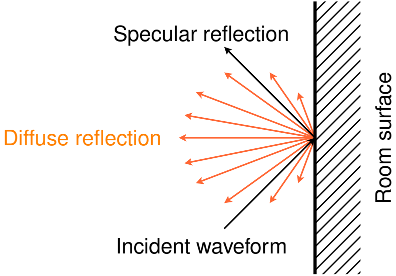

- The acoustic field can be approximated as *diffuse* (Schultz, 1971)
  - inside the time-frequency validity domain of the stochastic model
  - if source/sensors are at least a half-wavelength away from walls

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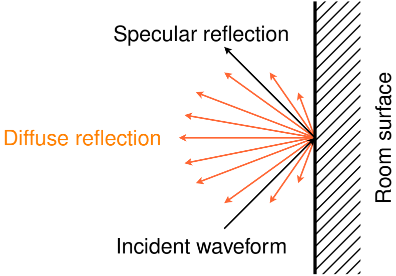

- The acoustic field can be approximated as *diffuse* (Schultz, 1971)
  - inside the time-frequency validity domain of the stochastic model
  - if source/sensors are at least a half-wavelength away from walls
- After many reflections, the acoustic field is *uniform* and *isotropic*

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Experiments

- Measured RIRs from C4DM database (169 RIRs, Fs=96 kHz)
  - Octagon room: 8 walls 7.5m length and domed ceiling 21m height
  - 13 x 13 sensor positions distributed on a uniform square grid
  - Space sampling of the omnidirectional microphone grid: $D = 1$m
  - Reverberation time: RT60 $\approx$ 2s

R. Stewart and M. Sandler. Database of omnidirectional and b-format room impulse responses. In *IEEE ICASSP*, pages 165–168, Center for Digital Music (C4DM), QMUL, London, March 2010

Emmanuel Vincent and Douglas R. Campbell. Roomsimove. GNU Public License, 2008. http://homepages.loria.fr/evincent/software/Roomsimove.zip

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  - 13 x 13 sensor positions distributed on a uniform square grid
  - Space sampling of the omnidirectional microphone grid: $D = 1$m
  - Reverberation time: RT60 $\approx$ 2s
- Synthetic RIRs from Roomsimove toolbox (400 RIRs, Fs=16 kHz)
  - Shoebox room: 4 x 5 x 2.5 m^3
  - Random source and sensor positions, random sensor orientations
  - Distance between the omnidirectional microphones: $D = 20$cm
  - Reverberation time: RT60 $\approx$ 0.1s

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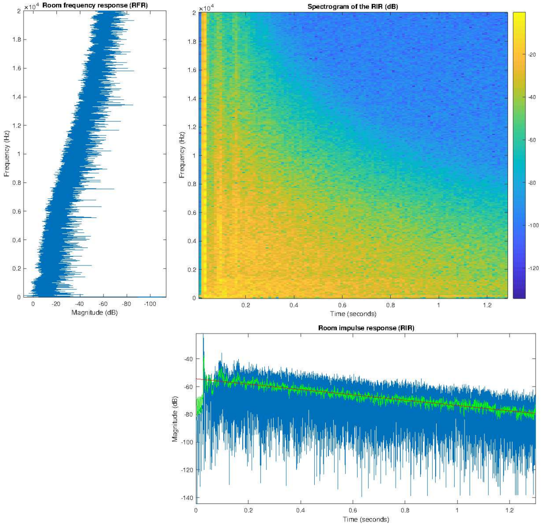
Time-frequency profile (C4DM database)



Part III

Review of reverberation models

Time domain

Schroeder (1962) and Moorer (1979): the RIR at microphone i is

$$h_i(t) = b_i(t) e^{-\alpha t} \mathbf{1}_{t \geq 0}$$

Manfred R. Schroeder. Frequency-correlation functions of frequency responses in rooms. *The Journal of the Acoustical Society of America*, 34(12):1819–1823, 1962

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$$h_i(t) = b_i(t) e^{-\alpha t} \mathbf{1}_{t \geq 0}$$

- $b_i(t)$ is a centered white Gaussian process
- $\alpha > 0$ is related to the reverberation time: $\mathrm{RT}_{60} = \frac{3 \ln(10)}{\alpha}$

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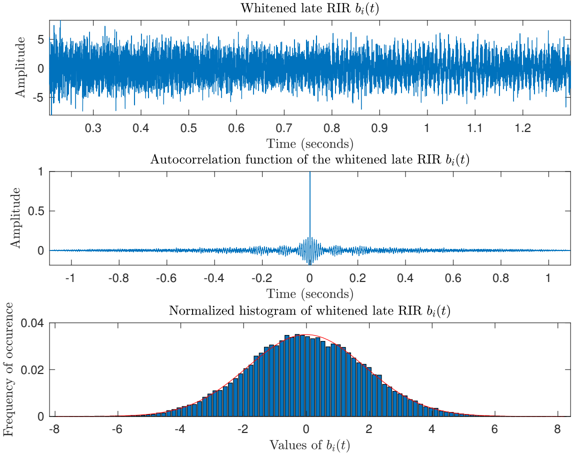
James A. Moorer. About this reverberation business. *Computer Music Journal*, 3(2):13–28, 1979

Validation of time model (C4DM database)



Frequency domain

- The RFR is the Fourier transform of the RIR:

$$\mathcal{F}_{h_i}(f) = \int_{t\in\mathbb{R}} h_i(t) e^{-2\imath\pi f t} dt$$

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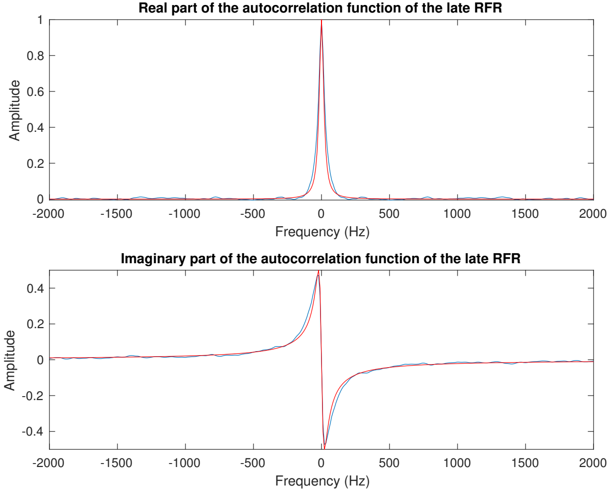
- Complex autocorrelation function of $\mathcal{F}_{h_i}(f)$:

$$\text{corr}\left[\mathcal{F}_{h_i}(f_1), \mathcal{F}_{h_i}(f_2)\right] = \frac{1}{1 + \imath\pi\frac{f_1 - f_2}{\alpha}}$$

Manfred R. Schroeder. Frequency-correlation functions of frequency responses in rooms. *The Journal of the Acoustical Society of America*, 34(12):1819–1823, 1962

Validation of spectral model (Roomsimove)



Space-frequency domain

- Correlation at frequency f between sensors (Cook et al., 1955):

$$\mathrm{corr}\left[\mathcal{F}_{h_1}(f),\, \mathcal{F}_{h_2}(f)\right] = \mathrm{sinc}\left(\frac{2\pi f D}{c}\right)$$

  - D is the distance between microphones
  - c is the speed of sound in the air ($\approx$ 343 m/s)

R. K. Cook, R. V. Waterhouse, R. D. Berendt, S. Edelman, and M. C. Thompson Jr. Measurement of correlation coefficients in reverberant sound fields.
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  - D is the distance between microphones
  - c is the speed of sound in the air ($\approx$ 343 m/s)
- Assumptions:
  - Plane waves (far field)
  - Isotropic incident waves (diffuse acoustic field)

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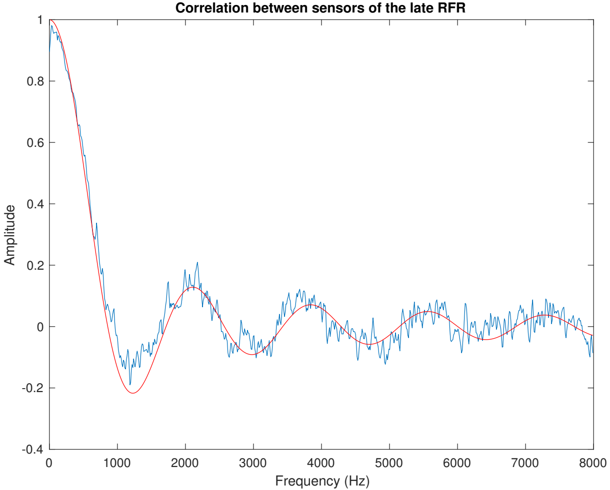
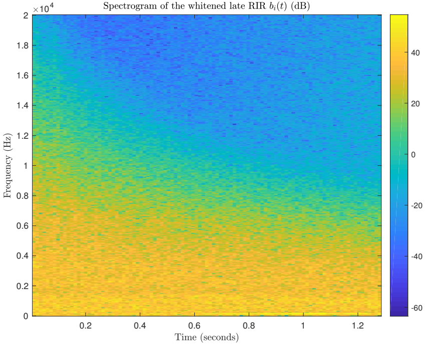
Validation of space model (Roomsimove)



Time-frequency domain

- Moorer (1979): the RIR at microphone i is $h_i(t) = b_i(t)e^{-\alpha t}\mathbf{1}_{t\geq 0}$ where $b_i(t)$ is a centered white Gaussian process
- Spectrogram of $b_i(t)$ (C4DM database):



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J. D. Polack. *La transmission de l'énergie sonore dans les salles.*
PhD thesis, Université du Maine, Le Mans, France, 1988

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- Polack (1988): the Wigner distribution of the RIR is

$$\mathcal{W}_{h_i,h_i}(t,f) = B(f)e^{-2\alpha t}\mathbf{1}_{t\geq 0}.$$

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- Wigner distribution of two 2[nd] order random processes ψ_1, ψ_2:

$$\mathcal{W}_{\psi_1,\psi_2}(t,f) = \int_{\mathbb{R}} \mathrm{cov}[\psi_1(t+\tfrac{u}{2}), \psi_2(t-\tfrac{u}{2})]e^{-2\imath\pi f u}du$$

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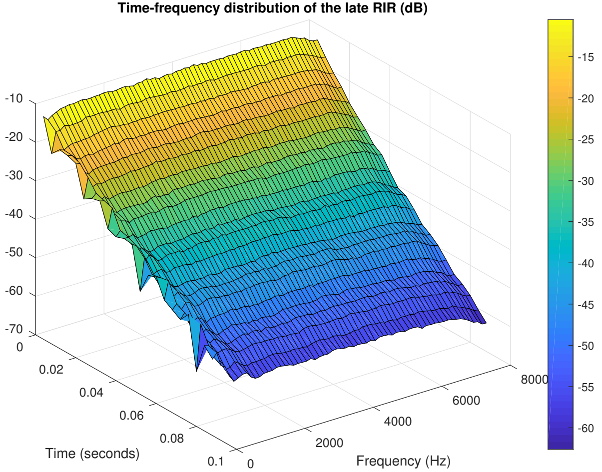
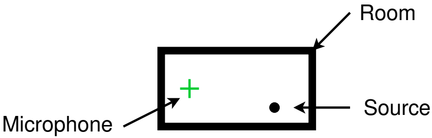
Validation of Polack's model (Roomsimove)



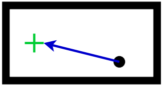
Part IV

Definition of the new stochastic model

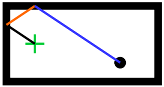
Source image principle (shoebox room)



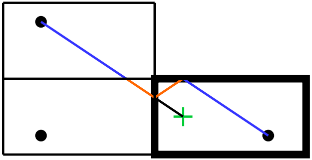
Source image principle (direct path)



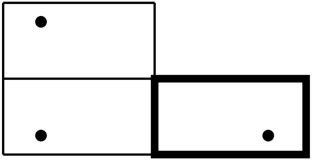
Source image principle (2 reflections)



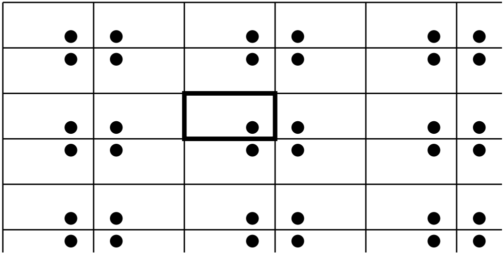
Source image principle (mirroring)



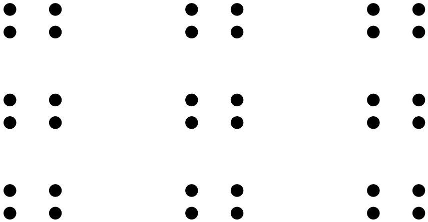
Source image principle (any sensor)



Source image principle (infinite space)



Source image principle (no wall)



Stochastic distribution of source images

- Source image principle: specular reflections in a shoebox room
  $\Rightarrow$ spatially uniform distribution of source images

Jean-Dominique Polack. Playing billiards in the concert hall: The mathematical foundations of geometrical room acoustics. *Applied Acoustics*, 38(2):235 – 244, 1993

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  - independent of the room geometry (Polack, 1993)

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  - holds even more in a *diffuse* (spatially uniform) acoustic field

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  - independent of the microphone position and true source position
  - independent of the room geometry (Polack, 1993)
  - holds even more in a *diffuse* (spatially uniform) acoustic field
- Assumption: microphone and source images are *omnidirectional*
  - The attenuation of sound waves is exponential w.r.t. the distance, *isotropic* and independent of frequency

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Unified stochastic reverberation model

- For a set of sensors at positions $\{\mathbf{x}_i\}_i \in \mathbb{R}^3$, the RIRs are

$$h_i(t) = \int_{\mathbf{x}\in\mathbb{R}^3} h(t, \|\mathbf{x} - \mathbf{x}_i\|_2)\, e^{-\frac{\alpha}{c}\|\mathbf{x}-\mathbf{x}_i\|_2}\, dN(\mathbf{x})$$

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  - $\alpha > 0$ is the attenuation coefficient (in Hz)

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  - $c > 0$ is the speed of sound in the air ($\approx$ 343 m/s)

Unified stochastic reverberation model

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  - $\alpha > 0$ is the attenuation coefficient (in Hz)
  - $c > 0$ is the speed of sound in the air ($\approx$ 343 m/s)
  - $h(t, r)$ is a coherent sum of *monochromatic spherical waves*:

$$h(t, r) = \int_{f\in\mathbb{R}} A(f)\, \frac{e^{2\imath\pi f\left(t-\frac{r}{c}\right)}}{r}\, df$$

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  - $dN(\boldsymbol{x}) \sim \mathcal{P}(\lambda d\boldsymbol{x})$ are independent Poisson increments
  - $\alpha > 0$ is the attenuation coefficient (in Hz)
  - $c > 0$ is the speed of sound in the air ($\approx$ 343 m/s)
  - $h(t, r)$ is a coherent sum of *monochromatic spherical waves*:

$$h(t, r) = \int_{f\in\mathbb{R}} A(f)\, \frac{e^{2\imath\pi f\left(t-\frac{r}{c}\right)}}{r}\, df$$

- We get $h_i(t) = e^{-\alpha(t-T)}\, b_i(t)$, $b_i(t) = \int_{\boldsymbol{x}\in\mathbb{R}^3} \frac{g\left(t-T-\frac{\|\boldsymbol{x}-\boldsymbol{x}_i\|_2}{c}\right)}{\|\boldsymbol{x}-\boldsymbol{x}_i\|_2} dN(\boldsymbol{x})$ with $g(t) \in L^2([-T, T])$ satisfying technical conditions

Part V

Statistical properties of the model

Space-time domain

- Asymptotic normality: with $h_i(t) = e^{-\alpha(t-T)}\, b_i(t)$, when $t \to +\infty$, $\boldsymbol{b}(t) = [b_i(t), b_j(t)]^\top$ converges to a *stationary Gaussian process*

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- At one sensor: $\forall t \geq 2T$, $b_i(t)$ is a centered wide sense stationary (WSS) process, of PSD

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  - When $t \to +\infty$, if $b_i(t)$ is white, we retrieve (Moorer, 1979)
- Between two sensors: $\forall t \geq 2T + \frac{D}{c}$, $\boldsymbol{b}(t) = [b_i(t), b_j(t)]^\top$ is a centered WSS process, of cross-PSD

$$B_{i,j}(f) \quad = \quad B(f)\,\mathrm{sinc}\left(\frac{2\pi f D}{c}\right) \quad \text{(new)}$$

Space-frequency domain

- Between two sensors: $\forall f_1, f_2 \in \mathbb{R}$,

$$\mathrm{corr}[\mathcal{F}_{h_i}(f_1), \mathcal{F}_{h_j}(f_2)] = \frac{e^{-\frac{\alpha D}{c} - 2\imath\pi(f_1 - f_2)(T + \frac{D}{2c})} \mathrm{sinc}(\frac{\pi(f_1+f_2)D}{c})}{1 + \imath\pi\frac{f_1 - f_2}{\alpha}} \quad \text{(new)}$$

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- At one sensor ($i = j$, $D = 0$) with $b_i(t)$ white ($T = 0$):

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- At one frequency ($f_1 = f_2 = f$), with no attenuation ($\alpha = 0$):

$$\mathrm{corr}[\mathcal{F}_{h_i}(f), \mathcal{F}_{h_j}(f)] = \mathrm{sinc}(\frac{2\pi f D}{c}) \quad \text{(Cook et al., 1955)}$$

Space-time-frequency domain

- Between two sensors: $\forall f \in \mathbb{R}$, $\forall t \geq 2T + \frac{D}{2c}$,

$$\mathcal{W}_{h_i,h_j}(t,f) \quad = \quad B(f)e^{-2\alpha(t-T)}\,\mathrm{sinc}(\tfrac{2\pi f D}{c}) \quad \text{(new)}$$

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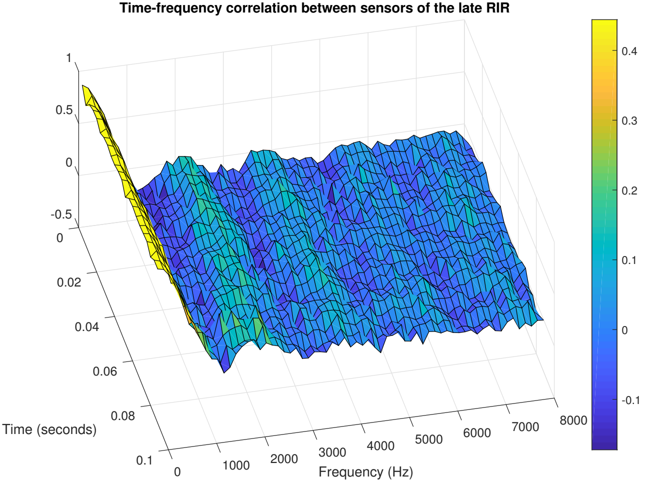
- Time-frequency correlation: $\forall f \in \mathbb{R}, \forall t \geq 2T + \frac{D}{2c}$,

$$\frac{\mathcal{W}_{h_i,h_j}(t,f)}{\mathcal{W}_{h_i,h_i}(t,f)} = \operatorname{sinc}\left(\frac{2\pi f D}{c}\right) \quad \text{(new)}$$

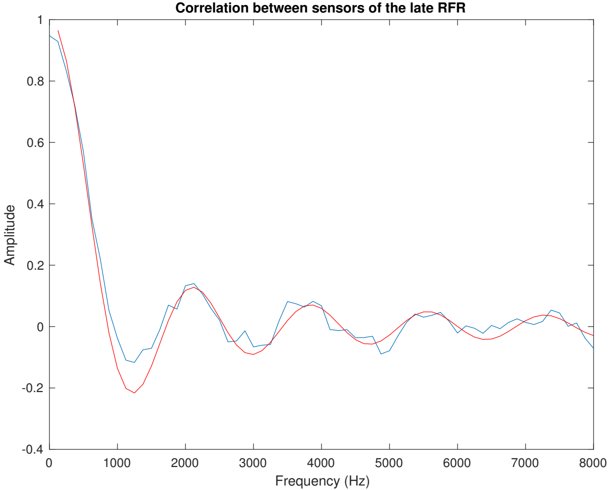
Part VI

Experimental validation

Time-frequency correlation (Roomsimove)



Projection on the frequency axis



Part VII

Conclusion

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- Summary
  - New reverberation model that unifies and generalizes known results

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    - Directional sources, directional microphones
    - Non-perfectly diffuse acoustic fields
    - Frequency-dependent attenuation α

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- Perspectives
  - Acoustics
    - Directional sources, directional microphones
    - Non-perfectly diffuse acoustic fields
    - Frequency-dependent attenuation α
  - Signal processing
    - Fast algorithm to estimate the model in discrete time
    - Applications: source separation, dereverberation,. . .

Thank you!


AAAAH?!
OUI, BELLE PIÈCE, HMM ! JE ME SUIS INSPIRÉ D'UN INSTRUMENT AFRICAIN... MAIS J'AI PERFECTIONNÉ...
LE PRINCIPE EST SIMPLE : UNE VIBRATION DU TONNERRE AVEC UNE RÉSONANCE MAXIMUM...
EUH...! MAGNI-FIQUE...

Work in progress

- Polack (1988): the RIR at microphone i is

$$h_i(t) = b_i(t)e^{-\alpha t}\mathbf{1}_{t\geq 0}$$

where $b_i(t)$ is a centered *stationary* Gaussian process, whose PSD $B(f)$ has slow variations

J. D. Polack. *La transmission de l'énergie sonore dans les salles.*
PhD thesis, Université du Maine, Le Mans, France, 1988

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- Then the Wigner distribution of the RIR is

$$\mathcal{W}_{h_i,h_i}(t,f) = B(f)e^{-2\alpha t}\mathbf{1}_{t\geq 0}.$$

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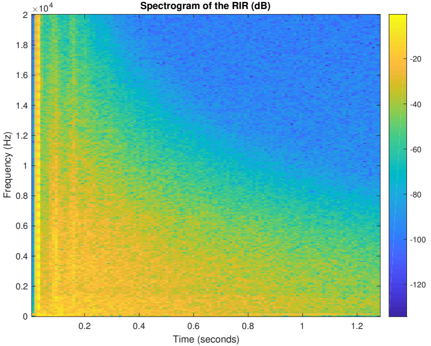
Work in progress (C4DM database)



- Polack (1988): the attenuation actually depends on the frequency:

$$\mathcal{W}_{h_i,h_i}(t,f) = B(f)e^{-2\alpha(f)t}\mathbf{1}_{t\geq 0}$$